Mathematical *versus* physical meaning of classical mechanics quantities

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Three principles of Newton's dynamics are considered as basic laws of mechanics theory, expressed in the definite theoretical language. There is presented a new analysis of the language. It is shown that all the laws give meanings of primary mechanics terms and that the sense of every term contains mathematical and physical contents mixed together. It is pointed out that in fact the complete axiom set of the mass point dynamics should include three principles of dynamics as well as axioms of differential and integral calculi. There is emphasized the significance of the relativity principle which sets the way of a new understanding of space, time, rest, motion, and source of motion (force).

The work should be considered as the original analysis of theoretical language itself and as the original study of determinants of the scientific character of physical theory. The scientific character is not legitimated exclusively by mathematical formulae, which allow calculating and measuring. It is guaranteed scarcely by the coherence of

theoretical language which delivers the quantitative and qualitative modeling of phenomena.

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1. Introductory remarks

P. Enders in his recent paper [1] shows that the anticipation of Faraday's and Maxwell's conception of the field one can find in Newton's *Mathematical Principles of Natural Philosophy*. He quotes the foundations of classical mechanics in Newton's original version and next translates words into words used today; especially there are repeated definitions of the following measured quantities: the quantity of matter (inertial mass), the quantity of motion (momentum), the absolute quantity of centripetal force (as gravitational or magnetic force), the accelerative quantity of centripetal force (acceleration caused be centripetal force), the motive quantity of centripetal force (weight proportional to gravitational mass).

Enders claims that Newton's definitions serve to supply the empirical (measured) notions with their mathematical interpretation. To develop this idea we would like to present the more fundamental achievement of Newton. He formulated his three principles of the massive point dynamics in the language containing as geometrical (mathematical) as physical meanings of the primary mechanical notions. Eventually these meanings are inseparable. In this way Newton created a new type of knowledge, i.e., the modern scientific knowledge, lingual entities of which correspond neither to the mathematical intuition nor to the abstraction of phenomena but to a complex of them.

Enders writes that Newton did not give “the mechanism for the propagation for the force (field)”. In the light of our analysis he could not do it because in his three principles of motion there is not introduced the wave motion. He described the motion of (massive) points. So, one can say, that the foretelling of the *wave-corpuscula* dualism appeared firstly in Newton's theory.

Now we shall consider three Newton's principles of dynamics with respect to mathematical and physical meanings of the primary mechanical concepts presented in the principles [2]. Our analysis will show the essential inseparability of mathematical intuition and phenomenal abstraction in the concepts of mechanics theory, and it will allow a better understanding why Newton could not consider simultaneously the mass point dynamics and the propagation of field waves.

2. The fundamental role of the Newtonian relativity principle

The first Principle of Newton's dynamics (I Principle) states:

> *Every body remains in the rest or it will move the uniform motion n a straight line, until the acting force does change this state.*

What it is discussing in the first Principle? Most generally, it is discussing the transition - of motion or rest. In this expression the rest is a state of the regular motion, in the limit of velocity $v = 0$. The force (external, outside, because acting on) which is being talked about in the I Principle, is the source of the transition – of state of motion, so directly – of very motion. The first Newton's Principle is called *the principle of relativity* of the motion; more precisely should be called the principle of relativity of the rectilinear uniform motion and the rest. The relativity, transferred into the structure of space in which the motion takes place, means that the rectilinear uniform motion (uniform translation) in the space is changing nothing; is simply moving all coordinates (with simultaneous even exchange of the numbering of coordinate in frames of reference), i. e. that the rectilinear uniform motion is the invariant of space.

Since in the I Principle there is spoken about the transition of motion, we must stress that the II Principle is *not independent* or even *not autonomous* principle, because it is necessary to ask: What is expressing quantitatively the change, i. e. what is a measure of the change of motion in time? It is the non-zero increase of momentum, i. e. of the product of (inertial) moving body mass m_I and velocity $\underline{v}$, in an interval of time Δt. And so the change is symbolized by means of non-zero $\Delta m_I \underline{v} / \Delta t$, $(\Delta m_I \underline{v} = m_I \Delta \underline{v} = m_I \Delta \underline{x}/\Delta t)$.

3. Is the I Principle scientific?

This quantitative formulating of the change is not bringing scientific plots alone from itself although it has *a translation into the measuring practice*. Very ability of performing the measurement or showing calculating patterns on the basis of measuring data is too little so that these treatments have *scientific character*, both in the meaning of understanding the cognition as the systematized conceptual knowledge and in the meaning of change, motion and force.

The I Principle does not explicitly include any mathematical formula and it seems to have only *qualitative* meaning. The very relativity of the motion declared in the first part of the I Principle has, however, the scientific interpretation, because it is an announcement of the mathematical structure of the space invariant towards *transformations* corresponding to the rectilinear uniform motion. In this case they are given by the group of spatial translations, called transformations of Galileo or Galilean automorphism group, that is of invariants of the Galileo's space. Therefore the I Principle is true *scientific* foundation of Newton's mechanics.

4. The significance of the second Newtonian principle in the geometrization of physics

The second principle of Newton's dynamics (II Principle) has the form:

> *The change of the motion is always proportional to the force acting on the body and it is directed along a straight line appointing direction of the effect of force, according to the sense of force vector.*

The II Principle is comprehending the change of motion, caused by an external force. In view of the established measure of the transition of the motion – the measure of the change of momentum, the change of motion is determined by the mathematical expression:

$$\Delta\, m_I \underline{v} / \Delta\, t = \kappa\, \underline{F},$$

where $\underline{F}$ means the force vector, κ – the constant of proportionality, $\kappa = 1$ in a chosen system of physical units. As the inertial mass m_I is a permanent characteristics of the body, keeping constant when moving, the above formula becomes the following

$$m_I \Delta\, \underline{v} / \Delta\, t = \underline{F}. \tag{1}$$

The change of motion is continuous. Thus the geometrization of motion, *which is postulated* in the I Principle, needs the continuity of space and time. *Since* the space and time are to be continuous, so every change of place in space and time is *continuous*. These changes can be expressed only as the motion in Newton's mechanics is picking a differential geometrical interpretation up, whereas the *physicality* of the motion is contained exclusively in the specific of force $\underline{F}$. In the differential geometry the formula of the II Principle of dynamics [3] is given as follows:

$$m_{\mathrm{I}}\, \mathrm{d}\underline{v}/\mathrm{d}\, t = \underline{F} \text{ or } m_{\mathrm{I}}\, \mathrm{d}^2 \underline{x}/\mathrm{d}t^2 = \underline{F} \tag{2}$$

In this way the motion is being represented by the continuous change in space and expressed with function $\underline{x}(t)$ which is to be smooth and continuous, i. e. differentiable. This function lets the theoretical determination of both past and future states of the motion, without the every measurement of state, provided at least one state of the motion (e. g. initial) is known, i. e. the value of spatial coordinates and velocity.

The I Principle yields the general frame of theoretical language of classical mechanics. In this frame the II Principle defines the velocity change accurately.

5. On the scientific impact of the second Newtonian principle

In which sense the II Principle (2) is a scientific formula? The formula alone is offering the *quantitative* description of motion attributes and the manner of *calculating* them. We wish to point out the important statement concerning Newton's mechanics, namely, that the formula must however belong to a wider conceptual apparatus, in which the measure of motion attribute will have theoretical interpretation, not whereas exclusively *measuring* ones or practical in phenomenal reality. When will it get this? - When it will be given to express the motion in theoretical language of (geometrical) changes of space and time, similarly to the transferring the I Principle (the principle of Galilean relativity) into the properties of space and time.

According to our analysis in the Newton's approach to scientific nature of knowledge neither any computational formulae delivering the quantitative description of phenomena (as (1)) *nor any methods of practical performance of measurement* will be enough for scientific status of knowledge.

6. The primary erms, inseparability and idealization

In the II Principle the quantification of phenomenon should be necessarily modeled in the abstract language of mathematical theory which is becoming the part (in the sense indicated by ourselves) of the scientific physical theory [4].

In the language of scientific physical theory such notions as space, time, mass and force are the primary terms. They should be understood as it is given in three Newton's principles. Time and space are *translated* directly into Euclidean differential geometry, whereas mass and force are *related to them* by means of the II Principle. In Newton's principles, however, the *massive point,* which is representing the moving body, is *manifesting itself* the theoretical fundamental inseparability of mathematical (geometrical) and physical *languages*. The geometrization of moving body path, introducing it as the one-dimensional line in the three-dimensional space, is *another* example of this inseparability.

So, in the scientific physical theory every notion is theoretical; every, in the comparison with its physical (phenomenal) equivalent, is an idealization [5]. An idealization of the body moving in space and time is an outstanding example: the massive point, the linear curve of motion path. Nothing, what is real in three-dimensional space and one-dimensional time, can be immediately identified with abstracts of points and lines. On the other hand, to give the accurate II Principle, it must consider the closely geometrical individual, i.e., the point, however marked with physicality – i.e., inertial mass.

7. The distinguished role of third Newtonian principle in the conservation of quantity of motion

The third principle of Newton's dynamics (III Principle) states:
The equivalent counteraction, but on the contrary directed, corresponds to every action, i.e. the mutual effects of two bodies on oneself are always equal and directed opposite.

The third Principle is usually called the law of action and reaction. This principle is of more general character *in comparison to previous ones*. Inasmuch as in the first and second Principle it was about attributes of motion. The III Principle is telling about the action as such, *without* the quality specification (not recalling already about quantitative characteristic) of action. It is telling about the conservation of motion in the action. The action is not defined here. It is fitting so to treat them commonly. And here we would have exclusively external character of power, exclusively an external source of the mechanical motion, and the new Newton's idea of exclusively physical action (not spreading other kinds of action, e. g. biological). In Hamiltonian or Lagrangean formulating of Newton's mechanics this way understood action will become the primary term, *not having any empirical and measurable interpretation*, i. e. no observable quantity corresponds to the action. But in Lagrangean or Hamiltonian integrals which express the action there is contained all the knowledge about dynamics of considered system of bodies and forces as well as about conservation laws.

8. Why mathematics is not sufficient for the modeling of a phenomenon?

The inseparability of mathematical intuition and phenomenal abstraction in the language of Newtons Principles of dynamics allows to answer the question raised in Enders' article. In the Newton's theory there is engaged the corpuscular picture of the world, i. e. massive points and forces acting on. All such conceptualized motions are described by means of differential geometry. The propagation of field is also a continuous phenomenon. It is, however, the motion conceptualized in another way, not reduced to massive points displacements. So, the *mathematics* (differential geometry) *alone is not sufficient to modeling the phenomenon*: The *physical abstract* in the form of chosen motion picture (of moving points or of propagating waves) is *necessary* to constitute the theoretical *physical* quantities.

According to our interpretation we can assume that this is the reason why Newton wrote on centripetal forces as follows: “For I here design only to give a mathematical notion of those forces, without considering their physical causes and seats” [6]. This statement is fully explained only in our presented approach, whereas the Enders' comments are of purely lingual nature and do not touch the essence of Newton's physics. He presents extensive quotations from Newton's "Principia" and makes translation of original Newton's words (of XVIIth) into modern language in physics. The example is Enders' suggestion that the above quoted Newton's statement entitles to conclude that: “It thus is incorrect to connect the physical meaning of 'action-at-distance' with Newton"[7]. Such one-statement declaration could not replace the complex chain of deduction in physics.

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References

[1] P. Enders, Precursors of force field in Newton's 'Principia', Apeiron **17** (1) (2010) 22-27.

[2] M. Zabierowski, The scientific method of Newtonian mechanics and the demarcation criterion (in Polish), Acta Academiae Paedagogicae Cracoviensis **60** (2008) 11-20.

[3] Our analysis concerns the today form of Newtonian principles. Therefore it is not here necessary to mention that in 1736 L. Euler firstly published Newton's equations in explicit form.

[4] K. Ajdukiewicz, Das Weltbild und die Begriffsapparatus, Erkenntnis **IV** (1934) 259-287; T. Grabińska, Philosophy in Science, Publ. Wrocław Technology Univ., Wrocław 2003.

[5] T. Grabińska, Idealization and reality, Proceedings of MSB'2000 Conference, Publ. Jagiellonian Univ., Kraków 2000, p. 315-323.

[6] I. Newton, Mathematical Principles of Natural Philosophy, London 1729, Definition VIII.

[7] Enders, op. cit., s. 25.