About some possible empirical evidences in favor of a cosmological time variation of the speed of light

Article *in* EPL (Europhysics Letters) · December 2009

DOI: 10.1209/0295-5075/88/59002 · Source: arXiv

yves-henri Sanejouand
French National Centre for Scientific Research

Empirical evidences in favor of a varying-speed-of-light

Yves-Henri Sanejouand[A,B]

[A] Laboratoire U3B, UMR 6204 of CNRS, Faculté des Sciences, 2 rue de la Houssinière, 44322 Nantes Cedex 3, France.
[B] Email: Yves-Henri.Sanejouand@univ-nantes.fr

Abstract: Some empirical evidences in favor of the hypothesis that the speed of light decreases by a few centimeters per second each year are examined. Lunar laser ranging data are found to be consistent with this hypothesis, which also provides a straightforward explanation for the so-called Pioneer anomaly, that is, a time-dependent blue-shift observed when analyzing radio tracking data from distant spacecrafts, as well as an alternative explanation for both the apparent time-dilation of remote events and the apparent acceleration of the Universe. The main argument against this hypothesis, namely, the constancy of fine-structure and Rydberg constants, is discussed. Both of them being combinations of several physical constants, their constancy imply that, if the speed of light is indeed time-dependent, then at least two other "fundamental constants" have to vary as well. This defines strong constraints, which will have to be fulfilled by future varying-speed-of-light theories.

Keywords: Lunar laser ranging – Lenght of day – Pioneer anomaly – Time dilation – Supernovae – Hubble's law – Cosmological constant – Fine Structure constant – Rydberg constant.

1 Introduction

During the twentieth century, the speed of light has reached the theoretical status of a "universal constant", a fixed value of $c_0 = 299{,}792{,}458$ m s^{-1} being chosen in 1983 as a basis for the international unit system.
In the present study, empirical evidences in favor of the hypothesis that the speed of light actually varies as a function of time are examined. It is by far not the first attempt to put forward such an hypothesis (Wold 1935; North 1965; Barrow and Magueijo 2000) but it is only recently that measurements accurate enough, on periods of time long enough, have allowed to witness several independent phenomenons in rather good agreement with it.

2 Main hypothesis

It is assumed herein that $c(t)$, the time-dependent speed of light, varies slowly on the considered timescales, so that it can be approximated by:

$$c(t) = c_0 + a_c t + \frac{1}{2}\dot{a}_c t^2 + \cdots$$

where a_c is the time derivative of $c(t)$, $\dot{a}_c$ the time derivative of a_c, and where c_0 is the value of the speed of light at $t = t_0 = 0$, *e.g.* when a series of measurements begins. Hereafter, for the sake of simplicity, only the two first terms of this expansion are retained. In other words, as proposed long ago (Wold 1935), it is assumed that $c(t)$ varies so slowly that it can be well approximated by:

$$c(t) = c_0 + a_c t \qquad (1)$$

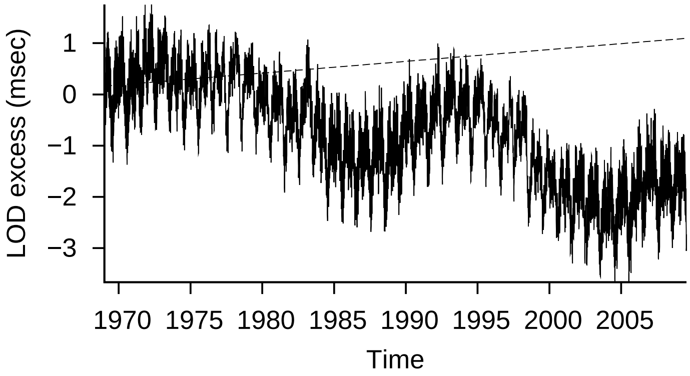

Figure 1: Length of day since 1969, that is, since Earth-Moon laser ranging data started to be collected. The dotted line shows the 2.3 msec cy^{-1} trend expected as a consequence of momentum conservation, when it is assumed that, using laser ranging, an actual increase of Earth-Moon distance is measured. LOD data comes from the EOP 05 CO4 series (Bizouard and Gambis 2009), as provided by the Earth Orientation Centre (http://hpiers.obspm.fr).

3 Lunar laser ranging

Thanks to reflectors left on Moon by Apollo and Lunokhod missions, using laser pulses, highly accurate measurements of δt_M, the time taken by light to go to the Moon and back to Earth, have been performed over the last fourty years (Dickey et al. 1994).
If d_M, the average Moon semi-major axis, is assumed to have *not* significantly changed over this timespan, then, as a consequence of (1):

$$\delta t_M = \frac{2d_M}{c(t)}$$

is expected to vary as a function of time, so that:

$$\dot{\delta t_M} = \frac{-2a_c d_M}{c_0^2} \tag{2}$$

As a matter of fact, a value of $\dot{\delta t_M} = 0.255 \pm 0.005$ nsec per year has been measured (Dickey et al. 1994). According to (2), this yields $a_c = -9.4\ 10^{-10}$m s^{-2}.
Since, nowadays, it is assumed that $c(t) = c_0$, the increase of δt_M is usually interpreted as an increase of d_M, of 3.82 ± 0.07 cm per year (Dickey et al. 1994). Such a steadily increase calls for an explanation, which is usually given as follows: since the Moon exerts a gravitational torque on the bulge of Earth, energy dissipation due to tidal friction yields a decrease of Earth rotation rate, which corresponds to a secular increase of the length of the day (LOD). In turn, as a consequence of momentum conservation, the Earth-Moon distance has to increase as well (Darwin 1879). But, in order to account for an increase of d_M of 3.8 cm per year, $\dot{T}_{LOD}$, the increase of LOD, has to be of 2.3 msec cy^{-1} (Stephenson and Morrison 1995), while current estimates are significantly smaller. Indeed, paleotidal values provided by late Neroproterozoic tidal rhythmites yield an average of $\dot{T}_{LOD} = 1.3$ msec cy^{-1} over the last 620 million years (Williams 2000), in close agreement with the value obtained by analyzing paleoclimate records over the last 3 million years, namely, 1.2 msec cy^{-1} (Lourens et al. 2001). Although, using an extensive compilation of ancient eclipses, a larger value of 1.70 ± 0.05 msec cy^{-1} for the last 2500 years has been obtained (Stephenson and Morrison 1995), it may prove to be not that relevant on other timescales since, for instance, fluctuations of several milliseconds have been observed over the last centuries, likely to be due to mantle-core interactions (Eubanks 1993) or events like the warm El Nino Southern Oscillation, which is accompanied by an excess in atmospheric angular momentum (Munk 2002). As a matter of fact, as shown in Fig. 1, since 1969, that is, since lunar laser ranging data have started to be collected, the mean LOD has *decreased* (Abarca del Rio et al. 2000). So, it seems likely that at least half of the observed increase of δt_M is *not* due to tidal forces. Instead, it could well indicate an actual decrease of the speed of light.

4 Pioneer anomaly

A straightforward way to check this later hypothesis is to emit an electromagnetic wave with a given frequency ν_0, and then to measure its wavelength as a function of time:

$$\lambda_{mes} = \frac{c(t)}{\nu_0}$$

since, as a consequence of (1), it should drift according to:

$$\lambda_{mes} = \lambda_0(1 + \frac{a_c}{c_0}t) \quad (3)$$

which, with $a_c < 0$, means that a blue-shift increasing linearly in time should be observed as if, when interpreted as a Doppler effect, the source were accelerating towards the observer.
As a matter of fact, such a time-dependent blue-shift may well have already been observed, by analyzing radio tracking data from Pioneer 10/11 spacecrafts (Anderson et al. 1998). During this series of experiments, a signal was emitted towards the spacecraft, up to 67 astronomical units (AU) away, at $\nu_0 = 2.292$ GHz, using a digitally controlled oscillator, sent back to Earth by the spacecraft transponder where λ_{mes}, the wavelength of the radiometric photons received[1], was measured with the antenna complexes and the low-noise maser amplifiers of the Deep Space Network (Anderson et al. 2002). An apparent anomalous, constant, acceleration, a_p, roughly directed towards the Sun was left unexplained, with $a_p = 8.74 \pm 1.33\ 10^{-10}$m s^{-2} (Anderson et al. 1998), in spite of extensive attempts to unravel its physical nature (Anderson et al. 2002; Nieto and Anderson 2005). In particular, it is unlikely to have a gravitational origin since such a constant acceleration, on top of Sun's attraction, would have been detected when analyzing orbits within the Solar System, noteworthy for Earth and Mars (Anderson et al. 1998) but also for, *e.g.*, trans-neptunian objects (Wallin et al. 2007). On the other hand, although directed heat radiation may well play a role (Anderson et al. 1998, 2002), it seems unlikely to account for more than 25% of the measured effect (Anderson and Nieto 2009).
Moreover, this anomaly was confirmed by at least two other independent analyses of the data, providing similar estimates for the effect, namely $a_p = 8.60 \pm 1.34\ 10^{-10}$m s^{-2} (Markwardt 2002) and $a_p = 8.4 \pm 0.1\ 10^{-10}$m s^{-2} (Levy et al. 2009). Interestingly, this later study confirmed that small amplitude, periodic variations, of the anomaly do occur (Anderson et al. 2002), the main component period being equal to Earth's sidereal rotation period, while a semi-annual component of similar magnitude is also exhibited (Levy et al. 2009), corresponding to a fluctuation of $\approx 0.2\ 10^{-10}$m s^{-2} (Turyshev et al. 2005).
These results are in good agreement with our hypothesis. Indeed, $a_c = -a_p$ yields a value for a_c which would explain 90% of the increase of δt_M, while assuming that a_p is directed towards the Sun, if it actually happens to be an apparent effect and, as such, is instead directed towards the Earth, introduces the following artefactual, periodic, fluctuation:

$$a_{mes} = a_p cos\alpha$$

where a_{mes} is the value taken into account in models used for interpreting the radiometric data (Anderson et al. 2002) and where α is the angle between spacecraft to Sun and spacecraft to Earth directions. When the spacecraft is far away from the observer, δa_p, the amplitude of the corresponding fluctuation, is approximately given by:

$$\delta a_p = \frac{1}{2} a_p \frac{d_E}{d_p}$$

where d_p is the Earth-spacecraft distance, d_E being Earth semi-major axis. So, according to our hypothesis, the amplitude of the semi-annual periodic component of a_p is expected to ly in a range including the reported value, since it has to decrease from $\delta a_p \approx 0.3\ 10^{-10}$m s^{-2}, when the spacecraft is 15 AU away from the Sun, that is, when the anomaly starts to show up in the radiometric data (Anderson et al. 2002), to $\delta a_p \approx 0.1\ 10^{-10}$m s^{-2}, when the spacecraft is 67 AU away from the Sun, that is, when the last useful data from Pioneer 10 were collected.

[1] Since the Pioneer spacecrafts beamed their radiometric signal at a power of eight watts (Anderson et al. 2002), photons are indeed detected one-by-one when the anomaly is exhibited, both spacecrafts being more than 10-15 AU away from the Sun (Anderson et al. 2002)

5 Time dilation

Both previous estimates of a_c (see Table 1) come from measurements performed within the Solar System, over a few decades. However, it has been noticed, then as a numerical coincidence, that a_p is nearly equal to $H_0 c_0$, where H_0 is the Hubble constant (Anderson et al. 2002). Within the frame of the present study, this suggests that the decrease of the speed of light at a rate of the order of magnitude of a_c may be revealed by studying phenomenons occuring over cosmological distances.

Indeed, as a consequence of the decrease of the speed of light, the timescale of remote events, for instance, is expected to be overestimated. To exhibit this effect in a clear-cut way, let us consider the case of a *static* Universe. Then, when two signals are emitted at times t_i and t_j, L, the distance between both is: $L = c(t_{em})T_0$, if it is assumed that during $T_0 = t_j - t_i$ the speed of light at $t = t_{em}$, $c(t_{em})$, does not change significantly. On the other hand, since (1) is *not* spatially dependent, L is expected to remain constant during the flight of the signals towards the observer who measures T_{mes}, the time delay between both, as:

$$T_{mes} = \frac{L}{c_0}$$

that is:

$$T_{mes} = \frac{c(t_{em})}{c_0} T_0$$

With (1), this yields:

$$\frac{T_{mes}}{T_0} = 1 - \frac{a_c \Delta t_g}{c_0} \tag{4}$$

where $\Delta t_g = t_{em} - t_0$ is the photon time-of-flight between the source and the observer and where the minus sign comes from the fact that c_0 is the value of the speed of light when the observation is performed.

On the other hand, up to ten years ago, z, the redshift of a galaxy, had been shown to be well described as a linear function of d_g, its distance, such that:

$$z = \frac{H_0 d_g}{c_0} \tag{5}$$

However, as an empirical law, (5) can also be written in the following form:

$$z = H_0 \Delta t_g \tag{6}$$

while with (6), (4) becomes:

$$\frac{T_{mes}}{T_0} = 1 - \frac{a_c}{H_0 c_0} z \tag{7}$$

As a matter of fact, it has been observed that light curves of distant supernovae are dilated in time (Hamuy et al. 1996; Leibundgut et al. 1996), according to:

$$\frac{T_{mes}}{T_0} = 1 + z \tag{8}$$

where T_0 and T_{mes} are the typical timescales of the event, as observed in the case of nearby and distant supernovae, respectively. Indeed, nowadays, a stretching by a $(1+z)$ factor of reference, nearby supernovae, light curves is included in all analyses of distant supernovae (Riess et al. 2004, 2007).

Such a phenomenon can be understood within the frame of standard cosmological models (Schrödinger 1939; Wilson 1939). However, if it is assumed that the decrease of the speed of light is responsible for most of this effect, (7) and (8) yield:

$$a_c = -H_0 c_0 \tag{9}$$

Note that with $H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1} (Freedman et al. 2001), $a_c = -7.0 \pm 0.8\ 10^{-10}$m s^{-2}, that is, a value in the range of both previous estimates (see Table 1).

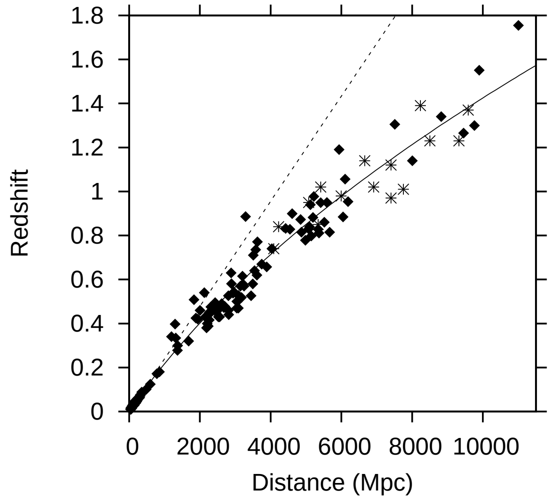

Figure 2: Redshift of type Ia supernovae as a function of their distance, in megaparsecs. Black diamonds: the 156 supernovae of the initially compiled "gold set" (Riess et al. 2004). Stars: 16 cases observed more recently, with the Hubble Space Telescope (Riess et al. 2007). Dotted line: Hubble's law. Plain line: a single-parameter fit of the data, performed using the relationship obtained following the varying-speed-of-light hypothesis discussed in the present study.

6 Supernovae redshifts

Moreover, under the additional, rather natural, hypothesis that (6) is a more generally valid form of Hubble's law than (5), as a consequence of the time-dependence of the speed of light, z is expected to be a non-linear function of d_g. Indeed, using (1), one gets:

$$d_g = c_0 \Delta t_g - \frac{1}{2} a_c \Delta t_g^2$$

This yields:

$$\Delta t_g = \frac{c_0}{a_c}\left(1 - \sqrt{1 - \frac{2 a_c d_g}{c_0^2}}\right) \tag{10}$$

and, with (6):

$$z = \frac{H_0 c_0}{a_c}\left(1 - \sqrt{1 - \frac{2 a_c d_g}{c_0^2}}\right) \tag{11}$$

which, for short distances, can be approximated by (5).

As a matter of fact, using the rather homogeneous type Ia supernovae subclass (Sne Ia) as standard candles, it was shown that, for large values of d_g, Hubble's law is not linear any more (Riess et al. 1998; Perlmutter et al. 1999). This is illustrated in Fig. 2 for a "gold set" of 182 Sne Ia (Riess et al. 2004, 2007), together with a least-square fit performed with (11) which yields $a_c = -6.6\ 10^{-10}$m s^{-2}. In practice, distances are obtained from extinction-corrected distance moduli, $m - M = 5\ log_{10} d_g + 25$, where m and M are the apparent and the absolute magnitudes of the supernovae, respectively (Riess et al. 2007).

The explanation nowadays given for the nonlinearity of Hubble's law rely on an acceleration of universe's expansion due to a non-zero, although very small, value of Λ, the cosmological constant (Riess et al. 1998; Perlmutter et al. 1999). However, this explanation looks like all previous attempts to introduce a non-zero Λ in the equations of General Relativity, namely, like an *ad hoc* one. Indeed, Λ was first added into these equations by Einstein himself, so as to obtain a static solution for the Universe as a whole (Einstein 1917), next kept by Lemaitre, in order to account for a then too large measured value of Hubble constant, with respect to the age of Earth (Lemaitre 1927), and it has now been reintroduced so as to explain why the Universe seems to accelerate, instead of the deceleration expected within the frame of standard cosmological models, as a consequence of gravitational forces. In all these cases, a non-zero value of Λ allowed to rescue a theory unable to explain a seemingly obvious fact. Although Λ helps improving the standard cosmological model, noteworthy within the frame of the "concordance model", note that this is at the cost of introducing both a "cosmic coincidence" (Zlatev et al. 1999) and a new kind of so-called "dark energy", of unknown origin but accounting for as much as 70% of universe's energy (Glanz 1998; Copeland et al. 2006).

Empirical fact	Implied value for a_c (m s^{-2})	Comments
Apparent increase of Earth-Moon distance	$-9.4 \pm 0.2\ 10^{-10}$	Tidal forces are expected to be partly responsible for this effect.
Apparent acceleration of Pioneer 10/11	$-8.7 \pm 1.3\ 10^{-10}$ $-8.6 \pm 1.3\ 10^{-10}$ $-8.4 \pm 0.1\ 10^{-10}$	(Anderson et al. 1998) (Markwardt 2002) (Levy et al. 2009)
Apparent time dilation of remote events	$-7.0 \pm 0.8\ 10^{-10}$	Depends upon the actual value of H_0.
Apparent acceleration of the Universe	$-6.6 \pm 0.7\ 10^{-10}$	Depends upon the actual value of H_0.

Table 1: Values obtained for a_c, the rate of change of the speed of light, through the analysis of four different kinds of experimental data, collected over two widely different timescales, namely, decades (top) and billions of years (bottom). H_0 is the Hubble constant.

Interestingly, the value of a_c obtained through the present analysis is found to be nearly equal to the previous one (see Section 5 or Table 1). Indeed, assuming that (9) is exact, (11) takes the following, appealingly simple, parameter-free form:

$$z = \sqrt{1 + \frac{2H_0 d_g}{c_0}} - 1$$

already advocated in a previous study (Sanejouand 2005). Note that since, in the case of a wave, $T_{mes} = \frac{\lambda_{mes}}{c_0}$ and $T_0 = \frac{\lambda_0}{c_0}$, while $z = \frac{\lambda_{mes} - \lambda_0}{\lambda_0}$, (6) can be obtained from (4) and (9). In other words, if (9) happens to be exact, then Hubble's law is itself a consequence of the decrease of the speed of light, as proposed seventy four years ago (Wold 1935).

7 Fine-structure constant

The speed of light plays a pivotal role in many physical phenomenons and, as such, its variations, even at a slow rate, are expected to have far reaching consequences. Noteworthy, c_0 is involved in several key combinations of physical constants, some of which are known with high accuracy. In particular, this is the case of α, the fine-structure constant:

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c_0}$$

where e is the electron charge, $\hbar$, the Planck constant and ϵ_0, the vacuum permitivity. Indeed, it has been shown that α depends little upon the redshift (Webb et al. 1999), if it does at all (Uzan 2003; Chand et al. 2004).
However, α may prove constant in spite of the time dependence of the speed of light if at least one among the other "fundamental constants" involved in α exhibits a complementary time-dependence. In the case of α, an obvious candidate is the vacuum permitivity since it is already known to be related to the speed of light, namely through:

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

where μ_0 is the vacuum permeability. Thus, the constancy of α would mean that Z_0, the characteristic impedance of vacuum:

$$Z_0 = \frac{1}{\epsilon_v(t) c(t)}$$

is the relevant fundamental constant, $\epsilon_v(t)$ being the time dependent vacuum permitivity.
Note that it has recently been proposed to redefine the international unit system so as to fix the values of both K_J and R_K, the Josephson and von Klitzing constants. Within such a frame, quantities that are nowadays assumed to have, *par définition*, fixed values, like vacuum permitivity and permeability, would become again quantities that have to be determined by experiment (Mills et al. 2006). Note also that if both α and Z_0 are actual fundamental

constants, then it has to be the case for R_K, since $R_K = \frac{Z_0}{2\alpha}$. Interestingly, like α, R_K belongs to the small set of physical constants known with a very high accuracy (Mohr et al. 2008).

8 Rydberg constant

However, introducing a time dependent vacuum permitivity (Sumner 1994) should not prove enough, since there is another combination of physical constants nowadays known to be *not* time-dependent, namely, R_y, the Rydberg constant (Peik et al. 2006):

$$R_y = \frac{m_0 c_0^2}{4\pi\hbar}\alpha^2$$

where m_0 is the electron mass. Likewise, the hypothesis of a varying speed of light would also prove consistent with this empirical fact if at least another "fundamental constant" is actually time-dependent, namely, either m_0 or $\hbar$. Since $\hbar$ is also involved in α, the additional hypothesis that the electron mass is time-dependent would be the simplest one.

9 Discussion and Conclusion

In the present study, four different kinds of experimental data have been shown to be consistent with the hypothesis that the speed of light decreases as a function of time. As summarized in Table 1, analyses of these data reveal that a_c, the rate of change of the speed of light, lies in a rather narrow range, namely, between -6.6 and -9.4 10^{-10}m s^{-2}, corresponding to a decrease of the speed of light of 2.1-3.0 cm s^{-1} per year. Note that the upper bound is likely to be overestimated since tidal forces are also expected to bring a significant contribution to the observed increase of the time taken by light to go to the Moon and back to Earth (Section 3). Note also that the data considered herein have been collected on two widely different timescales. Lunar laser ranging as well as Pioneer data have been determined over the last few decades, since 1969 and 1972, respectively, while the apparent time dilation of remote events and the Universe's acceleration were exhibited by analyzing light emitted billions of years ago, namely, by galaxies with $z \gg 0.1$ (see Fig. 2). This also suggests that a_c has not changed significantly over this timespan.
From an experimental point of view, the main argument against the hypothesis advocated in the present study is backed by the facts that both fine-structure and Rydberg constants have been shown to vary little in time, if at all (Peik et al. 2006). Indeed, this puts severe constraints on the development of a self-consistent theory in which the speed of light is varying in time since, as a consequence, some other "fundamental constants" have to vary accordingly.
However, building such a theory is beyond the scope of this paper. Although it could reveal hidden links between a wide range of physical phenomenons and, as a consequence, represents a challenge which is likely to arouse the interest of theoreticians, such a work may well await confirmation at the experimental level, as well as further clues, in order to be developped on firm grounds.

Acknowledgments

I thank William Sumner, Simon Mathieu, for stimulating discussions, Hélène Courtois, Jean-François Mangin, Jacques Poitevineau, Louis Riofrio, Antonio Alfonso Faus and Francesco Piazza, as well as some referees, for useful comments.

References

Abarca del Rio, R., Gambis, D., and Salstein, D. A. (2000). Interannual signals in length of day and atmospheric angular momentum. *Annales Geophysicae*, 18(3):347–364.

Anderson, J. D., Laing, P. A., Lau, E. L., Liu, A. S., Nieto, M. M., and Turyshev, S. G. (1998). Indication, from Pioneer 10/11, Galileo, and Ulysses data, of an apparent anomalous, weak, long-range acceleration. *Phys. Rev. let.*, 81(14):2858–2861.

Anderson, J. D., Laing, P. A., Lau, E. L., Liu, A. S., Nieto, M. M., and Turyshev, S. G. (2002). Study of the anomalous acceleration of Pioneer 10 and 11. *Phys. Rev. D.*, 65(8):082004.

Anderson, J. D. and Nieto, M. M. (2009). Astrometric Solar-System Anomalies. *arXiv gr-qc/0907.2469*.

Barrow, J. D. and Magueijo, J. (2000). Can a changing α explain the supernovae results ? *Ap. J.*, 532:L87–L90.

Bizouard, C. and Gambis, D. (2009). The Combined Solution C04 for Earth Orientation Parameters, recent improvments. *International Association of Geodesy Symposia Series*, 134:265.

Chand, H., Srianand, R., Petitjean, P., and Aracil, B. (2004). Probing the cosmological variation of the fine-structure constant: Results based on VLT-UVES sample. *A&A*, 417(3):853–871.

Copeland, E., Sami, M., and Tsujikawa, S. (2006). Dynamics of Dark Energy. *International Journal of Modern Physics D*, 15(11):1753–1935.

Darwin, G. H. (1879). On the Precession of a Viscous Spheroid, and on the Remote History of the Earth. *Phil. Trans. R. Soc. London*, 170:447–530.

Dickey, J. O., Bender, P. L., Faller, J. E., Newhall, X. X., Ricklefs, R. L., Riesi, J. G., Shelus, P. J., Veillet, C., Whipple, A. L., Wiant, J. R., Williams, J. G., and Yoder, C. F. (1994). Lunar laser ranging - A continuous legacy of the apollo program. *Science*, 265(5171):482–490.

Einstein, A. (1917). Kosmologische Betrachtungen zur allgemeinen Relativitatstheorie. *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, 1:l42–l52.

Eubanks, T. (1993). Variations in the orientation of the Earth. *Contributions of space geodesy to geodynamics: Earth dynamics, Geodyn. Ser*, 24:1–54.

Freedman, W. L., Madore, B. F., Gibson, B. K., Ferrarese, L., Kelson, D. D., Sakai, S., Mould, J. R., Kennicutt, R. C., Ford, H. C., Graham, J. A., Huchra, J. P., Hughes, S. M. G., Illingworth, G. D., Macri, L. M., and Stetson, P. B. (2001). Final results from the Hubble Space Telescope key project to measure the Hubble constant. *Ap. J.*, 553(1):47–72.

Glanz, J. (1998). Astronomy: Cosmic motion revealed. *Science*, 282(5397):2156–2157.

Hamuy, M., Phillips, M. M., Suntzeff, N. B., Schommer, R. A., Maza, J., Smith, R. C., Lira, P., and Aviles, R. (1996). The Morphology of Type IA Supernovae Light Curves. *Astron. J.*, 112(6):2438–2447.

Leibundgut, B., Schommer, R., Phillips, M., Riess, A., Schmidt, B., Spyromilio, J., Walsh, J., Suntzeff, N., Hamuy, M., Maza, J., Kirshner, R. P., Challis, P., Garnavich, P., Smith, R. C., Dressler, A., and Ciardullo, R. (1996). Time dilation in the light curve of the distant type Ia supernova SN 1995K. *Ap. J.*, 466(1):L21–L24.

Lemaitre, G. (1927). Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Annales de la Société Scientifique de Bruxelles*, 47:49–59.

Levy, A., Christophe, B., Bério, P., Métris, G., Courty, J. M., and Reynaud, S. (2009). Pioneer 10 Doppler data analysis: Disentangling periodic and secular anomalies. *Advances in Space Research*, 43(10):1538–1544.

Lourens, L. J., Wehausen, R., and Brumsack, H. J. (2001). Geological constraints on tidal dissipation and dynamical ellipticity of the Earth over the past three million years. *Nature*, 409(6823):1029–1033.

Markwardt, C. B. (2002). Independent confirmation of the Pioneer 10 anomalous acceleration. *arXiv gr-qc/0208046*.

Mills, I. M., Mohr, P. J., Quinn, T. J., Taylor, B. N., and Williams, E. R. (2006). Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation 1 (CI-2005). *Metrologia*, 43(3):227–246.

Mohr, P. J., Taylor, B. N., and Newell, D. B. (2008). CODATA recommended values of the fundamental physical constants: 2006. *Reviews of Modern Physics*, 80(2):633–730.

Munk, W. (2002). Twentieth century sea level: An enigma. *Proc. Natl. Acad. Sci. USA*, 99(10):6550–6555.

Nieto, M. M. and Anderson, J. D. (2005). Using early data to illuminate the Pioneer anomaly. *Classical and Quantum Gravity*, 22(24):5343–5354.

North, J. D. (1965). *The measure of the universe. A History of modern cosmology.* Oxford University Press.

Peik, E., Lipphardt, B., Schnatz, H., Tamm, C., Weyers, S., and Wynands, R. (2006). Laboratory Limits on Temporal Variations of Fundamental Constants: An Update. *arXiv physics/0611088.*

Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R. A., Nugent, P., Castro, P. G., Deustua, S., Fabbro, S., Goobar, A., Groom, D. E., Quimby, R., Lidman, C., Ellis, R. S., Irwin, M., McMahon, R. G., Ruiz-Lapuente, P., Walton, N., Schaefer, B., Boyle, B. J., Filippenko, A. V., and Couch, W. (1999). Ω and Λ from 42 high-redshift supernovae. *Ap. J.*, 517(2):565–586.

Riess, A. G., Filippenko, A. V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P. M., Gilliland, R. L., Hogan, C. J., Jha, S., Kirshner, R. P., Leibundgut, B., Phillips, M. M., Reiss, D., Schmidt, B. P., Schommer, R. A., Smith, R. C., Spyromilio, J., Stubbs, C., Suntzeff, N. B., and Tonry, J. (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.*, 116(3):1009–1038.

Riess, A. G., Strolger, L. G., Casertano, S., Ferguson, H. C., Mobasher, B., Gold, B., Challis, P. J., Filippenko, A. V., Jha, S., Li, W., Tonry, J., Foley, R., Kirshner, R. P., Dickinson, M., MacDonald, E., Eisenstein, D., Livio, M., Younger, J., Xu, C., Dahlén, T., and Stern, D. (2007). New Hubble Space Telescope Discoveries of Type Ia Supernovae at z $\geq$ 1: Narrowing Constraints on the Early Behavior of Dark Energy. *Ap. J.*, 659(1):98–121.

Riess, A. G., Strolger, L. G., Tonry, J., Casertano, S., Ferguson, H. C., Mobasher, B., Challis, P., Filippenko, A. V., Jha, S., Li, W. D., Chornock, R., Kirshner, R. P., Leibundgut, B., Dickinson, M., Livio, M., Giavalisco, M., Steidel, C., Benitez, T., and Tsvetanov, Z. (2004). Type Ia supernova discoveries at z $\geq$ 1 from the Hubble Space Telescope: Evidence for past deceleration and constraints on dark energy evolution. *Ap. J.*, 607(2):655–687.

Sanejouand, Y.-H. (2005). A simple varying-speed-of-light hypothesis is enough for explaining high-redshift supernovae data. *arXiv astro-ph/0509582*.

Schrödinger, E. (1939). The proper vibrations of the expanding universe. *Physica*, 6(9):899–912.

Stephenson, F. R. and Morrison, L. V. (1995). Long-term fluctuations in the Earth's rotation: 700 BC to AD 1990. *Philos. Trans. R. Soc. London A*, 351:165–202.

Sumner, W. Q. (1994). On the variation of vacuum permittivity in Friedmann universes. *Ap. J.*, 429(2):491–498.

Turyshev, S. G., Nieto, M. M., and Anderson, J. D. (2005). Study of the Pioneer anomaly: A problem set. *American Journal of Physics*, 73:1033–1044.

Uzan, J.-P. (2003). The fundamental constants and their variation: observational and theoretical status. *Rev. Mod. Phys.*, 75:403–455.

Wallin, J. F., Dixon, D. S., and Page, G. L. (2007). Testing gravity in the outer solar system: Results from trans-Neptunian objects. *Ap. J.*, 666:1296–1302.

Webb, J. K., Flambaum, V. V., Churchill, C. W., Drinkwater, M. J., and Barrow, J. D. (1999). Search for time variation of the fine structure constant. *Phys. Rev. lett.*, 82(5):884–887.

Williams, G. E. (2000). Geological constraints on the Precambrian history of earth's rotation and the moon's orbit. *Rev. Geophys.*, 38(1):37–59.

Wilson, O. C. (1939). Possible Applications of Supernovae to the Study of the Nebular Red Shifts. *Ap. J.*, 90:634.

Wold, P. I. (1935). On the Redward Shift of Spectral Lines of Nebulae. *Phys. Rev. E*, 47(3):217–219.

Zlatev, I., Wang, L., and Steinhardt, P. J. (1999). Quintessence, Cosmic Coincidence, and the Cosmological Constant. *Phys. Rev. let.*, 82(14):2858–2861.