Classical Origin of Quantum Spin

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A charged elementary particle fluctuates from its mean position because of the presence of zero-point fields in vacuum. Considering the complex oscillations of the particle as two general types of rotations, an attempt has been made to explore the classical origin of spin angular momentum. The oscillations of the particle in zero-point field can be transformed as rotations on complex plane and the spin angular momentum is identified with imaginary part of rotations. The quantum value of spin angular momentum has been determined within the purview of stochastic electrodynamics.

Keywords: Spin angular momentum, Zero-point energy, Stochastic Electrodynamics.

1. Introduction

Classically speaking the spin of a particle to certain extent is analogous to the rotation of the particle around its own axis. The spin is not a fundamental notion of quantum theory. The spin angular momentum was hypothesized by Uhlenbeck and Goudsmit [1] and independently by Bichowsky and Urey [2]. The spin angular momentum of only one half of a quantum unit of angular momentum was ascribed to the electron to account for the doublet fine structure in alkaline metals. The ratio between magnetic and mechanical moment of spinning electron is found to be equal to twice the corresponding ratio for the electron orbital motion. This is in agreement with the results obtained in quantum mechanics and accounts for the anomalous Zeeman effect. Over the years the half integral spin is defined as an inherent and fixed property of electron. However, the existence of spin can be derived from the fundamental postulates of quantum mechanics and the property of symmetry transformations. The spin angular momentum value $\pm\hbar/2$ appears naturally as a solution of the angular momentum commutation relation. The spin normally believed to have any classical correspondence but only regarded as a quantum phenomena. The spin angular momentum may be considered as a circulatory current of charged matter. The electron in the hydrogen atom does has neither definite position nor momentum but a definite angular momentum. Thus atomic electron may be assumed to possess complex motions and classically such motions may be described by rotations. The spin angular momentum is a kinematic property of massive elementary particles and it corresponds to a rotation group symmetry SU(2).

The Dirac theory of electron automatically includes the effect of spin which leads to the conclusion that the spin is a quantum relativistic effect. Applying Lorentz transformation to the Dirac spinar, in a special case of infinitesimal rotation, it has been shown that the total angular momentum is a generator of rotations. The total angular momentum is a sum of orbital and spin angular momenta. The two spin states are connected to the two values of the helicity operator. In non-relativistic theory the spin is incorporated by using the kinetic energy operator $H = (\sigma.p)(\sigma.p)/2m$ and into the general framework of relativistic quantum mechanics by using [(E/c) − (σ.p)] [(E/c) − (σ.p)] = mc^2. The coordinate operator in Dirac theory is given by [3]

$$x(t) = x(0) + c^2 p_k H^{-1} t + \left(\frac{ic\hbar}{2}\right)\left[\alpha(0) - cp_k H^{-1}\right] H^{-1} \exp\left(\frac{-2iHt}{\hbar}\right). \quad (1)$$

The third term in the above equation implies that the free electron executes very rapid oscillations in addition to the uniform rectilinear motion of the electron. This oscillatory motion is called *zitterbewegung*. Thus within the wave packet associated with the electron there exists a superposition of violent oscillations each with angular frequency $2mc^2/\hbar$. The particular oscillatory behaviour is due to interference between positive and negative energy components in the wave packet. By taking $E = mc^2 = \hbar\omega_0$, it is easy to see from Eq.(1) that the *zitterbewegung* frequency is $2\omega_0$. This frequency may be considered as the frequency of rotation of the *zitterbewegung* within the wave packet. In this connection it is worth mentioning the observation by Hestenes [4]. The spin angular momentum can be regarded as the angular momentum of *zitterbewegung* fluctuations. Thus spin may be connected to the *zitterbewegung* rotation within the wave packet.

The geometrical origin of spin has been suggested by Newman [5]. By applying complex translation to the coordinate in Minkowski space, the spin angular momentum has been identified as the imaginary part of complex displacement. The Dirac value of electron gyromagnetic ratio was obtained on purely classical arguments. In Swinger's oscillator model [6] of angular momentum there exists a connection between the algebra of angular momentum and two independent simple harmonic oscillators. These oscillators called as plus type and minus type, are denoted by creation and annihilation operators. The plus (minus) type oscillator is associated with spin up

+ ħ/2 (spin down –ħ/2) particle state. The meaning of plus type angular momentum is that it destroys one unit of spin down –ħ/2 and creates one unit of spin up +ħ/2 and thus the z-component of angular momentum is increased by ħ. Similarly the meaning of minus type angular momentum is that it destroys one unit of spin up +ħ/2 and creates one unit of spin down –ħ/2 and thus the z-component of angular momentum is decreased by ħ. Thus the spin is inherently connected to the field oscillators.

In the quantum mechanical treatment of electromagnetic radiation of the Plank oscillator, the radiation energy density relation contains an additional term equal to ħω/2 and this term does not vanish even at absolute zero and gives rise to zero-point energy (ZPE). The oscillatory behaviour of the particle is mainly attributed due to the vacuum fluctuations defined by the zero-point field (ZPF). The Planck's idea about ZPE was studied quite later by Marshall in terms of classical stochastic electrodynamics (SED) [7] and found the connection between the behaviour of classical and quantum oscillator. Boyer showed that for a harmonic oscillator, the fluctuations produced by ZPF on the position of the particle are exactly in agreement with quantum theory [8]. The ZPE associated with a charged particle can be obtained by using classical SED. This energy may be assumed to be confined within the wave packet of the particle.

With this understanding an attempt has been made to find the classical origin of quantum spin. In section 2 we deal with the classical origin of spin angular momentum. In section 3 considering the energy of rotation as the ZPE associated with the particle, the value of spin angular momentum is obtained. The discussion of the results is presented in section 4.

2. Classical perspective of quantum spin

Let us consider a charged elementary particle of mass m and charge e under rotation. The charged particle may be considered as a symmetric spherical ball of radius r_0. The radius vector r is a function of time and can be expressed as $r(t) = r_0 \exp(+i\theta) = r_0 \exp(+i\omega t)$, where $\omega = d\theta/dt$ is the angular frequency of rotation. Here we have considered the rotation in the counterclockwise direction and the angular frequency of rotation is $+\omega$. However, if the rotation is considered in the clockwise direction, the radius vector can be expressed as $r(t) = r_0 \exp(-i\theta) = r_0 \exp(-i\omega t)$ with a frequency of rotation $-\omega$. The projection of any rotation can be visualized as an oscillation about a mean point. Thus the radius vector oscillations are linear in one dimension. Similarly the oscillations of a particle can be visualized as a rotation defined by a radius vector r(t). The oscillations that represent the counterclockwise rotation of the particle are associated with positive frequency.

$$r_{AC}(t) = r_0 \exp(+i\omega t) = r_0 (\cos \omega t + i \sin \omega t) \tag{2}$$

Similarly the oscillations that represent clockwise rotation of the particle are associated with the negative frequency.

$$r_C(t) = r_0 \exp(-i\omega t) = r_0 (\cos \omega t - i \sin \omega t) \tag{3}$$

We express the rotations expressed in Eqs.(2) and (3) as a combination of r_{AC} and r_C in the following manner.

$$r_+(t) = \left[\frac{r_{AC}(t) + r_C(t)}{2}\right] = r_0 \cos \omega t \tag{4}$$

$$r_-(t) = \left[\frac{r_{AC}(t) - r_C(t)}{2}\right] = i r_0 \sin \omega t \tag{5}$$

One can get the radius of rotation from Eqs.(4) and (5) by finding the mean square values.

$$\left|r(t)\right|^2 = \left|r_+(t)\right|^2 + \left|r_-(t)\right|^2 \quad \text{and} \quad \left|r(t)\right| = r_0 \tag{6}$$

Thus the decomposition shows that the radius of rotation can be represented by two separate modes of oscillations. The mean values of $r_+(t)$ and $r_-(t)$ over a complete cycle are zero. However, the average values $r_+(t)$ and $r_-(t)$ found over an angle π are having different values.

$$\left\langle r_+(t)\right\rangle_{av} = \frac{r_0}{\pi}\int_0^{\pi}\cos\theta.d\theta = 0 \tag{7}$$

$$\left\langle r_-(t)\right\rangle_{av} = \frac{r_0}{\pi}\int_0^{\pi}\sin\theta.d\theta = \frac{2}{\pi}r_0 \tag{8}$$

These two values give $\langle r(t)\rangle_{av} = 2r_0/\pi$. Physically when a particle is rotated half way in one direction and half way in the opposite direction means the rotation is considered as null. Or in other words the oscillations sum up to produce a standing wave. Thus we have $\langle r_+(t)\rangle_{av}$ is zero and the function $r_+(t)$ is an even mode of $r(t)$ as $r_+(-t) = r_+(t)$. An important concept arises with Eq.(5) as we shall see now. Eq.(5) also represents a radius vector with a half rotation in direction with frequency $+\omega$ and a half rotation in clockwise direction with frequency $-\omega$. However the effect of $-r_C$ is to produce a half rotation in the counterclockwise direction again. Thus physically speaking the frequency of the $r_-(t)$ must be twice the frequency of rotation. The function $r_-(t)$ is an odd mode of $r(t)$ as $r_-(-t) = -r_-(t)$. Now, these two modes of rotations may be considered to represent two modes of complex oscillations of the particle. The oscillatory behaviour of the particle is mainly attributed to the presence of vacuum fluctuations defined by the ZPF. In the limit when ωt is small Eqs.(4) and (5) can be approximated as

$$r_+(t) = r_0 \qquad \text{and} \qquad r_-(t) = r_0\omega_0 t\ . \tag{9}$$

Where, ω_0 is the fundamental frequency of oscillations of the particle under consideration. The velocities associated with the two modes can be written from the Eq.(9) as

$$v_+(t) = v_+(-t) = r_0\omega_0 \tag{10}$$

$$v_-(t) - v_-(-t) = 2r_0\omega_0 \qquad \text{and} \qquad v_-(-t) - v_-(t) = -2r_0\omega_0\ . \tag{11}$$

When the velocity of the particle changes from v(t) to v(–t) the difference in these two velocities must be proportional to the spin frequency. The change in velocity $v_S = \pm[v_-(t) - v_-(-t)] = \pm 2r_0\omega_0$ can be identified to produce the spin frequency $\omega_S = v_S/r_0 = \pm 2\omega_0$. Thus the spin frequency can be negative or positive depending on the change in velocity and hence the spin angular momentum is two fold in nature.

Now we are in a position to define two types angular momentum of the particle under rotation. The angular momentum of the particle under rotation is $L = mv_+(t)r_+(t)$ with a constant angular frequency $\omega_L = \omega_0$. When the radius of rotation is expressed with oscillation mode $r_+(t)$ we write the angular momentum as

$$L = m\omega_L r_0 r_+(t) = m\omega_0 r_0^2\ . \tag{12}$$

The spin angular momentum is assumed to be produced only due to the mode $r_-(t)$. Since the frequency of oscillations of the mode $r_-(t)$ is equal to twice the value of ω_0, we define the spin angular momentum S with spin frequency $\omega_S = v_S/r_0 = \pm 2\omega_0$. In analogy with Eq.(12) the spin angular momentum can be defined as

$$S = \tfrac{1}{2} m\omega_S r_0 r_-\left(t\right) = \pm m\omega_0 r_0 r_-\left(t\right). \tag{13}$$

Now, using Eq.(9) we can find the classical equation of motion of spin angular momentum.

$$\frac{dS}{dt} = \pm \tfrac{1}{4} m\omega_S^2 r_0^2 = \pm\Delta E_{spin} \tag{14}$$

If the average energy of rotation of the particle is $E_{rot} = L\omega_L/2$, the angular momentum L can be expressed in terms of energy of rotation as

$$L = \eta \frac{2E_{rot}}{\omega_L} = \eta \frac{2E_{rot}}{\omega_0}. \tag{15}$$

Where, a factor $\eta = \pi/2$ is introduced to express r_0 in terms of $\langle r(t)\rangle_{av}$. In the same way the spin angular momentum can also be expressed as

$$S = \eta \frac{2E_{rot}}{\omega_S} = \pm\eta \frac{2E_{rot}}{2\omega_0} = \pm\eta \frac{E_{rot}}{\omega_0}. \tag{16}$$

The significance of classical Eqs.(15) and (16) will be soon revealed when the energy of rotation is identified with the energy of fluctuations of the particle due to ZPF. However, first we try to understand the expressions in the quantum mechanical treatment of angular momentum. In quantum theory the angular momentum conservation is expressed by the Bohr-Sommerfeld quantum condition $L = n_\theta \hbar$, where n_θ is the angular momentum quantum number. With the substitution $L = \hbar$ in Eq.(15), the energy of rotation can be written as $E_{rot} = \hbar\omega_0/\pi$ and using this value of energy of rotation in Eq.(16), the observed value of spin angular momentum can be obtained as $S = \pm\hbar/2$. Thus we have the correct value of spin angular momentum in any direction. In the next section the energy absorbed by a charged particle immersed in ZPF will be derived in the purview of SED and the value of spin angular momentum will be obtained.

3. Stochastic determination of spin angular momentum

The electromagnetic ZPF in classical form can be expressed in terms of superposition of plane waves. For each direction of propagation given by propagation vector **k**, there exist two mutually orthogonal polarization states and the ZPF is a sum of two polarization states [8]. The electric component of electromagnetic ZPF can be expressed as

$$\mathbf{E}^{ZP}\left(\mathbf{x},\mathrm{t}\right)=\sum_{\lambda=1}^{2}\int \mathrm{d}^3\mathrm{k}\;\boldsymbol{\varepsilon}\left(\mathbf{k},\lambda\right)H\left(\omega\right)\cos\left[\mathbf{k.x}-\omega t-\theta\left(\mathbf{k},\lambda\right)\right] \quad (17)$$

Where $\varepsilon(k,\lambda)$ is the polarization vector which is a function of k and polarization index $\lambda = 1,2$. The phase angle $\theta(k,\lambda)$ is introduced to generate random fluctuations of the ZPF which is in accordance with the random phases introduced by Plank and Einstein and Hopf. The spectral function $H(\omega)$ is introduced to represent the magnitude of ZPE and it can be evaluated as $H^2(\omega) = \hbar\omega/2\pi^2$ in the stochastic treatment [9].

Consider a particle of mass m and charge e in the homogeneous and isotropic space filled with electromagnetic ZPF. Such a particle is assumed to oscillate from its mean position due to the random fluctuations of the ZPF. The classical equation of motion of a charged oscillating particle under external electromagnetic field is given by [10]

$$\frac{\mathrm{d}^2\mathbf{x}\left(\mathrm{t}\right)}{\mathrm{dt}^2}-\Gamma_a\frac{\mathrm{d}^3\mathbf{x}\left(\mathrm{t}\right)}{\mathrm{dt}^3}+\omega_0^2\mathbf{x}\left(\mathrm{t}\right)=\frac{\mathrm{e}}{\mathrm{m}}\mathrm{E}^{ZP}\left(\mathbf{x},\mathrm{t}\right). \quad (18)$$

Where, Γ_a is the radiation damping term and ω_0 is the frequency of oscillations of the particle under consideration. This equation is in general known as Abraham-Lorentz equation of motion. The charged particles at the level of their charge spread may respond to high frequency spectrum of ZPF. The particle field interaction can be visualized by considering free particle oscillator immersed in ZPF. In the case of a free charged particle, to a first approximation, as the spectral density of ZPF is proportional to ω^3, the binding term in the equation of motion of the charged particle can be neglected. As the radiation damping term is small, the equation of motion can be written in an asymptotic approximation without both damping and radiation terms.

$$\frac{d^2\mathbf{x}(t)}{dt^2} = \frac{e}{m}\mathbf{E}^{ZP}(x,t) \tag{19}$$

The energy absorbed by the charged particle immersed in ZPF is then obtained using the method followed by Puthoff [11]. The velocity v(t) of the particle is obtained by integrating the Eq.(29). Substituting the expression for $E^{ZP}(x,t)$ from Eq.(17), one can find $|v|^2$ by using the complex conjugate of velocity $v^*(t) = v(-t)$. The required integration is then carried out by taking the upper limit to a cut-off frequency equal to ω_0 and taking the stochastic average, $\langle |v|^2 \rangle$ can be obtained.

$$\left\langle |v|^2 \right\rangle = \frac{2\Gamma_a \hbar \omega_0^2}{\pi m} \tag{20}$$

The kinetic energy of the oscillating particle in terms of its average velocity can be written as $\Delta E_k = m\langle |v|^2 \rangle/2$ and the ZPE associated with particle is then obtained by substituting $\langle |v|^2 \rangle$ from Eq.(20).

$$\Delta E_k = \frac{\Gamma_a \hbar \omega_0^2}{\pi} \tag{21}$$

Now applying the classical limit $\Gamma_a\omega_0 \sim 1$ and assuming that the ZPE associated with the charged particle is equal to the energy of rotation E_{rot}, the quantum value of angular momentum can be obtained from Eq.(15).

$$L = \eta \frac{2\Delta E_k}{\omega_0} = \hbar \tag{22}$$

And using Eq.(16) the quantum value of spin angular momentum can be determined.

$$S = \pm\eta \frac{2\Delta E_k}{\omega_S} = \pm\frac{\hbar}{2} \tag{23}$$

Thus the spin of the particle is connected to the ZPE associated with the charged fundamental particle and the spin value is obtained within the classical theory.

4. Discussion

Simple harmonic oscillations may be considered as a combination of two equal and opposite circular rotations. A charged particle immersed in the ZPF may be considered to possess complex oscillations. These complex oscillations may be described to a better advantage in terms of modes of rotation $r_+(t)$ and $r_-(t)$ considered in the classical explanation of quantum spin. The modes of rotation $r_+(t)$ and $r_-(t)$ can also be expressed as a combination of scalar and vector products of two arbitrary vectors say **u** and **v** with magnitude $(r_0)^{1/2}$ and separated by an angle θ. Then $r_+(t) = \mathbf{u}.\mathbf{v} = uv\cos\theta = r_0\cos\theta$ and $\mathbf{e}r_-(t) = \mathbf{u}\times\mathbf{v} = \mathbf{e}uv\sin\theta = \mathbf{e}r_0\sin\theta$, where **e** is a unit vector perpendicular to the rotation plane of **u** and **v**. It is also possible to express r(t) as a complex quantity in terms of $r_+(t)$ and $r_-(t)$ and one can express $r(t) = r_+(t)+ir_-(t)$ for positive frequency and counterclockwise rotation and the complex conjugate $r^*(t) = r_+(t)-ir_-(t)$ for negative frequency clockwise rotation. The scalar product gives the real part of r(t) and the vector product the imaginary part of r(t). The vector product represents the rotation perpendicular to the spin direction. Thus when r(t) is transformed onto a complex plane, the imaginary part $r_-(t)$ is connected with the spin angular momentum S and the real part $r_+(t)$ to the angular momentum L. The presence of unit imaginary in these equations itself implies the presence of spin of the particle. In converting r(t) into components **u.v** and **u×v** products and representing them in the form of complex quantities seems to have some analogy (or similarity) with the spacetime algebra introduced by Hestens [4]. However the spacetime algebra treatment is completely different which uses the Dirac matrices. In the Newman's [5] interpretation the spin is identified as S = mcY, where Y is the imaginary part of complex displacement and c the velocity of light. To give correct value of spin this length Y must be equal to $\lambda_C/4\pi$, where λ_C is the Compton wavelength. In the present context as $\lambda_C = \lambda_0$, the inverse of displacement Y is proportional to the spin frequency ω_S.

There seems to be some similarity between Plank oscillator and rigid rotator problems in quantum mechanics in regard to half integral quantum numbers. The energy of Plank oscillator is equal to $(n + ½) \hbar\omega$. The energy of a symmetric rigid rotator is given by $\hbar^2 n_r(n_r+1)/2I$ [12]. Where, I and n_r are the moment of inertia of the rotator and rotational quantum number respectively. Substituting $I\omega = n_r\hbar$, one can find the ground state energy of the rotator is just equal to $\hbar\omega/2$. Thus the ground state energies of plank oscillator and rigid rotator are same. From this one can conclude that the oscillations and rotations in quantum mechanics give similar results. Thus it is legitimate to consider E_{rot} in Eq.(16) as equal to the ZPE associated with the charged particle.

The oscillations of the particle produced due to the presence of ZPF are the main reason for the indeterminacy of position and momentum of the particle and also considered to be the foundation for the origin of wave nature of the particle in general. Thus the particle we have considered is assumed to be confined in a volume of space with dimensions of the order of charge radius of the particle and the rigid rotator that is considered is actually the wave packet. The ZPE associated with particle may be considered to be confined to the volume of this wave packet. From the equation of motion of spin given by Eq.(14) one can also express spin angular momentum as

$$S = \pm \int \Delta E_{spin}\, dt. \tag{24}$$

This equation gives the geometrical origin of spin. The spin is thus equal to the area under the energy-time curve over a time interval which corresponds to the size of the wave packet. Thus the spin angular momentum is in one way the consequence of energy and time uncertainty relation.

The energy of rotation may be classically interpreted as the charge rotation within the region of space surrounding the particle and the dimension of this region of space may be assumed to be of the order of the charge radius which is almost equal to λ_0. In section 3 the equation of motion of the charged particle is considered in an asymptotic approximation. Considering all the terms in the equation of motion of charged particle and using the upper cut-off frequency equal to ω_0, the energy absorbed by the particle has been found to be almost same as the one obtained in Eq.(21).

In the Dirac theory of electron the gyromagnetic ratio is shown to be g = 2. In the treatment presented in section 2 the ratio ω_S/ω_L gives the same value of g. However, the g value is experimentally found to be more than 2 and this excess value is known as anomalous magnetic moment. In quantum electrodynamics or quantum field theories in general, by considering vacuum fluctuations of the fields a correction to the mechanical mass has been introduced [13]. The observed mass is expressed as $m = m_0 + \delta m$, where m_0 is called the non-electromagnetic and δm the electromagnetic mass. The radiation reaction of the charged particle or the self energy of the particle is normally known as the electromagnetic mass of the particle. In the classical theory of stochastic electrodynamics, it is believed that the ZPF fluctuations are responsible for the presence of electromagnetic mass of the charged particle. The ZPE associated with particle is assumed to give a correction to mass of the particle. And when $m \to m+\delta m$ in Eq.(13), one can write the slightly increased spin angular frequency as

$$\omega_S \to \left(1+\frac{\delta m}{m}\right)2\omega_0 . \tag{25}$$

Then the gyromagnetic ratio can be expressed as

$$\frac{g}{2} = \frac{\omega_S}{2\omega_0} = 1+\frac{\delta m}{m} = 1+a_l . \tag{26}$$

Where, a_l is the anomalous magnetic moment of a lepton. Since the average fluctuations connected with the orbital angular momentum are zero, as shown in connection with $r_+(t)$ mode in Eq.(7), one need not consider any mass correction term connected with ω_L. Thus only the measured value of ω_S must be slightly more than $2\omega_0$ and the increased value of ω_S due to mass correction accounts for the anomalous magnetic moment.

Finally we find the spin angular momentum is a consequence of ZPF fluctuations. When these fluctuations are expressed as rotations on complex plane, the spin angular momentum is connected with imaginary part of rotations. The classical origin of spin arises mainly because of classical nature of ZPF.

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