Electron Impedances

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It is only recently, and particularly with the quantum Hall effect and the development of nanoelectronics, that impedances on the scale of molecules, atoms and single electrons have gained attention. In what follows the possibility that characteristic impedances might be defined for the photon and the single free electron is explored is some detail, the premise being that the concepts of electrical and mechanical impedances are relevant to the elementary particle. The scale invariant quantum Hall impedance is pivotal in this exploration, as is the two body problem and Mach's principle.

To understand the electron would be enough - Einstein

Introduction

In this note both the photon and the electron are viewed as quantum resonators [1]. Like all resonators, they have characteristic mode impedances.

We begin with the photon far and near field impedances, with the transition from far to near field taken to be at the electron reduced Compton wavelength, the Compton radius. The energy of a photon of this wavelength is the rest mass energy of the electron.

A simple derivation of the scale invariant quantum Hall impedance from the uncertainty principle is presented, reinforcing the notion that the concept of impedance is meaningful at the level of the elementary particle.

This is followed by a second derivation of the quantum Hall impedance. The mass of the electron enters here, in terms of a mechanical impedance determined by equating the inertial and Lorentz forces, where the inertial force is determined from Mach's principle as applied to the two body problem.

Seeking symmetry between electric and magnetic, two additional scale invariant impedances are derived in terms of the dual of the Lorentz force, from the motion of magnetic charge in electric fields. One of the two is numerically equal to the quantum Hall impedance, and the other is a factor of $1/2\alpha$ larger, where α is the fine structure constant. These derivations require the introduction of the electric flux quantum. In the process we note that the sought electric/magnetic symmetry is broken both topologically and electromagnetically. Neither broken symmetry appears to be documented in the literature.

One last derivation of the quantum Hall impedance follows from application of the concept of mechanical impedance to the ground state of the hydrogen atom. The appearance of scale invariance here, in the hydrogen atom, is at least a little surprising, as is the appearance of the quantum Hall impedance.

Additional electron impedances are derived for Coulomb and dipole interactions of magnetic and electric monopoles and dipoles. The resulting impedances are plotted as a function of space scale, followed by a brief discussion.

The possibility of observation of an additional quantum Hall impedance of a few ohms is introduced.

1. The Photon Impedances

The impedance of free space is defined as

$$Z_0 := \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad Z_0 = 3.7673031346 \times 10^2 \,\text{ohm}$$

where μ_0 and ε_0 are the free space magnetic and electric permeabilties/permittivities [2]. This definition says nothing about the region of interest, the scale-dependent near field. The near field is more a property of the photon than of space, the transition between far and near fields being inversely dependent on the photon energy rather than being a characteristic of space.

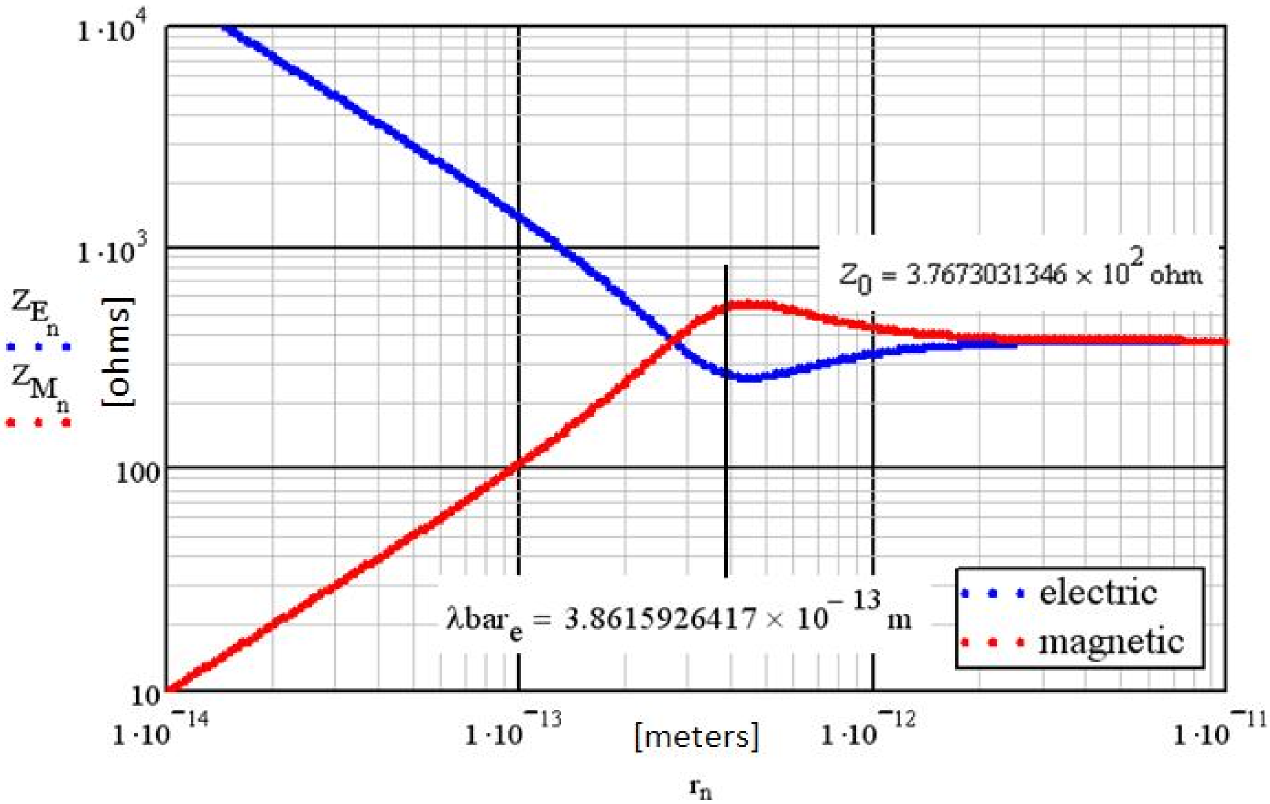
Here we choose the boundary between near and far fields to be the Compton radius of the electron, the scale at which the photon energy is equal to the rest mass of the electron. Ignoring for now the phases likely needed for a proper quantum mechanical treatment, the electric and magnetic dipole impedances [3] can be calculated in multiples of the Compton radius

$$\lambda\text{bar}_e := \frac{\text{hbar}}{m_e \cdot c} \qquad \lambda\text{bar}_e = 3.861592642 \times 10^{-13}\,\text{m}$$

and an index parameter r that permits to plot these impedances as a function of scale.

$$Z_{E_n} := \frac{Z_0 \cdot \left| 1 + \frac{\lambda\text{bar}_e}{j \cdot r_n} + \frac{\lambda\text{bar}_e^{\,2}}{\left(j \cdot r_n\right)^2} \right|}{\left| 1 + \frac{\lambda\text{bar}_e}{j \cdot r_n} \right|} \qquad Z_{M_n} := \frac{Z_0 \cdot \left| 1 + \frac{\lambda\text{bar}_e}{j \cdot r_n} \right|}{\left| 1 + \frac{\lambda\text{bar}_e}{j \cdot r_n} + \frac{\lambda\text{bar}_e^{\,2}}{\left(j \cdot r_n\right)^2} \right|}$$

Such a plot is shown in the figure below.



2. Quantum Hall Impedance from the Uncertainty Principle

The relevance of the quantum Hall impedance to the single electron is explored in the literature [4-10]. The most convincing argument is perhaps the derivation from the uncertainty principle [4], as reproduced below.

Time and energy (or frequency) are conjugate variables. They are Fourier transform duals. Their product is angular momentum, whose minimum is defined by the Heisenberg uncertainty relation

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

This can be written as

$$\frac{\Delta E}{e}\frac{\Delta t}{e} \geq \frac{\hbar}{2e^2}$$

where $\frac{\Delta E}{e} = V$ can be interpreted as a voltage and $\frac{\Delta t}{e} = \frac{1}{I}$ as the inverse of a current. We then have

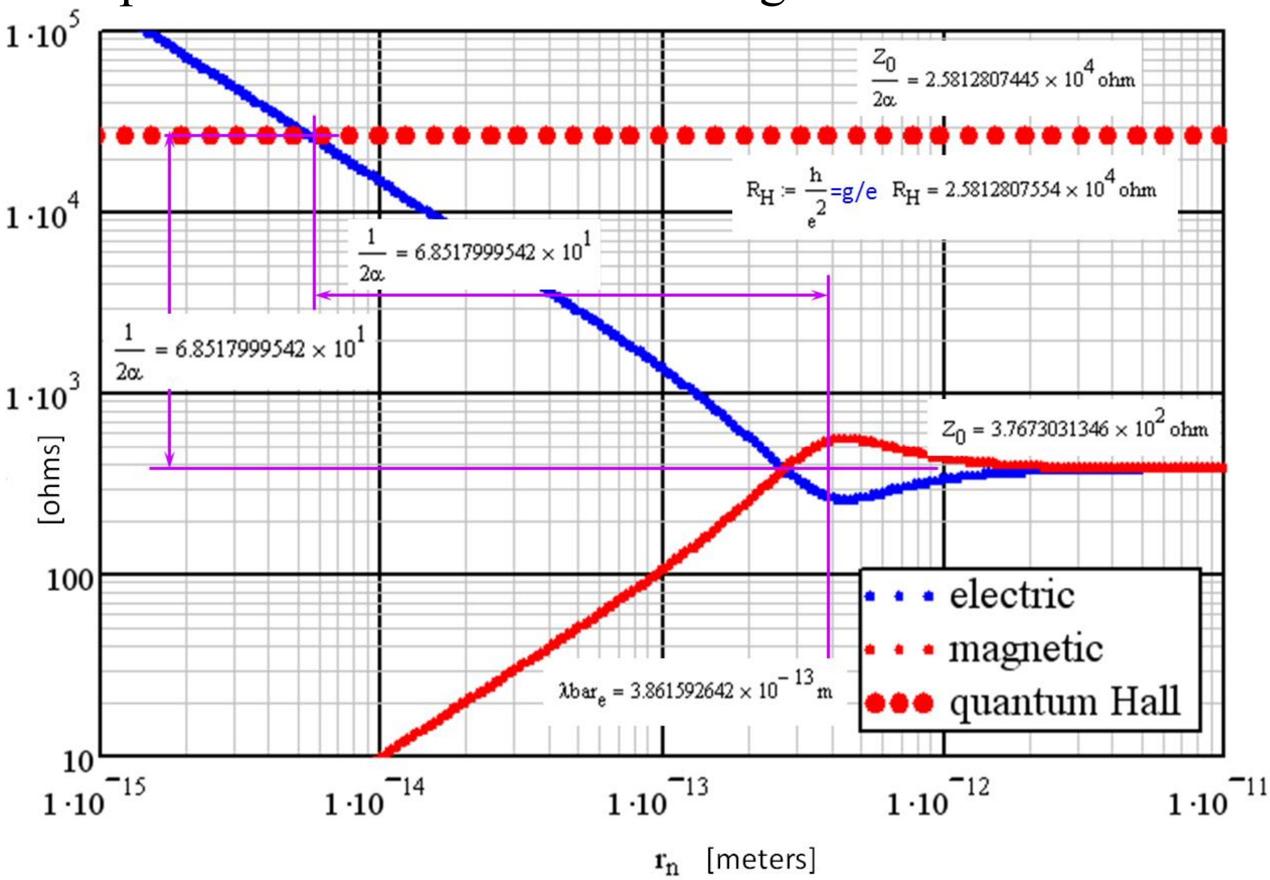
$$\frac{V}{I} = \frac{\hbar}{2e^2} = Z$$

which differs from the quantum Hall impedance

$$R_H = \frac{\hbar}{e^2} = 2.581280756 \cdot 10^4 \cdot ohms$$

by a spin-related factor of two.

The quantum Hall impedance is plotted with the photon impedances in the following figure. As the figure shows, it equals the photon impedance at twice the electromagnetic radius.



3. Mechanical Derivation of the Quantum Hall Impedance

The method of calculating mechanical impedances for the various possible forces arises from the application of Mach's principle to the two body problem [11]. That earlier discussion presents the basics of the problem, and is appended to this note. The reader is encouraged to consult it before continuing.

Summarizing the results presented there, the inertial force in the case of uniform circular motion can be written as

$$\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} = m\omega^2\vec{r} + \vec{v}\frac{dm}{dt}$$

Given the logical constraints of a rigorously defined two body problem it is not possible to observe angular velocity. However, as discussed in the appendix there is some sense in which the possibility remains to observe radial velocity. We then write the inertial force as

$$\vec{F} = \vec{v}\frac{dm}{dt}$$

Implicit in this is the recognition that the concepts of dimension and direction become less familiar in the context of the rigorous two body problem, as does the concept of mass. When considering time variation of mass, we might ask how this relates to whatever it is that fluctuates in the quantum mechanical wave equation.

In other words, what waves in the deBroglie matter wave?

We seek to equate this inertial force with the Lorentz force

$$\vec{F} = e\left(\vec{E} + \vec{v}\times\vec{B}\right) + g\left(\vec{B} - \vec{v}\times\vec{E}\right)$$

For the moment we ignore the magnetic charge term, as well as the first electric charge term. We then have

$$\vec{F} = e(\vec{v} \times \vec{B})$$

This Lorentz force term occupies an odd niche between conservative and non-conservative forces. It is velocity dependent, like all non-conservative forces. However, it is conservative in the sense that it is not dissipative. The force and the resulting velocity are orthogonal. It can do no work. It can transfer no energy, at least in the classical sense. This is related to the scale invariance of this impedance, and has interesting implications for mode coupling.

Equating these two forces, the inertial and the Lorentz, and for the moment ignoring the vectorial character in consideration of the constraints imposed by the rigorous two body problem, we have

$$F = v\frac{dm}{dt} = evB$$

which yields

$$\frac{dm}{dt} = eB$$

The units in this equation are [kg/sec]. These are the units of mechanical impedance [12]. Mechanical force is measured in units of [kg-m/sec^2]. This is the impedance times the relative velocity of the two objects sharing the force. By this definition there is no force if there is no relative movement. There is no deBroglie wave if there is no relative velocity [13,14].

The magnetic field intensity in the above equation can be determined by taking the magnetic flux quantum to be confined to the Compton radius of the electron. Here we take the flux quantum to be that associated with a single free electron [15]

$$\Phi_B = \frac{h}{e}$$

In a brief aside, we note that by the Dirac relation $eg=h$ the above definition of the flux quantum can be extended to

$$\Phi_B = \frac{h}{e} = g$$

This exposes the first anomaly [15,16], a topological anomaly. The units of the flux quantum are [tesla-m^2]. It can be pictured as a disc of a given radius, say for instance the Compton radius, penetrated perpendicularly by magnetic field lines that extend to infinity in opposite directions without diverging. The term 'spinor' is at least somewhat applicable to such an object. It is something between two and three dimensions. We see that it is numerically equal to the magnetic charge, a monopole whose field lines extend radially outward to infinity in all directions. The concept of space is somehow broken in this. Whether the breakage is simply in the SI system of units, which "…were developed in a framework that would facilitate relating the standard units of mechanics to electromagnetism…" [17], or indicative of some deeper subtlety is not immediately obvious. In any case it is integrated into our larger conceptual schema, into the highly refined picture of the physical world that we carry around in our heads, and must be given consideration.

Continuing on with the mechanical derivation of the quantum Hall impedance, from the definition of the flux quantum we can write

$$B = \frac{\Phi_B}{\pi \cdot \bar{\lambda}^2} = \frac{h}{\pi \cdot e\bar{\lambda}^2}$$

so that

$$\frac{dm}{dt} = eB = \frac{h}{\lambda^2}$$

where, in light of the uncertainty regarding the concept of dimensionality in the rigorous two body problem, we have ignored the factor of π in the denominator. Relative to the fine structure constant, factors of two, three, four and π often emerge in the present study of impedance matching. Their various origins remain to be fully clarified. For the moment the simplest expedient is to omit them whenever possible, in the hope that an eventual clear understanding of the physics will permit to insert them in their proper places.

The conversion factor from mechanical to electrical impedance is the inverse of line charge density squared. Taking the line charge density to be that of the charge quantum at the reduced Compton wavelength of the electron, we then have

$$\frac{\lambda^2}{e^2} \cdot \frac{dm}{dt} = \frac{\lambda^2}{e^2} \frac{h}{\lambda^2} = \frac{h}{e^2} = R_H = \frac{g}{e}$$

This completes the derivation.

4. Quantum Hall Impedance and the Electric Flux Quanta

The two electric flux quanta were defined in earlier notes [15,16]. Here we present those definitions in terms of both electric and magnetic charge, along with the magnetic flux quantum.

$$\Phi_B = \frac{h}{e} = g = 4.1356673326 \cdot 10^{-15} tesla \cdot m^2$$

$$\Phi_{E1} = \frac{h \cdot c}{e} = g \cdot c = 1.2398418751 mV \cdot mm$$

$$\Phi_{E2} = \frac{e}{\varepsilon_0} = \frac{h}{\varepsilon_0 g} = 0.0180951265 mV \cdot mm$$

Again we find a broken symmetry [15,16], this time not topological but rather of electromagnetism. There are two electric flux quanta, and only one magnetic. What does one do with the extra electric flux quantum? How does it fit? We again suggest [ref] that it is somehow related to the difficulties regarding the gauge invariance of the photon and the removal of the longitudinal component via the Ward identity, and hope to address this further at some future time.

We also note that the mechanism which results in two electric flux quanta also applies to the electric dipole moment. The electric partner of the magnetic dipole moment has not one but two numerical values.

Returning to the full expression for the Lorentz force,

$$\vec{F} = e\left(\vec{E} + \vec{v} \times \vec{B}\right) + g\left(\vec{B} - \vec{v} \times \vec{E}\right)$$

and using the same procedure as presented previously, we can now calculate the mechanical impedances associated with the force

$$\vec{F} = -g\left(\vec{B} - \vec{v} \times \vec{E}\right)$$

which gives $\frac{dm}{dt} = gE$

In this case there are two impedances, one for each of the two electric field strengths that result from having two electric flux quanta. Again, these impedances are scale invariant. The weaker of the two is again equal to the quantum Hall impedance. The stronger is larger by the ever more ubiquitous factor of $1/2\alpha$.

One might also note that, despite our efforts to the contrary for the sake of simplicity, our system of units intrudes again. To be dimensionally correct the previous expression reads

$$\frac{dm}{dt} = \frac{gE}{\mu_0 c^2}$$

5. Quantum Hall Impedance and the Hydrogen Atom

The derivation is shown in Appendix I. Here we only mention that the step between equations (3) and (4) is bridged by making the appropriate substitutions for the deBroglie wavelength

$$v_{rad} = \frac{h}{mr}$$

and Bohr radius

$$r = a_0 = \frac{4\pi\varepsilon_0 h^2}{me^2}$$

The result is again the quantum Hall impedance. As mentioned earlier, the appearance of scale invariance here, in the hydrogen atom, is at least a little surprising, as is the appearance of the quantum Hall impedance.

At this point we are perhaps losing track of just how many ways this quantum Hall impedance can be derived.

2. The Gang of Eight

To proceed further in defining electron impedances, it is necessary to clarify previous assumptions and make some additional assumptions regarding the structure of both the photon and the electron [15,16]. We consider

Three basic topologies:

- flux quantum (~spinor)
- charge quantum (monopole)
- dipole quantum (dipole)

Two types of charge:

- electric
- magnetic

Two broken symmetries, one topological and one electromagnetic, that result in:

- one each magnetic dipole and magnetic flux quantum
- two each electric dipole and electric flux quantum

This gives a total of eight basic entities, or perhaps ten if one considers the observed magnetic flux quantum and magnetic dipole to be degenerate states.

The definitions and numerical values of the Gang of Eight are tabulated below, as well as the field strengths of the flux quanta when confined to the Compton radius of the electron.

$$e := 1.602176487 \cdot 10^{-19} \cdot \text{coul}$$

$$g := \frac{h}{e} \qquad g = 4.1356673326 \times 10^{-15}\ \text{tesla} \cdot \text{m}^2$$

$$\Phi_B := \frac{h}{e} \qquad \Phi_B = 4.1356673326 \times 10^{-15}\ \text{tesla} \cdot \text{m}^2 \qquad B := \frac{\Phi_B}{\pi \cdot \lambda\text{bar}_e^{\ 2}} \qquad B = 8.828009833 \times 10^{9}\ \text{tesla}$$

$$\Phi_{E1} := \frac{h \cdot c}{e} \qquad \Phi_{E1} = 1.2398418751 \times 10^{0}\ \text{mvolt} \cdot \text{mm} \qquad E_1 := \frac{\Phi_{E1}}{\pi \cdot \lambda\text{bar}_e^{\ 2}} \qquad E_1 = 2.6465707671 \times 10^{18}\ \frac{\text{volt}}{\text{m}}$$

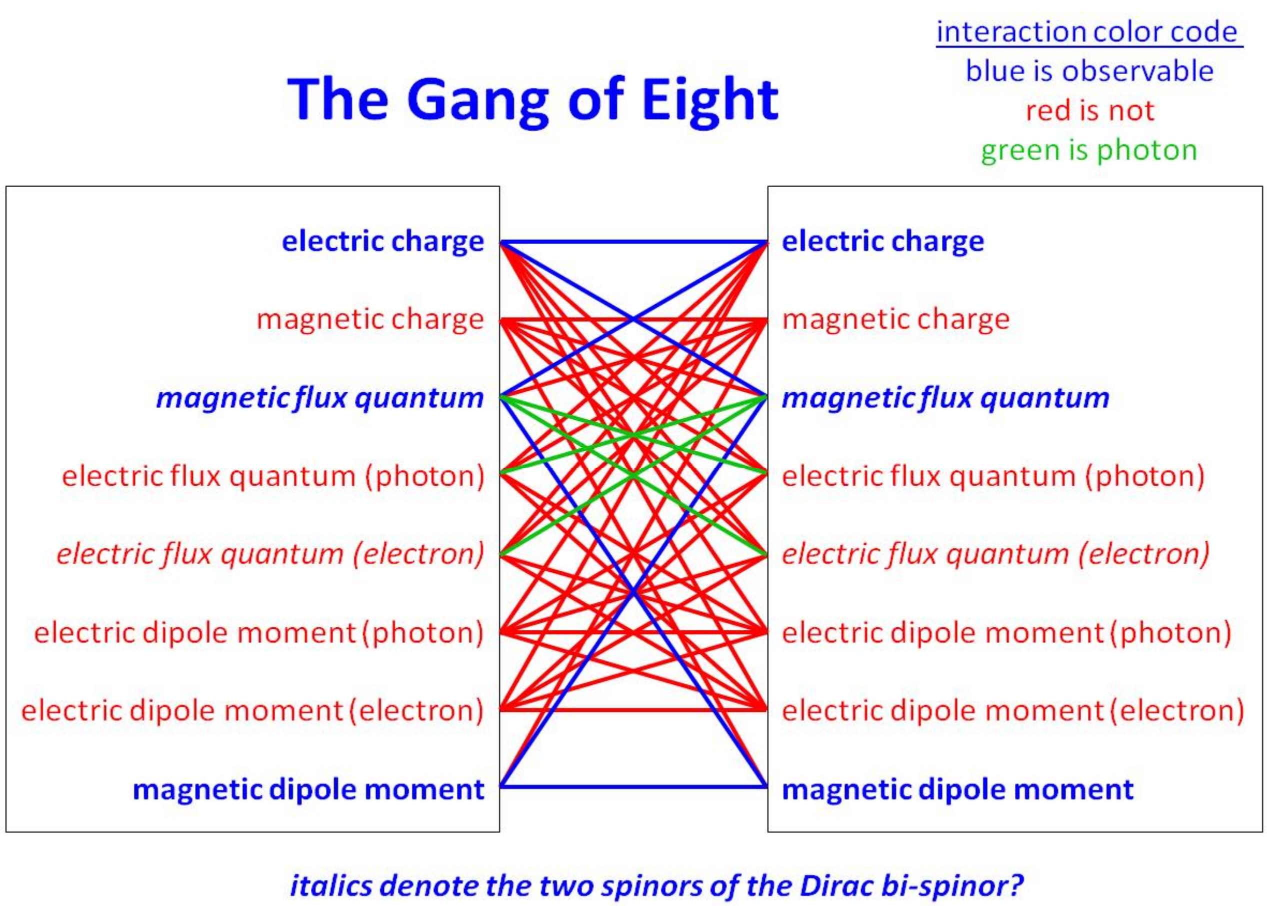
$$\Phi_{E2} := \frac{e}{\varepsilon_0} \qquad \Phi_{E2} = 1.809512651 \times 10^{-2}\ \text{volt} \cdot \mu\text{m} \qquad E_2 := \frac{\Phi_{E2}}{\pi \cdot \lambda\text{bar}_e^{\ 2}} \qquad E_2 = 3.8625919811 \times 10^{16}\ \frac{\text{volt}}{\text{m}}$$

$$\mu_B := \frac{e \cdot \lambda\text{bar}_e \cdot c}{2} \qquad \mu_B = 9.2740091365 \times 10^{-24}\ \frac{\text{joule}}{\text{tesla}}$$

$$d_{Bohr1} := \frac{g \cdot \text{hbar}}{\mu_0 \cdot m_e \cdot c^2} \qquad d_{Bohr1} = 4.2391764 \times 10^{-30}\ \text{m coul}$$

$$d_{Bohr2} := e \cdot \lambda\text{bar}_e \qquad d_{Bohr2} = 6.1869529329 \times 10^{-32}\ \text{m coul}$$

The interactions can be represented graphically, as shown in the figure below.



The electric and magnetic flux quanta comprise the photon and some portion of its interaction with the electron. From these one can present reasonable rationales that permit to calculate the masses of the electron, muon, pion, and nucleon with remarkable accuracy [15]. The calculated mass of the electron is correct at the nine significant digit limit of experimental accuracy, the muon at a part in one thousand, the pion at two parts in ten thousand and the nucleon at seven parts in one hundred thousand.

The role of the remaining members of the Gang of Eight is not yet fully clear. At the Compton radius the energies associated with some of the interactions pictured in the above graphic are shown below.

$$2\mu_B \cdot B = 1.02199782 \times 10^0\,\mathrm{MeV}$$

$$d_{Bohr2} \cdot E_2 = 1.4915756772 \times 10^1\,\mathrm{KeV} \qquad d_{Bohr1} \cdot E_2 = 1.02199782 \times 10^0\,\mathrm{MeV} \qquad d_{Bohr1} \cdot E_1 = 7.0025246458 \times 10^1\,\mathrm{MeV}$$

$$d_{Bohr2} \cdot E_1 = 1.02199782 \times 10^0\,\mathrm{MeV}$$

$$e \cdot E_2 \cdot \lambda bar_e = 1.4915756772 \times 10^1\,\mathrm{KeV} \qquad e \cdot E_1 \cdot \lambda bar_e = 1.02199782 \times 10^0\,\mathrm{MeV} \qquad \frac{g}{\mu_0} \cdot B \cdot \lambda bar_e = 7.0025246458 \times 10^1\,\mathrm{MeV}$$

$$\pi \cdot \varepsilon_0 \lambda bar_e^{\,3} \cdot E_2^{\,2} = 1.4915756772 \times 10^1\,\mathrm{KeV} \qquad \pi \cdot \varepsilon_0 \lambda bar_e^{\,3} \cdot E_1 \cdot E_2 = 1.02199782 \times 10^0\,\mathrm{MeV} \qquad \pi \cdot \varepsilon_0 \lambda bar_e^{\,3} \cdot E_1^{\,2} = 7.0025246458 \times 10^1\,\mathrm{MeV}$$

$$\pi\,\lambda bar_e^{\,3} \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_2 \cdot B = 1.02199782 \times 10^0\,\mathrm{MeV} \qquad \pi\,\lambda bar_e^{\,3} \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_1 \cdot B = 7.0025246458 \times 10^1\,\mathrm{MeV}$$

$$\frac{\pi\,\lambda bar_e^{\,3}}{\mu_0} \cdot B^2 = 7.0025246458 \times 10^1\,\mathrm{MeV}$$

The rightmost two of the three columns in this figure are familiar, corresponding to the 70MeV mass quantum [16,18-20] and, at the limit of experimental precision, twice the mass of the electron. The leftmost column indicates the presence of a 14.9KeV mass quantum, or perhaps half that, 7.45KeV. The absence of this mass quantum from the experimental evidence is notable, and will be commented on later in this note.

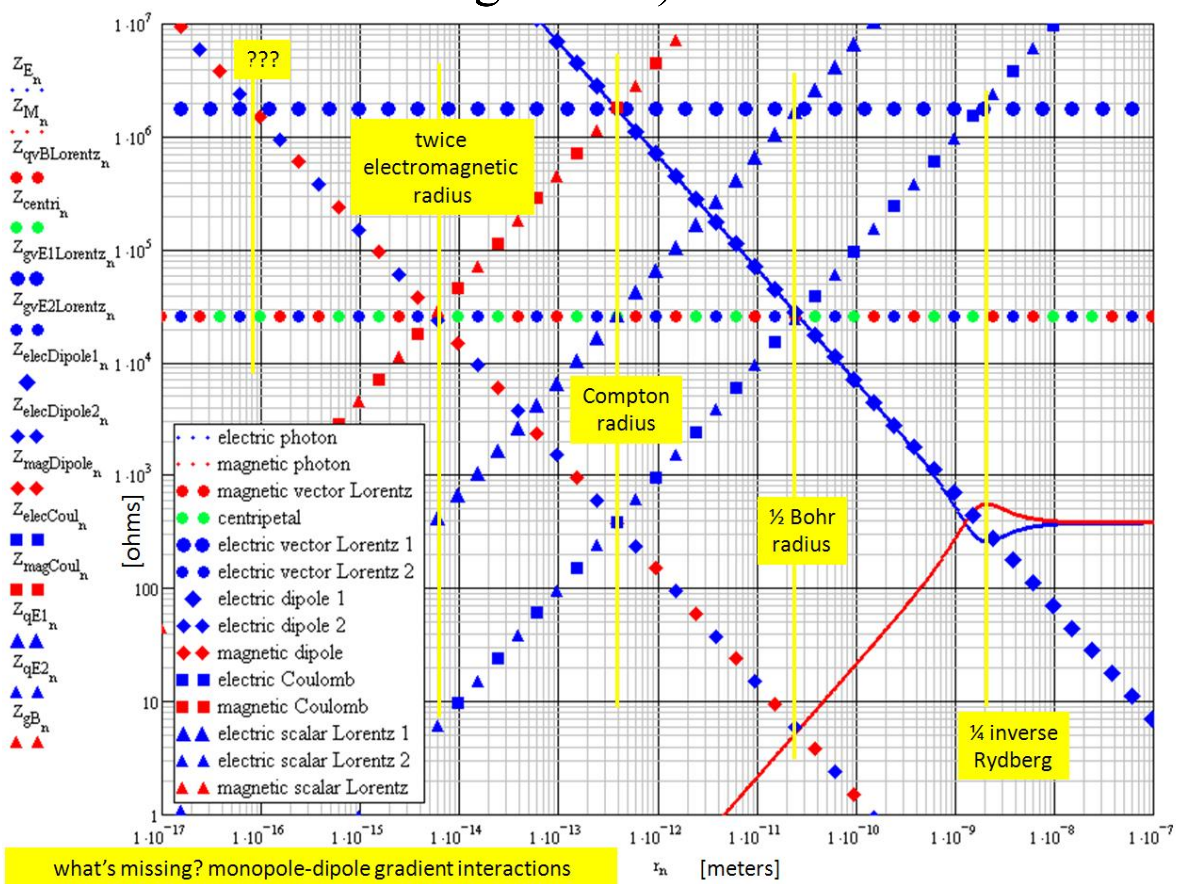
6. The Remaining Impedances

The Gang of Eight interacts via the

- Coulomb impedances – one magnetic, one electric
- vector Lorentz impedances – one magnetic, two electric
- scalar Lorentz impedances – one magnetic, two electric
- dipole – dipole impedances – one magnetic, two electric
- charge – dipole impedances – one magnetic, two electric

From this list we can see that the broken symmetry of the electric dipole and the electric flux quantum is reflected in the impedances.

The present note does not address the last item in the list, the charge-dipole impedances. Impedances for the rest of the interactions have been calculated using the methods outlined earlier, and are plotted in the following figure. In that figure the photon energy is the 13.6eV ionization energy of the ground state hydrogen atom (again with factors of 2 and 4 floating around).



The obvious question – what is one to make of a plot like this? Exploration of any possible meaning of the concept of impedance as applied to an elementary particle is yet at a very early stage. Here we hazard a few comments, with the understanding that what is presented here is, for the most part, speculative.

a) Scaling

Before examining the plot on the previous page in any detail, it is necessary to address the overall distance scaling. The reference dimension is the Compton radius. From that scale everything would be much neater if the impedance junctions were spaced in powers of $1/\alpha$ rather $1/2\alpha$. One wants the impedance crossing to be at the Bohr radius, not half the Bohr radius. And similarly, one wants the wavelength of the photon whose energy is 13.6eV to be at the inverse Rydberg, not one fourth the inverse Rydberg. The present author has devoted some effort to trying to understand the origin of this scale compression factor. As improbable as it may seem, the possibility that it results from the mapping of the bosonic photon onto the fermionic electron seems to be the best explanation so far devised. In any case, the reader is advised to not be too confused by the factors of two and four, but rather mind the ~tildes and focus on understanding the physics.

b) The Potentials

In the plot the dipole impedances become large as we go to progressively smaller length scales, whereas the Coulomb and scalar Lorentz impedances become small, and the centripetal and vector Lorentz impedances are scale invariant. In terms of the corresponding potentials, the dipole potential varies as $1/r^3$, whereas the Coulomb and scalar Lorentz potentials vary as $1/r$. The centripetal potential is inverse square [21], as are the vector Lorentz potentials. In the experience of the author, inverse square potentials are fairly obscure, with limited and confusing insight to be gained by consulting the literature.

c) What Couples to What?

Visual inspection of the plot leads to the following conclusions: In the scale-dependent impedances, electric couples to electric, and magnetic couples to magnetic. There is no electromagnetic coupling. Blue couples to blue, and red couples to red. The coupling is introduced by the scale invariant impedances.

d) Riding the Photon

If we imagine that we are riding on the photon, entering the impedance plot from the lower right at ~377 ohm, the first thing we encounter is the larger of the two electric dipole impedances. This impedance corresponds to the larger of the two electric flux quanta, derived from flux quantization in the photon [15]. Our 13.6eV photon is well matched to that impedance. The diagram suggests that the energy of the photon is somehow transferred to corresponding dipole mode.

It should also be noted that in addition there is a confluence of the larger of the two scale invariant electric vector Lorentz impedances with the Coulomb and the smaller of the two electric scalar Lorentz impedances at the ~inverse Rydberg. While the impedance is mismatched by a factor of $1/4\alpha^2$, the interaction of the modes corresponding to these three impedances opens the possibility that at least some energy will be transferred.

e) Riding the Flux Quanta

As can be seen in the impedance plot, at the ~inverse Rydberg the electric and magnetic flux quanta that comprise the 13.6eV photon decouple, at least in their impedances. The electric flux quantum is coupled to the electric dipole moment. The magnetic flux quantum appears to be flying free. One might consider that, were it not captured an instant later by the magnetic dipole moment at the ~Bohr radius, it would continue on almost forever, neutrino-like [15].

At the ~Bohr radius the energy is delivered to the electron. The magnetic flux quantum couples through the magnetic dipole moment at low impedance [22]. The interaction of the electric flux quantum is much more complex. It appears to involve all of the electrical impedances, at both the higher and highest scale invariant levels.

The presence of the Coulomb impedance suggests a monopole mode, a breathing mode. This mode seems to be absent from the literature. We speculate that it is related to the powers of $1/2\alpha$ scaling problem.

f) The 14.9KeV mass quantum

Earlier the absence of the 14.9KeV mass quantum (or perhaps half that, 7.45KeV) was noted. Examining the impedance plot, we might now gain some understanding of this. The wavelength of a 14.9KeV photon is the Bohr radius. There is no impedance/mode for the photon to match with at that length scale. However, the mismatched modes certainly deserve some attention.

7. Series and Parallel Impedances

In Section 2 it was asserted that the quantum Hall and electric photon impedances are equal at twice the electromagnetic radius. Actually, this is not quite true. The photon impedances at that scale can be calculated as follows:

$$Z_{E2\alpha} := \frac{Z_0 \cdot \left| 1 + \frac{1}{j \cdot 2\alpha} + \frac{1}{(j \cdot 2\alpha)^2} \right|}{\left| 1 + \frac{1}{j \cdot 2\alpha} \right|} \qquad Z_{M2\alpha} := \frac{Z_0 \cdot \left| 1 + \frac{1}{j \cdot 2\alpha} \right|}{\left| 1 + \frac{1}{j \cdot 2\alpha} + \frac{1}{(j \cdot 2\alpha)^2} \right|}$$

$$Z_{E2\alpha} = 2.5807310457 \times 10^4 \text{ ohm} \qquad Z_{M2\alpha} = 5.4994389794 \times 10^0 \text{ ohm}$$

where α is the fine structure constant.

The point here is that, at the scale of the electromagnetic radius of the electron, the quantum Hall impedance is the sum of the electric and magnetic impedances (with perhaps a factor of two floating around).

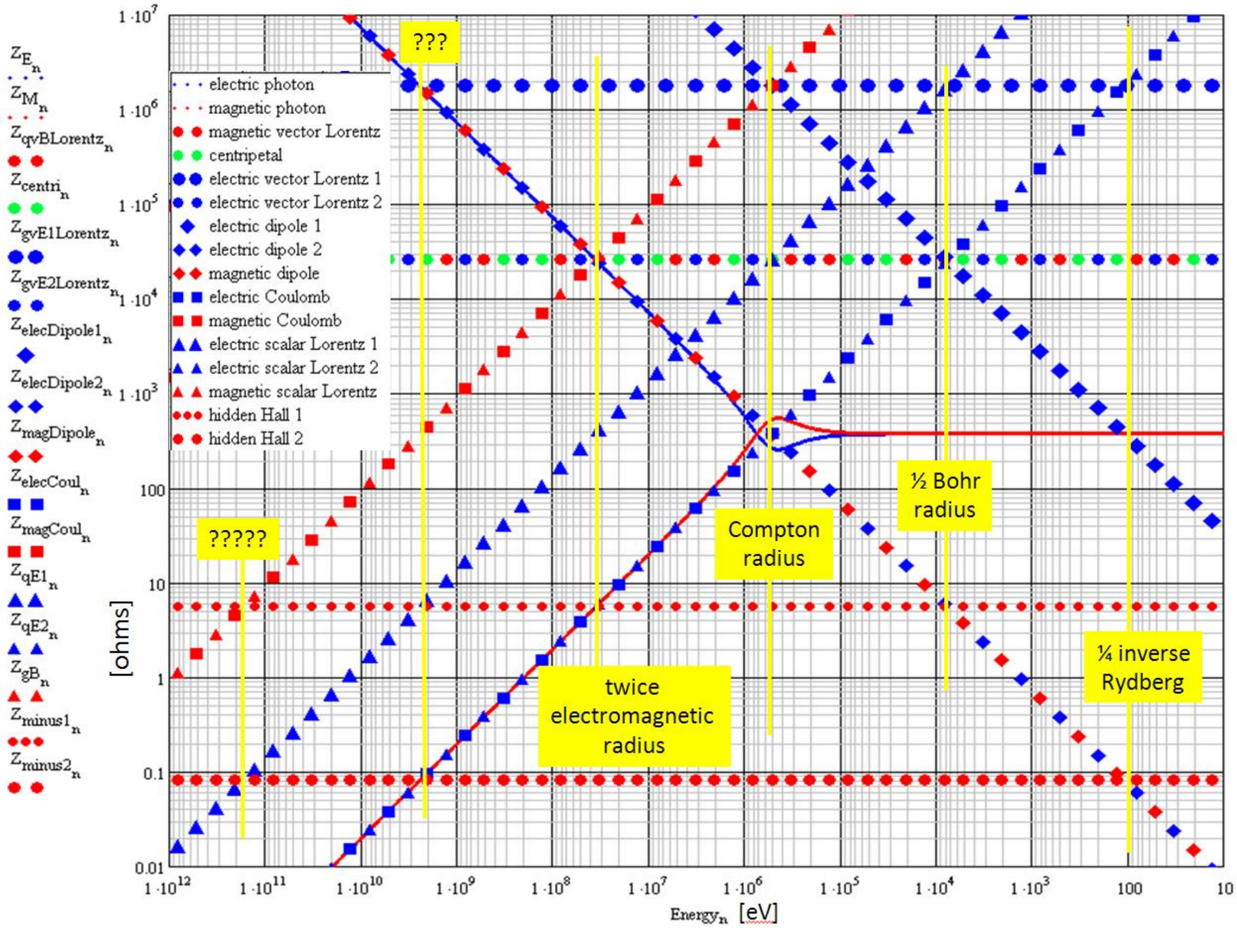
$$Z_{E2\alpha} + Z_{M2\alpha} = 2.5812809896 \times 10^4 \text{ ohm}$$

$$R_H = 2.5812807554 \times 10^4 \text{ ohm}$$

This may be interpreted as having the two impedances in series, which suggests that there may well be another quantum Hall impedance, the parallel impedance.

$$\frac{1}{\frac{1}{Z_{E2\alpha}} + \frac{1}{Z_{M2\alpha}}} = 5.4982673197 \times 10^0 \text{ ohm}$$

From here it is a relatively small step to suggest that the impedance plot should be symmetric with respect to electric and magnetic coupling. Such a circumstance is illustrated in the plot on the next page. The reader is reminded that the charge-monopole impedance is absent from the plot.



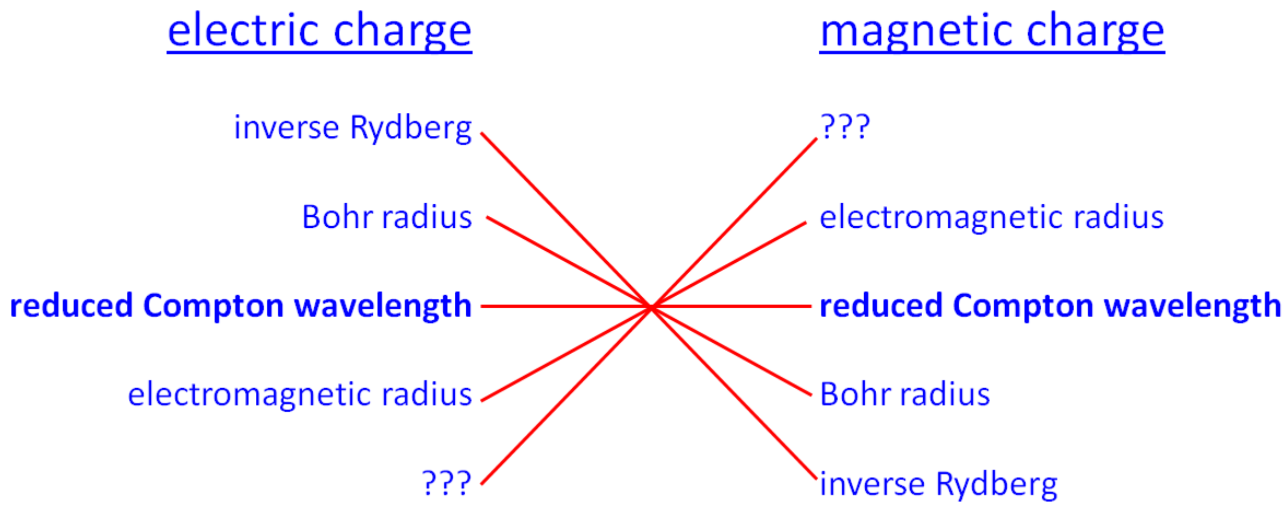
The horizontal scale in this plot is in units of electron volts, rather than meters. It is numerically correct at the .511MeV Compton radius, but varies by factors of two and four as one moves away in powers of 1/2α, towards either the larger or the smaller.

In our present understanding there are three stable particles. The photon and the electron appear here. It is not yet obvious how the proton couples to this plot.

The phenomenon of the unstable particles can be viewed as energy being passed between modes via this impedance network. Such a model requires a more complete understanding of how the modes are confined [24].

8. The Missing Monopole and Dark Matter

A simple explanation exists for the absence of the magnetic monopole from the experimental evidence [23]. To quote from the summary of that paper, "…such a particle has a classical radius larger than its Compton or Bohr radius." This is illustrated schematically below.



Due to the factor of α difference in coupling strengths and the consequent reversed hierarchy of characteristic lengths, the magnetic monopole couples extremely weakly to the photon. The energetics and impedance matches are wrong.

It could be that the magnetic monopole is everywhere. We just can't 'see' it. One wonders whether a similar argument can be advanced for the electric flux quanta and dipole moments, and whether the unstable particle spectrum is comprised at least in part by transient excitations of this 'hidden sector' via the impedance pathways presented here.

9. Conclusions

If any of the above is more than numerical tautology, then the lesson here is that one can't localize the electron beyond a certain limit, defined by its angular momentum. When you try to understand it, you must think of it in terms of every possible way basic electrodynamic objects can interact at the length scale defined by its mass, by its Compton radius, keeping in mind the amplitudes *and phases* of the impedances.

Ignoring the photon impedances, and arbitrarily taking the number of independent impedances to be found at the quantum Hall impedance at three, the Compton radius is distinguished by the fact that it is at the conjunction of ~**twelve** impedances, one more than the Bohr radius. The electromagnetic radius numbers nine. The flow of energy between the associated modes is incredibly complex, but not insoluble.

Finally, it is difficult to ignore the temptation to point out that out there just beyond the lower left hand corner of the last impedance plot, at about 0.1 ohm and 10TeV, there is a conjunction of three magnetic impedances. As can be seen from the central apex of the plot, these impedances couple with the 10TeV 'photon' via the very first impedance it sees, the electron dipole impedance, then again at the Compton radius and then most strongly at the scale labeled '???'.

10. Acknowledgements

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APPENDIX

The Two Body and Mach's Principle

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THE TWO BODY PROBLEM AND MACH'S PRINCIPLE

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The classical analysis of the two-body problem is frequently complicated by the introduction of a system of co-ordinates which is independent of either of the bodies. The validity of such an analysis rests upon the premise that the co-ordinate frame does not interact with the physical system via any known physical laws, and that one is therefore free to choose whatever reference frame seems most useful.

A strong epistemological argument might be advanced against this reasoning. If sufficently rigorous constraints are placed upon the spatial properties of the interacting bodies, the introduction of an independent observer will have a radical effect upon the form of the equations which describe the interaction, to the extent that strongly differing concepts might be developed regarding such fundamental things as space, time, and matter. Newton understood the importance of such considerations, as they caused the delay of the publication of the theory of gravitation some twenty years for lack of a calculus to prove that a spherical mass might be regarded as a point mass.

A familiar example of the two-body problem is the small mass m_1 which is gravitationally bound to a much larger mass m_2. We consider the case of uniform circular motion with m_2 as the center of mass. We require that relativistic corrections be vanishingly small, and in addition that the bodies m_1 and m_2 be 'faceless' and therefore incapable of offering orientational information, possessing intrinsic angular momentum, or generating magnetic fields.

The law of gravitation states that

$$F = dp_1/dt = Gm_1m_2/r^2$$

The common treatment of this problem superposes an independent co-ordinate frame upon the rotating system and determines that dp_1/dt is equal to the centripetal force m_1w^2r.

A technique more in harmony with the spirit of physics would identify the observer with one of the interacting bodies. We might suppose that we are the body m_2, observing the body m_1 at the distance r. We know from prior experience that the force Gm_1m_2/r^2 exists between us and the body m_1, and that this force is equal and opposite to the centripetal force m_1w^2r. A problem arises when we realize that this centripetal force term is meaningless, that given the initial conditions imposed upon this problem it becomes impossible to observe the angular velocity w without a third object of reference. It becomes neccessary to re-examine the law of force.

$$F = d(mv)/dt = m\omega^2 r + v\,dm/dt$$

The first term has no meaning and must be discarded. The second term would also seem to be meaningless. We have no reason to suspect that m_1 varies in time, and nothing in our initial conditions seems to require that m_1 be a point mass, a circumstance which would deprive us of the ability to observe radial velocity. Either we accept the second force term as counter-balancing the gravitational attraction or we regard the whole situation as senseless. Nothing in the initial conditions requires that the problem is senseless, so we write

$$v_{rad}\,dm_1/dt = Gm_1m_2/r^2 \quad \text{or}$$

$$dm_1/dt = (Gm_1m_2/v_{rad})(1/r^2) \qquad (1)$$

In writing this we note that it was neccessary to take $v=v_{rad}$ to maintain the co-linearity of forces. The quantity Gm_1m_2/v_{rad} has units of angular momentum, which suggests

$$dm_1/dt = L/r^2 \qquad (2)$$

We find that the analysis of a simple rotational two-body problem, given the specified constraints, leads to an unfamiliar result. The body m_1 does not succumb to the gravitational attraction and come crashing against us because it possesses a radial velocity we do not observe, and further that it's mass changes for no apparent reason. This situation has some of the tone of the peculiarities we encounter in quantum mechanics. An intuitive understanding of the meaning of equation (1) for a macroscopic system (such as the moon

4

orbiting the earth) can be gained by substituting in the appropriate numerical values, which leads to the conclusion that the sum of the infinitesimal changes dm_1 over one period of revolution is equal to the total mass $m_{1\odot} \times 2\pi$.

The constrained situation presented above is not to be found in reality. The closest we might come is to consider a system composed of elementary particles. The Bohr model of the hydrogen atom is a familiar example. As before, we consider uniform circular motion (n=1), we consider the proton to be the center of mass, and we require that relativistic corrections be negligible and that the intrinsic angular momentum and magnetic fields of the particles be ignored. The law of force is

$$dp/dt = q^2/4\pi e_0 r^2$$

Following the line of reasoning previously developed, we write this as

$$dm_e/dt = (q^2/4\pi e_0 v_{rad})(1/r^2) \qquad (3)$$

which for the Bohr atom n=1 yields

$$dm_e/dt = \hbar/r^2 \qquad (4)$$

a result which is similar in form to the previously analyzed gravitationally bound system.

Several comments should be made on the above results. First, it is interesting to note that the lowest state of the hydrogen atom has no angular momentum, and we are therefore deprived of the simple correspondence between dm/dt and m which was evidenced in the macroscopic system.

Second, in order to obtain any of equations (1) thru (4) it was neccessary that we set $v=v_{rad}$. Although it might be possible to develop some justification for doing this (i.e. space is somehow 'folded' at atomic dimensions with respect to atoms composed of two spinless constituents) it would be more logical to suggest that such a thing would become meaningful at the level of individual elementary particles.

Third, in order to obtain any of the above development we were forced to consider only 'faceless' objects. We can confront this in either of two ways. On the one hand, we can suppose that in the process of isolating a rotating system from the reference reality and examining the interaction in terms of known laws we have begun to develop a technique whereby Mach's principle might be applied. The task would then be to step from the two body problem to a single spinning particle and derive a formulation of Mach's principle, and encapsulate the formulation in such a way that it might be carried back into the two body problem. This would seem to require that we have an understanding of how the particle's properties relate to each other, a condition which seems to render the task impossible at our present level of understanding. Alternatively, it might be that on the quantum level spin-related effects are not amenable to Mach's principle. Such a circumstance would lead to some unusual results. For example, it would be neccessary to consider light as composed of an electric component which experiences the influence of

Mach's principle in one way and a magnetic component which experiences this in another.

Fourth, the only place in theory where we find that mass changes in time in a way that might be related to the changes required by equations (1) thru (4) is in the theory of relativity. Preliminary examination suggests that the establishment of such a relationship might be possible. For example, we might consider the bending of light by a gravitational field. In this case we take v_{rad}=c and write

$$dm_1/dt = Gm_1m_2/cr^2 \quad \text{or}$$

$$dm_1/m_1 = Gm_2dt/cr^2$$

If we take dt=r/c we then have

$$dm_1/m_1 = Gm_2/c^2r$$

Multiplying numerator and denominator of the left side by c and allowing that p=mc, we have

$$dp/p = Gm/c^2r$$

which seems to be the correct form for the deflection of light in a gravitational field.

Finally, I would like to suggest that the local inertial frame of reference of a two body problem, supplemented by the requirement of the 'facelessness' of the constituents, be termed the 'Mach frame' of the problem.

As a last example of the application of the Mach frame, we look at a simple derivation of escape velocity. We equate the differential changes in kinetic and potential energies:

$$d(m_1v2/2)/dt = d(m_1gR)/dt$$

where the mass m_1 is initially at rest at the surface of a

7

body of radius R which generates a gravitational acceleration g. Expanding this equation, we have

$$(m/2)dv^2/dt + (v^2/2)dm/dt = mgdR/dt + mRdg/dt + gRdm/dt$$

We now examine the situation in terms of the understanding developed in the previous examples. We consider that there is a sense in which the escaping body does not leave the surface of the body of radius R, that the escape is simply the act of passing through the 'fold' we have previously mentioned. On the right side of the equation the terms dR/dt and dg/dt are then equal to zero. If we require that the mass m experience no acceleration in passing through this fold, what remains on the left side is once again the product of a velocity we cannot observe with a mass change we have no reason to suspect exists.

$$(v^2/2)dm/dt = gRdm/dt$$

Nonetheless, the correct result follows

$$v^2 = 2gR$$