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THE CASE FOR AN EXPONENTIAL RED SHIFT LAW

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THE conclusions summarized here form part of a much more complete theoretical investigation which will be published elsewhere. Five arguments are given for believing that the Hubble red shift law is a linear approximation to a strictly exponential law.

(1) In many respects the most satisfactory of the cosmological models based on solutions of Einstein's gravitational field equations is the steady-state de Sitter model. It has been considered that this model cannot contain matter; but in the more detailed analysis it will be shown that, for the case of the Universe, the energy–momentum tensor takes the form of the cosmical term $\Omega g_{\alpha\beta}$. Tangherlini[1] has also regarded $\Omega g_{\alpha\beta}$ as an energy–momentum tensor; but the proof given in the complete analysis reveals that this form of tensor applies only to the Universe as a whole, which is the smallest perfectly isolated system because of inertial force. Accepting, then, the de Sitter Universe, and transforming the metric into the Robertson steady-state form:

$$\mathrm{d}s^2 = -\,e^{2kt}(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2) + c^2\mathrm{d}t^2 \qquad (1)$$

where $k = c/R_u$, R_u being the radius of the Universe, one deduces for the cosmological red shift:

$$\frac{\nu_2}{\nu_1} = e^{-r/R_u} \qquad (2)$$

where ν_1 and ν_2 are the frequencies of a photon at emission and reception. If $\delta\nu$ is the red shift for a distance r such that $r \ll R_u$ then:

$$\frac{\nu - \delta\nu}{\nu} \quad 1 - \frac{r}{R_u} + \ldots \qquad (3)$$

Thus the linear Hubble law will be valid only provided that $r \ll R_u$. This deduction of the red shift proves that the shift is some sort of gravitational effect since only by virtue of gravitational fields can the metric depart from the Galilean form.

(2) A very strong argument that radiation must be absorbed by the Universe in some manner comes from the absorber theory of radiation proposed by Wheeler and Feynman[2]. This complete and satisfying theory was developed to give physical interpretation of Dirac's prescription for calculating the force of radiative reaction. According to Wheeler and Feynman, an accelerated charge radiates only if other charges are present to absorb the radiation. They show that all other charges of the Universe, considered to be set in motion by the retarded inductive field of the accelerated charge, give an advanced field at the position of the accelerated charge which is equal to one-half the retarded minus one-half the advanced field generated by the accelerated charge itself. The advanced inductive field of Universe charges is the physical origin of Dirac's field. This advanced field combines with the half-retarded, half-advanced field of the accelerated source charge to give for the total disturbance the full retarded field which we observe.

In proving these conclusions, Wheeler and Feynman perform a Fourier analysis of the acceleration of the source charge, rejecting however the zero-order Fourier component as contrary to experience. In the more detailed work it will be shown that this term is just correct to account for the inertial force on the electron. Inertial force then becomes a component of the force of radiative reaction. It is of purely electromagnetic origin in the case of the electron, and it acts instantaneously. This removes the only objection to Mach's principle, and at the same time resolves the old problem of whether or not a uniformly accelerated charge radiates[3].

The necessary and sufficient condition for Wheeler and Feynman's theory is that all radiation emitted by any accelerated charge should be completely absorbed by the Universe.

(3) A mechanism for the absorption of radiation by the Universe is provided by the hypothesis of Finlay-Freundlich[4], who invokes a photon–photon interaction to explain the empirical relation:

$$\delta\nu = A\nu l T^4 \qquad (4)$$

where $\delta\nu$ is the red shift of a photon of frequency, ν, which has a path-length, l, in a medium of radiation temperature, T. With the reduced estimate of $A(2 \times 10^{-32}$ cm.$^{-1}$ deg.$^{-4})$ made by Melvin[5], most of the criticisms of Freundlich's evidence (unexplained red shifts in stellar radiation fields) disappear. In addition, recent rocket experiments[6] have revealed unexpectedly large red shifts and broadening of the Lyman-α ultra-violet line which might be attributed to the Freundlich effect. It has been estimated[7] that, due to coherent scatterings, a Lyman-α photon has a path-length of the order of 10^{20} cm. in a planetary nebula before escaping. An unexplained red shift at the limb of the solar disk has now been unambiguously demonstrated[8]. Conservation of momentum in a photon–photon interaction may account for the excess deflexion of starlight passing close to the Sun over the deflexion predicted by general relativity[9]. If the energy lost by the starlight due to red shifting in the Sun's radiation field, $h\delta\nu$, is gained by the outward flux of solar radiation then the excess deflexion should be of order $\delta\nu/\nu$. Inserting for $\delta\nu$ twice the difference between the observed and the predicted gravitational red shift at the solar limb, one deduces an excess deflexion of 0·25″ of arc, in good agreement with observation.

The explanation of the Freundlich photon–photon interaction may be closely related to the gravitational red shift phenomenon, and also to weak nuclear interactions. Only a gravitational interaction between photons seems capable of accounting for the gradual nature of the Freundlich red shift. Suppose that photon energy can be quantized gravitationally as a field of gravitons according to:

$$h\nu = n\overline{h\nu_0} \qquad (5)$$

where $\overline{h\nu_0}$ represents a constant energy which must be exceedingly minute. The photon–photon interaction is then an interaction between two fields of gravitons, say, n and n'. Assuming that the probability per unit time that a graviton is lost by the ν-photon and gained by the ν'-photon is proportional to nn', one deduces a red shift relation of the form (4). At the same time, by allowing only $\Delta n = \pm 1$ transitions, one constrains the photon to decay gradually.

The only possible significance of $\overline{h\nu_0}$ is the rest energy of a pair of particles, and the only particles can be a neutrino, antineutrino pair. The association of a rest energy with the neutrino can be justified if it is assumed that the energy, $\overline{h\nu_0}$, represents, at the same time, a basic uncertainty in all energy due to the impossibility of measuring life-times longer than the age of the Universe, R_u/c. Then, from the uncertainty principle:

$$(R_u/c)(\overline{h\nu_0}) \sim \tfrac{1}{2}\hbar \qquad (6)$$

giving a neutrino mass of 10^{-66} gm. It is implied, now, that there is no such thing as a continuum of energy. Perhaps the neutrino plays a part in gravitation similar to that of the electron in electromagnetism.

Of course, any photon decay or absorption process will give a strictly exponential law. From (4):

$$d\nu = A\nu dl T_0^4 \tag{7}$$

which on integration gives equation (2), with $AT_0^4 = 1/R_u$. Hence, for the intergalactic radiation temperature, T_0, one deduces a value of 10° K., with the new estimate of A. This explanation of the cosmological red shift is consistent with the prediction of the de Sitter model which merely states that the effect is gravitational in character.

(4) As well as the exchange of a graviton between two photons in the Freundlich effect (virtual creation and annihilation of a neutrino pair) one expects the actual creation of neutrinos by annihilation of gravitons in the presence of a gravitational field of matter. This process may be regarded as a microscopic interpretation of the gravitational red shift predicted by Einstein. It may also be regarded as the first step in the creation of matter from radiation in the Universe. The rate of creation will then be greatest in the gravitational field of galactic halos. The rate of loss of radiant energy due to red shifting in a volume, V, of the Universe is HUV, where H is Hubble's constant and U is the mean density of radiant energy in the universe. For an intergalactic radiation temperature of 10° K., this gives 3×10^{-28} V. erg sec.$^{-1}$. Since this is of the order of the rate at which radiant energy is being produced in stars one may postulate a steady-state universe. On the local scale there will be continual evolution of galaxies due to a radiation → matter → radiation cycle, the period of which is evidently about 4×10^{17} sec. Obler's paradox does not arise.

In the detailed work to which I have alluded, it is shown that when two electrons approach each other to distances less than e^2/mc^2, where m is the electron mass, then the inertia of either electron is due more to the inductive field of the other electron than to distant charges of the universe. The electromagnetic field then becomes non-linear. Spin–spin couplings between electrons or positrons obey an energy law, $\hbar c/r$. Strong coupling in the nucleus emerges as a zero-order (in the perturbation series) electromagnetic interaction between electrons; Coulomb force becomes a first-order exchange force between electrons in the field of the Universe, in accordance with Wheeler and Feynman's point of view. The energy which an electron possesses in the field of all charges in the Universe (both local and distant) as a result of all orders of electromagnetic interaction is a constant in the rest frame of the electron; this implies that the Coulomb force saturates in the nucleus, and the zero-order spin–spin force becomes negligible outside the nucleus. On this basis it is argued that mesons are assemblies of spin–spin coupled electrons and positrons. For example, the muon consists of three elementary particles, and the pion four. Each elementary charge contributes $\sim (137/2)mc^2$ to the energy of the meson. Evidently, then, matter creation demands a mechanism for producing electrons and positrons from neutrinos.

Another suggestion which is made is that the Universe, bounded by the radius, R_u, is an elementary particle in a super-universe, which will be an isolated system on an enormously greater scale and will involve scaled-up constants. An infinite system of such isolated systems at different levels is defined, and spin momentum, $\alpha^n \hbar$, where α is an enormous dimensionless constant, is associated with the n^{th} level system. The Universe, the electron, and the neutrino are identified with the systems, $n = +1$, $n = 0$, and $n = -1$, respectively. The only observable neutrino interaction will be spin coupling with the Universe because this interaction will have energy of the order of $\hbar^2$. This is consistent with vector–axial symmetry of neutrino coupling, and also with the non-conservation of parity in weak interactions since the Universe, as an elementary particle, should define a specific parity. Evidently a neutrino field of sufficient energy ($2mc^2$) automatically becomes an electron pair. It is noteworthy that in addition to an electromagnetic system of de Broglie waves affording a statistical prediction of electron pair creation (based on h) one expects a gravitational sub-system of de Broglie waves which will afford a prediction of neutrino pair creation (based on $\bar{h} = \alpha^{-1} h$).

(5) A deviation from the linear Hubble law in the case of the more distant sources has for long been suspected but has been difficult to confirm because of uncertainty about the correcting factors to be applied in the measurement of the distance of a source. It now appears that confirmation may come from number-intensity counts of sources.

Regarding the e^{2kt} factor in the de Sitter metric (1) as a continuous evolution factor because a well-known transformation[10] to a new time origin leaves (1) unchanged, and assuming that, on the average, the distribution of galaxies in various stages of evolution at different depths of the universe is constant, one may use Euclidean geometry for number-intensity counts. As one moves out in depth, and hence backwards in time, one assumes that the time origin is adjusted so as to make e^{2kt} effectively a constant. Then the number of sources with intensity greater than S^* (the corrected value) will be:

$$N_S = \frac{4}{3}\pi N_0 \left(\frac{P_0}{S^*}\right)^{3/2} \tag{8}$$

where P_0 is the luminosity per steradian of the average source in a defined frequency interval. To relate S^* to the observed intensity, S, one has to correct for loss of energy due to red shifting and for the fact that one observes the source in a different frequency interval. Assuming that the source spectrum is $P_\nu \propto \nu^{-x}$, correction for these two effects gives, using (2):

$$S = \left(\frac{\nu_2}{\nu_1}\right)^{1+x} = e^{(r/R_u)(1+x)}\, S^* \tag{9}$$

and hence from (8):

$$\log N_S = -\frac{3}{2}\log S - (1+x)(r/R_u) + \text{constant} \tag{10}$$

In the case of optical sources Hubble[11] has found that the observations fit a relation identical to (10), if $x = 0{\cdot}6$. In the case of radio sources, Ryle's[12] group have recently found a pronounced departure from the 3/2 power law for $r \sim R_u$ and their observations also fit (10) very well.

1 Tangherlini, F. R., *Nuovo Cimento*, **20**, 1 (1961).
2 Wheeler, J. A., and Feynman, R. P., *Rev. Mod. Phys.*, **17**, 157 (1945).
3 Fulton, T., and Rohrlich, F., *Annal Phys.*, **9**, 499 (1960).
4 Finlay-Freundlich, E., *Phil. Mag.*, **45**, 303 (1954).
5 Melvin, M. A., *Phys. Rev.*, **98**, 884 (1955).
6 Bryan, E. T., Chubb, F. A., and Friedman, H., *Space Research*, edit. by Kallman H., 599 (North Holland, 1960).
7 Ambartsumyan, V. A., *Theoretical Astrophysics*, 438 (Pergamon Press, 1958).
8 Adam, M. G., *Mon. Not. Roy. Astro. Soc.*, **119**, 460 (1959).
9 Finlay-Freundlich, E., *Vistas in Astronomy*, **1**, 239 (Pergamon Press, (1955).
10 Tolman, R. C., *Relativity, Thermodynamics and Cosmology*, 348 (Oxford Univ. Press, 1934).
11 Hubble, E., *The Realm of the Nebulae*, 196 (Dover, 1958).
12 Scott, P. F., and Ryle, M., *Mon. Not. Roy. Astro. Soc.*, **122**, 416 (1961).

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