Simple Considerations on the Cosmological Redshift

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Generally, the cosmological redshift is considered as an indication of the expansion of the universe. However, we are going to consider, using very elemental arguments, that this physical phenomenon could be due to the gravitational field, and that the universe could be static and flat.

Key words: Light redshift, gravity.

1. Introduction

In the theory of the Big Bang, the redshift of the light emitted from distant galaxies is interpreted as a Doppler effect and then considered as an indication of the expansion of the universe, following the law of Hubble.

However, we are going to consider, using very simple arguments, that the nature of that redshift could be related with the gravity and not with that expansion.

2. The light redshift as a Doppler effect

We suppose a light source that emits a photon and that it is moving with relative constant velocity v with respect to an observer, then from the special relativity theory (SRT) of Einstein we have that

$$\Delta t' = \Delta t \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} \tag{2.1}$$

being $\Delta t'$ and Δt the intervals of time in the moving and rest frames, respectively, and c the velocity of the light in the vacuum. From (2.1), which is the equation of the dilatation of the time, considering frequencies, we have

$$\frac{1}{\nu'} = \frac{1}{\nu}\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

$$\nu = \nu'\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} \tag{2.2}$$

being ν' and ν the light frequencies emitted and observed of the photon, respectively. Developing (2.2)

$$\nu = \nu'\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} = \frac{\nu'}{c}\left(c^2 - v^2\right)^{\frac{1}{2}} = \frac{\nu'}{c}\left(\left(c+v\right)\left(c-v\right)\right)^{\frac{1}{2}} = \nu'\frac{c+v}{c}\left(\frac{c-v}{c+v}\right)^{\frac{1}{2}}$$

For $v << c$

$$\nu = \nu'\left(\frac{c-v}{c+v}\right)^{\frac{1}{2}} \tag{2.3}$$

If the light source is moving away from the observer, v is positive, $\nu < \nu'$, and the light is red shifted. If the light source is moving toward the observer, v is negative, $\nu > \nu'$, and the light is blue shifted.

3. Gravitational redshift

For a particle of mass m in a gravitational field with potential φ we have that

$$E = m\,c^2 \tag{3.1}$$

$$V = m\varphi \tag{3.2}$$

being E and V the relativistic and potential energies of the particle, respectively. Therefore, for a photon (assuming an "effective mass" $m = \frac{E}{c^2}$)

$$E = h\nu \tag{3.3}$$

$$V = \frac{h\nu}{c^2}\varphi \tag{3.4}$$

being h the constant of Planck. As the gravitational field is conservative, then

$$T + V = h\nu + \frac{h\nu}{c^2}\varphi = const. \tag{3.5}$$

$$h\nu_0 + \frac{h\nu_0}{c^2}\varphi_0 = h\nu + \frac{h\nu}{c^2}\varphi \tag{3.6}$$

$$\nu_0\left(1 + \frac{\varphi_0}{c^2}\right) = \nu\left(1 + \frac{\varphi}{c^2}\right)$$

$$\frac{\nu_0}{\nu} = \frac{c^2 + \varphi}{c^2 + \varphi_0} \tag{3.7}$$

where $T = h\nu$ is the kinetic energy of the photon, ν and ν_0 the light frequencies emitted and observed, respectively, φ and φ_0 the potentials in the points of emission and observation, respectively. The gravitational potential φ varies with the inverse of the distance and always is $\varphi < 0$, only $\varphi(\infty) = 0$. If $|\varphi_0| < |\varphi|$, then $\nu_0 < \nu$, and the light is red shifted, and $h\nu_0 < h\nu$. As *V* increases (it is less negative) then *T* decreases. But, if $|\varphi_0| > |\varphi|$, then $\nu_0 > \nu$, and the light is blue shifted. Now, introducing the light redshift parameter *z*, we have

$$z = \frac{\nu - \nu_0}{\nu_0} = \frac{\nu}{\nu_0} - 1 \tag{3.8}$$

where $z > 0$ when $\nu_0 < \nu$; $z + 1 = \frac{\nu}{\nu_0}$, $\frac{\nu_0}{\nu} = \frac{1}{z+1}$,

$$\frac{\nu_0 - \nu}{\nu} = \frac{\nu_0}{\nu} - 1 = \frac{1}{z+1} - 1 = -\frac{z}{z+1} \tag{3.9}$$

$$\frac{\nu_0 - \nu}{\nu} = \frac{\nu_0}{\nu} - 1 = \frac{c^2 + \varphi}{c^2 + \varphi_0} - 1 = \frac{\varphi - \varphi_0}{c^2 + \varphi_0} \tag{3.10}$$

For $|\varphi_0| << c^2$

$$\frac{\varphi - \varphi_0}{c^2} = \frac{\nu_0 - \nu}{\nu} = -\frac{z}{z+1}$$

$$\varphi - \varphi_0 = c^2 \frac{\nu_0 - \nu}{\nu} = -c^2 \frac{z}{z+1} \tag{3.11}$$

4. The cosmological redshift

Within the framework of the general relativity theory (GRT) of Einstein, we assume that in a gravitational field the interval *ds* has the form

$$ds^2 = c^2\, d\tau^2 - a^2(\tau)\, b^2 \left(dx^2 + dy^2 + dz^2\right) = c^2\, d\tau^2 - a^2(\tau)\, b^2\, dr^2 \quad (4.1)$$

being τ the proper time, $a(\tau)$ the scale factor, *b* a constant to infer, *x*, *y* and *z* the space coordinates and *r* the distance.

In [1] (p. 103), *b* is an integration constant. Note that for $b = 1$, (4.1) is the square of the space-time interval of Robertson and Walker with $k = 0$, being *k* the curvature constant [2]. In [1] (p. 103), $k = 0$ is the only possible value.

From, $ds^2 = 0$, we obtain that the velocity of the light is

$$\frac{dr}{d\tau} = \frac{c}{a(\tau)\, b} \quad (4.2)$$

And then

$$\frac{d\tau}{a(\tau)} = \frac{b\, dr}{c}$$

Now, from [1] (pp. 105-106), we put the observation point at the origin of the reference system, $r = 0$. We consider two photons emitted from the point *r* in the times τ and $\tau + d\tau$ and that arrive at the point $r = 0$ in the times τ_0 and $\tau_0 + d\tau_0$, respectively. Then, for the photon emitted at the moment τ and that arrives at the moment τ_0 we have that

$$\int_{\tau}^{\tau_0} \frac{d\tau}{a(\tau)} = \int_{r}^{0} \frac{b\, dr}{c} = \frac{b}{c} \int_{r}^{0} dr = \frac{b}{c} [r]_r^0 = \frac{b}{c}(0 - r) = -\frac{b\, r}{c}$$

Also, for the photon emitted at the moment $\tau + d\tau$ and that arrives at the moment $\tau_0 + d\tau_0$ we have that

$$\int_{\tau+d\tau}^{\tau_0+d\tau_0} \frac{d\tau}{a(\tau)} = \int_{r}^{0} \frac{b\,dr}{c} = -\frac{b\,r}{c}$$

Consequently

$$\int_{\tau}^{\tau_0} \frac{d\tau}{a(\tau)} = \int_{\tau+d\tau}^{\tau_0+d\tau_0} \frac{d\tau}{a(\tau)}$$

$$F(\tau_0) - F(\tau) = F(\tau_0 + d\tau_0) - F(\tau + d\tau)$$

$$F(\tau + d\tau) - F(\tau) = F(\tau_0 + d\tau_0) - F(\tau_0)$$

$$d\tau\; F'(\tau) = d\tau_0\; F'(\tau_0)$$

$$\frac{d\tau}{a(\tau)} = \frac{d\tau_0}{a(\tau_0)}$$

And using the light angular frequencies, $\omega = \dfrac{d\theta}{d\tau}$ and $\omega_0 = \dfrac{d\theta}{d\tau_0}$, being θ the angle, we have

$$\frac{d\theta}{a(\tau)\,\omega} = \frac{d\theta}{a(\tau_0)\,\omega_0}$$

$$a(\tau)\,\omega = a(\tau_0)\,\omega_0$$

$$\omega = \frac{a(\tau_0)}{a(\tau)}\,\omega_0 \tag{4.3}$$

Hence, the light angular frequency emitted $\omega = 2\pi\nu$ is not equal to the light angular frequency observed $\omega_0 = 2\pi\nu_0$. Now, for the light redshift parameter, we have

$$z = \frac{\omega - \omega_0}{\omega_0} = \frac{\omega}{\omega_0} - 1 = \frac{a(\tau_0)}{a(\tau)} - 1 \qquad (4.4)$$

And $z > 0$ if $a(\tau_0) > a(\tau)$. Also, $z > 0$ if $\omega > \omega_0$, that is, if $h\nu > h\nu_0$.

But also we have that

$$ds^2 = c^2\, d\tau^2 - a^2(\tau)\, b^2 \left(dx^2 + dy^2 + dz^2\right) = c^2\, d\tau^2 - a^2(\tau)\, b^2\, dr^2$$

$$= c^2\, d\tau^2 - d\ell^2 \qquad (4.5)$$

being $d\ell$ the length of an "arc" measured in the surface of "radius" $a(\tau)$ and dr is the length of an "angle" multiplied by b, that is, $arc = radius \times angle$ or $d\ell = a(\tau)\, b\, dr$.

Note that now the velocity of the light would be

$$\frac{b\, dr}{d\tau} = \frac{c}{a(\tau)} \qquad (4.6)$$

In the Big Bang model, $a(\tau)$ increases with the time, and as $\tau_0 > \tau$, then $a(\tau_0) > a(\tau)$, therefore $z > 0$ and $\omega > \omega_0$ and also $d\ell = a(\tau)\, b\, dr$ increases, hence the universe expands, and the light is red shifted.

On the other hand, for the stationarity of (4.5), it would be required that $a(\tau)\, b = const.$, which implies that $b = 0$. Also, as the scale factor $a(\tau)$ is considered like a "curvature radius" it is infinite for a flat surface [3] (p. 479).

Then, in our flat $\left(a\left(\tau\right)=\infty \right)$ and stationary $\left(b=0 \right)$ model, $\frac{a\left(\tau_0\right)}{a\left(\tau\right)}=\frac{\infty}{\infty}=$ *undetermined* and also $d\ell = a\left(\tau\right) b\, dr = \infty\, 0\, dr =$ *undetermined*, therefore the universe does not expand. Then, $z>0$ only because the gravity can make that $\omega > \omega_0$ (when $\left|\varphi\right| > \left|\varphi_0\right|$), equation (3.11), which is derived for a weak gravitational field in the framework of the GRT in [3] (p. 349).

Now, from [3] (p. 486), we put $\frac{\omega}{\omega_0}=\frac{a\left(\tau_0\right)}{a\left(\tau\right)}$ in the form $\frac{\omega}{\omega_0}=\frac{a\left(\tau\right)}{a\left(\tau-\Delta\tau\right)}$, therefore

$$\frac{\omega_0}{\omega}=\frac{a\left(\tau-\Delta\tau\right)}{a\left(\tau\right)}=1-\frac{1}{a\left(\tau\right)}\frac{da\left(\tau\right)}{d\tau}\Delta\tau=1-H\frac{\Delta\ell}{c}$$

where $H=\frac{1}{a\left(\tau\right)}\frac{da\left(\tau\right)}{d\tau}$ is the constant of Hubble and $\Delta\ell = c\,\Delta\tau$.

Then

$$\frac{\omega_0-\omega}{\omega}=\frac{\omega_0}{\omega}-1=-\frac{H\,\Delta\ell}{c} \tag{4.7}$$

and making

$$\frac{\omega_0-\omega}{\omega}=-\frac{v}{c} \tag{4.8}$$

we have the law of Hubble

$$v=H\,\Delta\ell \tag{4.9}$$

being *v* the velocity of recession.

Note that for $v << c$, the Lorentz transformation: $x' = (x - vt)\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$, $t' = \left(t - \frac{v}{c^2}x\right)\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$, is converted into the Galileo transformation: $x' = x - vt$, $t' = t$, hence $\frac{dx'}{dt} = \frac{dx}{dt} - v$, and for the case of the light, $c' = c - v$. Then, for $v << c$ (which is in accordance with a weak gravitational field)

$$\frac{\omega_0 - \omega}{\omega} = \frac{2\pi\nu_0 - 2\pi\nu}{2\pi\nu} = \frac{\nu_0 - \nu}{\nu} = \frac{\frac{c-v}{\lambda} - \frac{c}{\lambda}}{\frac{c}{\lambda}} = -\frac{v}{c}$$

being λ the wavelength.

And from (3.9) and (4.8)

$$\frac{z}{z+1} = \frac{v}{c} \tag{4.10}$$

For $z << 1$, we have, from (4.10), $z = \frac{v}{c}$ and $v << c$, and, from (3.11), $\varphi - \varphi_0 = -c^2\ z$.

In the Big Bang model, the valid equation is (4.10), with (4.4) and (4.9), but in our flat ($a(\tau) = \infty$) and stationary ($b = 0$) model, the valid equation would be (3.11), with (3.8).

Note that for $b = 0$

$$v = H\ \Delta\ell = \frac{1}{a(\tau)}\frac{da(\tau)}{d\tau}a(\tau)\,b\ \Delta r = \frac{da(\tau)}{d\tau}b\ \Delta r = \frac{da(\tau)}{d\tau}0\ \Delta r = 0$$

(4.11)

which implies that the model is also static.

The equation (4.10) gives the value of the cosmological redshift due to the expansion of the universe, in which the space itself is expanding following the law of Hubble. But, the equation (3.11) gives the value of the light redshift caused by the force of attraction of the gravity, in a static universe.

Now, from [3] (pp. 479-480), in the context of the limit of the flat model of Friedmann, as for our model, $a(\tau)\,b = const.$ and $a(\tau) = \infty$ and $b = 0$, we substitute $a(\tau)\,b$ by a time function $e(\tau)$ and put the interval *ds* like

$$ds^2 = c^2\, d\tau^2 - e^2(\tau)\left(dx^2 + dy^2 + dz^2\right) \tag{4.12}$$

and for each τ, $e(\tau)$ has a constant value that can be reduced to the unity by means of a simple transformation of coordinates (proportional to $\dfrac{1}{const.}$). For low values of the pressure *P* it is found that $\rho\, e^3(\tau) = const.$, being $\rho = \dfrac{const.}{\tau^2}$ the mass density, then $e(\tau) = const.\,\tau^{\frac{2}{3}}$. And, for low values of τ it is $P = \dfrac{\rho\, c^2}{3}$, and it is found that $\rho\, c^2\, e^4(\tau) = const.$, then $e(\tau) = const.\,\tau^{\frac{1}{2}}$. For $\tau = 0$, $e(0) = 0$ and $\rho(0) = \infty$, which would be the mass density of a volume of matter of radius zero.

Therefore, with these equations our flat ($a(\tau) = \infty$), stationary ($b = 0$) and static ($v = 0$) model of the universe would be in the limit of the flat model of Friedmann ($\tau = \infty$, $R(\infty) = \infty$ and

$v = v_e(\infty) = \left(\frac{2GM}{R(\infty)}\right)^{\frac{1}{2}} = \left(\frac{2GM}{\infty}\right)^{\frac{1}{2}} = 0$, being G the gravitational constant of Newton, and $v_e = \left(\frac{2GM}{R(\tau)}\right)^{\frac{1}{2}}$, $R(\tau)$ and M the escape velocity, the radius and the mass of the universe, respectively).

5. Discussion

For $v << c$, we can consider that the light is red shifted because its velocity would be $c - v$, then the kinetic energy of the photon, $h\nu$, decreases. But v can be a relative velocity away from the observer (Doppler effect), or the recession velocity (Hubble law), or a velocity related with the force of attraction of the gravity and obtained by comparison between (3.11) and (4.10)

$$v = -\frac{\varphi - \varphi_0}{c} \tag{5.1}$$

If the gravitational field is much weaker in the point of observation than in the point of emission, then $|\varphi_0| << |\varphi|$ and $v = -\frac{\varphi}{c} = \frac{GM}{R\,c}$, being M and R the mass and the radius of the light source, respectively.

Now, for two supposed identical galaxies, at rest, at distances d_1 and d_2, with apparent radii R_1 and R_2, and for $z << 1$, we have, for an expanding universe, that

$$\frac{v_1}{v_2} = \frac{H\ d_1}{H\ d_2} = \frac{d_1}{d_2} \tag{5.2}$$

and, for a static universe, when $|\varphi_0| << |\varphi|$, that

$$\frac{v_1}{v_2} = \frac{\frac{GM}{R_1\ c}}{\frac{GM}{R_2\ c}} = \frac{R_2}{R_1} \tag{5.3}$$

but, as

$$\frac{R_2}{R_1} = \frac{d_1}{d_2} \tag{5.4}$$

then, both velocities, $v = H\ d$ and $v = \frac{GM}{R\ c} = \frac{GM}{R\ d\ c}\ d$, would give the same value of the redshift, $z = \frac{v}{c}$. Therefore, both red shifts would be interchangeable apparently, but the one due to the recession velocity exists only if the universe is expanding, while the one caused by the velocity related with the gravity exists always. This could be favorable to the static universe model. Although, for high values of the z parameter, it would be required to use a velocity for high gravitational fields.

Note that the velocity $v = -\frac{\varphi}{c} = \frac{GM}{R\ c}$ would be the escape velocity of a photon, because from (3.5), and adapting the escape velocity concept for a photon (with $m = \frac{E}{c^2} = \frac{h\nu}{c^2}$), we have

$$\frac{h\ v_{eph}}{\lambda} + \frac{h\ c}{\lambda\ c^2}\varphi = 0$$

$$v_{eph} = -\frac{\varphi}{c} = \frac{GM}{R\,c} \qquad (5.5)$$

v_{eph} being the escape velocity of a photon of wavelength λ emitted by a light source of mass M and radius R.

6. Conclusion

We have seen first the light redshift as a relativistic (SRT) Doppler effect, where the light is red shifted by a relative constant velocity away from the observer. After, we have seen the gravitational redshift, where the light is red shifted by the gravity and the law of conservation of the energy. And for last, in a GRT context, we have seen the cosmological redshift, in two models: the Big Bang and in our flat, stationary and static as a limit of the flat model of Friedmann. In the Big Bang model the light is red shifted by the expansion of the universe, in which the space itself is expanding following the law of Hubble. In our proposed flat, stationary and static model the light is red shifted by the force of attraction of the gravity, and the expansion of the universe would not be real but only apparent. In both models the values of the cosmological redshift are the same. Hence, the expansion of the universe would not be the only explanation of this physical phenomenon.

References

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