Unified Field Theory – Part II of Paper I

Gravitational, Electromagnetic, Weak & the Strong Force.

Golden Gadzirayi Nyambuya
North-West University -Potchefstroom Campus,
School of Physics - Unit for Space Research,
Potchefstroom, Republic of South Africa.
Email: gadzirai@gmail.com

The present paper is a continuation of the earlier paper Nyambuya (2007b) and the reader is advised to have both this paper and the present as they constitute a single unit paper. The paper was split into two for reasons of space and hence thus this paper starts off from section 10 and Nyambuya (2007b) ends with section 9.

10 Electromagnetic Force

Equations 42 and 54 reproduces the first and second group of Maxwell's equations exactly. The first group of equations emerges provided a suitable definition of J_μ is found. The task of finding this suitable definition will be left for Paper II where a connection between and QM and Classical Physics is sought. In order to obtain the Electromagnetic field tensor from equation 48, we must have $g^*_\mu = e$ for all $\mu = 0, 1, 2, 3$ where e is a constant, hence $g^*_{\mu\nu} = 0$. The resulting field tensor is then given by

$$\mathrm{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{55}$$

which as is well known, is invariant under the Gauge transformation

$$A_\mu \longmapsto A_\mu + \partial_\mu \chi, \tag{56}$$

which for all purposes is Maxwellian Electromagnetic field tensor and the condition equation 56 is said to be $U(1)$ Gauge and its effect of leaving the tensor equation 55 means the Maxwell's theory is $U(1)$ gauge invariant. The above transformation equation 56 leaves the Lagrangian

$$\mathcal{L} = \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}, \tag{57}$$

invariant. This Lagrangian is in actual fact the kinetic energy of the Electromagnetic field. When one looks at equation 42 and compares this to the exact Maxwell equation, one sees immediately that the Lorentz Gauge Condition $\partial^\mu A_\mu = 0$ is in fact automatic, that is, it is in-built into the system of equations.

At this point it is important to make the realization that the quantity $\mathcal{L} = \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}$ is in actual fact the field energy and is a scalar. Its scalar nature makes compulsory the requirement that it be invariant under any-kind of transformation. If the field changes in such a way that the field energy remains invariant, this would constitute a gauge transformation and there will exist different sets of these transformations which in actual fact are different rotational states of spacetime. A gauge transformation is a transformation that changes the field in such a way that the field energy remains invariant. The case $g^*_\mu = e$ for all μ causes the energy field $\mathcal{L} = \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}$ to be obey $U(1)$ symmetry. We shall generalize this, that for any configuration or setting of the g^*_μ's, the quantity $\mathcal{L} = \mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}$ must remain invariant, this as shall be shown in the next sections, leads to the energy field to obeying $SU(2)$ and $SU(3)$ and also $SU(4)$ symmetry hence representing the Weak, Strong and a new force which I shall coin the Super force which is $SU(4)$ invariant respectively.

11 Strong Force

We proceed further to identify the equations describing $SU(3)$ Gauge Fields which off cause is the Strong force. Now, if just one of the g^*_μ is zero and the rest are none-zero the $g^*_{\mu\nu} = g^S_{\mu\nu}$ will have none-zero values. With this setting, then we will have as-well three none-zero components of the vector A_μ. Let us write this Vector as G_μ instead of A_μ because this makes one to think of the Electromagnetic force. There are only four ways in which to distribute the zero amongst the indices of g^*_μ, that is $(0, g^*_1, g^*_2, g^*_3)$, $(g^*_0, 0, g^*_2, g^*_3)$, $(g^*_0, g^*_1, 0, g^*_3)$ and $(g^*_0, g^*_1, g^*_2, 0)$ hence four combinations of the fields, that is $(0, G_1, G_2, G_3)$, $(G_0, 0, G_2, G_3)$, $(G_0, G_1, 0, G_3)$ and $(G_0, G_1, G_2, 0)$, let us write these in a more compact way as G^i_μ where is this case $i = 1, 2, 3, 4$. For each of the components G^i_μ, if we assume that we can exchange the components without exchanging the position of the zero, for example $(0, A_1, A_2, A_3)$, $(0, A_1, A_3, A_2)$, $(0, A_1, A_2, A_3)$, $(0, A_1, A_3, A_2)$, $(0, A_1, A_2, A_3)$ and $(0, A_1, A_3, A_2)$, there we will have six combinations of the Gauge Fields, let us add another index for this, that is G^{ij}_μ where $j = 1, 2, 3, ..., 6$ we will have for the field tensor

$$\mathrm{F}^{ij}_{\mu\nu} = \partial_\mu G^{ij}_\nu - \partial_\nu G^{ij}_\mu + g^S_{\mu\nu} G^{ij}_\mu G^{ij}_\nu . \tag{58}$$

Now somewhat handwavingly, I shall try to fit this to what we already know. We know that there are six quarks (u,d,s,c,b,t), the i then can be thought of as representing the flavors u,d,s,c,b,t. Quarks have color and if we are to identify the number j with quark color, we run into a little problem because i runs from 1 to 4 meaning there should be four colors but we only know of three colors – Red, Green and Blue. The solution to this would be that the present theory is predicting a forth color of the quarks which we can call the neutral color charge. We shall not worry much about this for now until Paper VI thus we shall proceed and take $i = 1, 2, 3, 4$ to represent the color of the quarks and accept the prediction of the theory that there must exists a forth color for the quarks.

Now if the field energy, $\mathcal{L} = \mathrm{F}^{ij}_{\mu\nu}\mathrm{F}^{ij\mu\nu}$ is to be invariant under some appropriate Gauge transformation, then the Gauge Fields G^{ij}_μ should submit to the decomposition

$$G^{ij}_\mu = T^{ik}_S G^{ijk}_\mu , \tag{59}$$

where G^{ijk} are the generators of the $SU(3)$ group and T^{ik}_S are 3×3 Gell-Mann matrices written in four dimensions and $k = 1, 2, 3, ..., 8$.

The normal 3×3 Gell-Mann matrices are given

$$
\begin{array}{ccc}
\frac{1}{2}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
\frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} & \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\
\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} & \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} &
\end{array}, \tag{60}
$$

and there will be as many 4×4 Gell-Mann matrices as there are the i's and these will be obtained by appropriately placing zeros in the rows and columns of the the three dimensional Gell-Mann matrices. For the Gauge Field resulting form the the case $g_0^* = 0$ we have

$$T_S^{1k} =$$

$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$
$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \quad \frac{1}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \tag{61}$$

and for the case $g_1^* = 0$ we have

$$T_S^{2k} =$$

$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$
$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \quad \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \tag{62}$$

and for the case $g_2^* = 0$ we have

$$T_S^{3k} =$$

$$\frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} ,$$

$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \quad \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \tag{63}$$

and for the case $g_3^* = 0$ we have

$$T_S^{4k} =$$

$$\frac{1}{2}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$
$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{64}$$

and these satisfy the Clifford Algebra

$$\left[T_S^{im}, T_S^{in}\right] = i f_S^{mnl} T^{il}, \tag{65}$$

where the f_S^{klm} are the usual structural constants suitable for $SU(3)$ Gauge transformations. Now the Gauge Fields G_μ^{ijk} represents the mediating gauge Bosons of the Strong force and these will have the field tensor given by

$$\mathrm{F}_{\mu\nu}^{ijk} = \partial_\mu G_\nu^{ijk} - \partial_\nu G_\mu^{ijk} + g_S^k G_\mu^{ijk} G_\nu^{ijk}, \tag{66}$$

where g_S^k are the appropriate gauge coupling constants and from this the appropriate Gauge transformation

$$\partial_\mu \longmapsto \partial_\mu + g_S^k G_\mu^{ijk}, \tag{67}$$

and

$$G_{\mu}^{ijk} \longmapsto G_{\mu}^{ijk} - \frac{1}{g_S^k}\partial_{\mu}\theta^{ij,k} + f_S^{klm}A_{\mu}^{ijl}\theta^{ijm}, \tag{68}$$

that leaves the Lagrangian $\mathcal{L} = \mathrm{F}_{\mu\nu}^{ij}\mathrm{F}^{ij\mu\nu}$ invariant as long as one commits to mind that $\mathrm{F}_{\mu\nu}^{ij} = \sum_k \mathrm{F}_{\mu\nu}^{ijk}$. Certainly, this kind of mathematics describes the Strong force.

SU(3) symmetry arise because one of the g_{μ}^{*} takes a zero value and the rest have none-zero values that lead to a finite g_S. It is possible to show that instead of this g_{ν}^{*} taking a zero value, it could take a finite value such that $g_{\mu\nu}^{*} = g_S$ only for three Gauge Fields and the other Gauge Field having a finite $g_{\mu\nu}^{*}$ but this being different from g_S. In this case, the invariance of the the Lagrangian $\mathcal{L}$ is preserved by a combined symmetry of SU(3) and U(1) meaning to say, the force field unifies with the Electromagnetic field. I will not present this unification in the paper for the sack of keeping the present paper as short as possible thus I shall leave this for a Paper VI.

12 Weak Force

Following the above procedure, if just two of the g_{μ}^{*} are equal to zero and the rest are none-zero such that for some of the terms $g_{\mu\nu}^{*} = g_{\mu\nu}^{W}$ one is at least none-zero, then we will have as well two none-zero components of the vector A_{μ}. Let us write this vector as W_{μ} for the same reason as before. There will be only six ways in which to distribute the zero amongst the indices of g_{μ}^{*} (that is $C_2^4 = 4!/2!2!$) hence there will be six combinations of the fields W_{μ}; let us write these in a more compact way as W_{μ}^{i} where in this case $i = 1, 2, 3, 4, 5, 6$. For each of the components W_{μ}^{i}, if we assume that we can exchange the components without exchanging the position of the zeros, there will be only two combinations of the Gauge Fields W^i_μ, let us add another index for this, that is W^{ij}_μ where $j = 1, 2$ we will have for the field tensor given by

$$\mathrm{F}^{ij}_{\mu\nu} = \partial_\mu W^{ij}_\nu - \partial_\nu W^{ij}_\mu + g^W_{\mu\nu} W^{ij}_\mu W^{ij}_\nu. \tag{69}$$

Since there are six particles that take part in the Weak interaction, the anti/electron $e^\pm$, anti/moun $\mu^\pm$, the anti/tau $\tau^\pm$, the anti/electron-neutrino ν_e, the anti/moun-neutrino ν_μ and the anti/tau-neutrino ν_τ, then the i's must represent these 6 states and the j most certainly represent the handedness of a particle since for the neutrinos we have the right-handed and left-handed neutrinos or the spin state since there can be two spin states, the spin-up and spin-down. If this (handedness) is to be extended to the other three particles ($e^\pm$, $\mu^\pm$, $\tau^\pm$), then, they too must have this handedness property. All we know about these particles ($e^\pm$, $\mu^\pm$, $\tau^\pm$) is that they can have either a spin-up or spin-down state and they show on property of handedness. On spin states, all the fermion particles observed to date have spin $1/2$ and none has been observed with spin $-1/2$. If the ($e^\pm$, $\mu^\pm$, $\tau^\pm$) share the handedness property, then, there ought to be an explanation of why these show no handedness property. The fact that they don't show this property does not mean they don't have it, it simple means they all posses just one of these two states in that part of our observable Universe. Further on, if j represents the spin state, then there ought to be a good reason why we do not observe the spin $-1/2$ state. There is need to look at these matters in a separate paper and this will be done in Paper VI.

Now if the field energy, $\mathcal{L} = \mathrm{F}^{ij}_{\mu\nu}\mathrm{F}^{ij\nu\nu}$ is to be invariant under some Gauge transformation, then the Gauge Fields W^{ij}_μ should, likewise submit to the decomposition

$$W_\mu^{ij} = T_W^{ik} W_\mu^{ijk}, \tag{70}$$

where the W^{ijk} are the generators of the $SU(2)$ group and T_W^{ik} are 2×2 Pauli matrices written in four dimensions and $k = 1, 2, 3$. The normal 2×2 Pauli matrices are given

$$\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{71}$$

where for the case $g_0^* = g_1^* = 0$ we have

$$T_W^{1k} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{72}$$

and for the case $g_0^* = g_2^* = 0$ we have

$$T_W^{2k} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{73}$$

and for the case $g_0^* = g_3^* = 0$ we have

$$T_W^{3k} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{74}$$

and for the case $g_1^* = g_2^* = 0$ we have

$$T_W^{4k} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{75}$$

and for the case $g_1^* = g_3^* = 0$ we have

$$T_W^{5k} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{76}$$

and for the case $g_2^* = g_3^* = 0$ we have

$$T_W^{6k} = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{77}$$

and these like wise satisfy the Clifford Algebra

$$\left[T_W^{im}, T_W^{in}\right] = i f_W^{mnl} T_W^{il}, \tag{78}$$

where the f_W^{klm} are the usual structural constants suitable for the $SU(2)$. Now the Gauge Fields W_μ^{ijk} represents the mediating gauge Bosons of the Strong force and these will have the field tensor given by

$$\mathrm{F}_{\mu\nu}^{ijk} = \partial_\mu W_\nu^{ijk} - \partial_\nu W_\mu^{ijk} + g_W^k W_\mu^{ijk} W_\nu^{ijk}, \tag{79}$$

where g_W^k are the appropriate gauge coupling constants and from this the appropriate Gauge transformation

$$\partial_\mu \longmapsto \partial_\mu + g_W^k W_\mu^{ijk}, \tag{80}$$

and

$$W_\mu^{ijk} \longmapsto W_\mu^{ijk} - \frac{1}{g_W^k}\partial_\mu \theta^{ijk} + f_w^{klm} W_\mu^{ijl} \theta^{ijm}, \tag{81}$$

and likewise that leaves the Lagrangian $\mathcal{L} = \mathrm{F}_{\mu\nu}^{ij}\mathrm{F}^{ij\mu\nu}$ invariant. Certainly, this kind of mathematics describes the Weak force. It should be said that there is need to fully explore these equations in more detail. As for case of the Strong force it is possible to show that the the Weak force unifies with Electromagnetic force under a $SU(2)\times U(1)$ symmetry and also that it unifies simultaneously with the Strong and Electromagnetic force under a $SU(2) \times SU(2)$. This unification will be shown in Paper VI. The reader must note that the present expedition is not to fully explore the theory but to show that the theory has in it the in-built structure to explain the nuclear forces otherwise the present reading would be lengthy and tedious for the reader.

13 Super Force

Now, if all the $g^{*}_{\mu} \neq 0$ such that $g^{*}_{\mu\nu} = g^{SS}_{\mu\nu}$ is a none-zero for at least one of the indices, then for this setting there will be sixteen combinations of the Gauge Fields A_{μ} and as before, lets write these as S^{i}_{μ} where $i = 1, 2, 3, ..., 16$. For the Lagrangian $\mathcal{L} = \mathrm{F}^{i}_{\mu\nu}\mathrm{F}^{i\mu\nu}$ to remain invariant, the Gauge Fields S^{i}_{μ} must submit to $SU(4)$ symmetry meaning to say there will be 16 intermediating Gauge Fields S^{ik}_{μ} where $k = 1, 2, 3, ..., 16$. We will have $S^{i}_{\mu} = T^{k}_{SS}S^{ik}_{\mu}$ where T^{k}_{SS} is a set of 16 4×4 matrices spanning the space of all 4×4 matrices. If γ^{μ} are the Dirac Gamma matrices, then these 16 matrices are $(\gamma^{\mu}, \mathrm{I}, \gamma^{5}, \sigma^{\mu\nu}, \gamma^{\mu}\gamma^{5},)$ where $\sigma^{\mu\nu} = \gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}$ and $\gamma^{5} = i\Pi^{3}_{0}\gamma^{\mu} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}$ and I is the 4×4 identity matrix. The field tensor of these intermediating Gauge Bosons is

$$\mathrm{F}^{ik}_{\mu\nu} = \partial_{\mu}S^{ik}_{\nu} - \partial_{\nu}S^{ik}_{\mu} + g^{SS}_{\mu\nu}S^{ik}_{\mu}S^{ik}_{\nu}, \tag{82}$$

where g^{k}_{SS} are the appropriate gauge coupling constants and from this the appropriate Gauge transformation are

$$\partial_{\mu} \longmapsto \partial_{\mu} + g^{k}_{SS}S^{ik}_{\mu}, \tag{83}$$

and

$$S^{ik}_{\mu} \longmapsto S^{ik}_{\mu} - \frac{1}{g^{k}_{SS}}\partial_{\mu}\theta^{ik} + f^{klm}_{SS}S^{il}_{\mu}\theta^{im}, \tag{84}$$

and these likewise leave the Lagrangian $\mathcal{L} = \mathrm{F}^{i}_{\mu\nu}\mathrm{F}^{i\mu\nu}$ invariant where $\mathrm{F}^{i}_{\mu\nu} = \sum_{k}\mathrm{F}^{ik}_{\mu\nu}$. The $SU(4)$ can be broken into $SU(3) \times U(1)$, $SU(3) \times U(1)$ and $SU(2) \times SU(2)$ symmetry. This will be shown in Paper VI. The prediction of the $SU(4)$ force is in actual fact the ground on with the validity of the theory can be tested. It is possible to calculate the energy range at with one expects to find the Super force. The message is clear that we should expect some surprises at the Large Hadron Collider which is currently under construction and is scheduled to begin operation sometime in 2008.

14 New Geodesic Law

Lastly, I could like to address the problem raised in the section "Problem & Quest" of the geodesic law namely that it is neither invariant nor covariant under a change of the system of coordinates and/or change in the frame of reference. The geodesic law equation 11 is derived (upon making proper algebraic operations) from the scalar function called the Lagrangian

$$\mathcal{L} = g_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}, \tag{85}$$

from the Lagrangian equation of motion namely

$$\frac{d}{ds}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}^{\mu}}\right) - \frac{\partial\mathcal{L}}{\partial x^{\mu}} = 0, \tag{86}$$

where in general the Lagrangian function $\mathcal{L} = \mathcal{L}(x^{\mu}, \dot{x}^{\mu}) = T(\dot{x}^{\mu}) - V(x^{\mu})$, T is the kinetic term and V is the potential term. We note that the Lagrangian function equation 85 contains just a kinetic term and not the potential term. We need to find a potential term and this term must fulfill the Law of Equivalence. To reach that end, first we note that even though the object J^{μ} is not a vector, the object $\int J^{\mu}dx_{\mu}$ is a scalar and at the same time a function of the x^{μ}. Like the kinetic term, the object $\int J^{\mu}dx_{\mu}$ is wholly a part of the fabric of spacetime in that it is a function of $g_{\mu\nu}$ and x^μ. If we set this term to be that potential term, that is $V = \int J^\mu dx_\mu$ and substitute the resulting Lagrangian into the Lagrangian equation of motion and then making proper algebraic operations as is done in order to arrive at the geodesic equation 11, one arrives at the equation

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\nu}\frac{dx^\alpha}{ds}\frac{dx^\nu}{ds} + \frac{1}{2}J^\mu = 0. \tag{87}$$

This equation is invariant under both a change of the system of coordinates and the frame of reference as long as it is understood that $(dx^\mu/ds)(dx_\mu/ds) = 1$, hence it will satisfy the Law of Equivalence. I propose this equation as the appropriate geodesic equation of motion.

15 Discussion & Conclusions

I have shown in this reading that it is possible to describe all the known forces of Nature using a 4*D* geometric theory that needs not the addition of extra dimensions as is the case with Strings Theories. This has been achieved first by demanding that the connections as we know them from Riemannian Geometry must have a tensor form to avoid once and forever the problem that the GTR faces – that of privileged systems of coordinates. This achievement is based on a new geometry that I have coined the Reimann-Hilbert Spacetime and this geometry without a doubt needs a thorough mathematical investigation. For example there is no mathematical justification as to why the object equation 49 takes the form it takes except that it allows us to obtain the Yang-Mills Gauge Fields ([35]). Another achievement of this geometry is that it has enabled us to achieve one thing that Einstein sought in a unified theory – that is, the material field must be part and parcel of the fabric of spacetime. Einstein is quoted as having said the left handside of his equation is like marble and the right handside is like wood and that he found wood so ugly that his dream was to turn wood into marble. These feelings of Einstein against his own GTR are better summed up in his own words in a letter to Georges Lemaître (1894-1966) the Belgian Roman Catholic priest on September 26 1947:

> *"I have found it very ugly that the field equation should be composed of two logically independent terms which are connected by addition. About justification of such feelings concerning logical simplicity is to difficult to argue. I can not help to feel and I am unable to believe that such an ugly thing should be realized in nature."*

Einstein hoped that the final theory must be such that the ponderable material function (ψ) must emerge from the geometry of the theory – this off course has been achieved in the present theory. It will be shown in Paper II & IV that ψ is the four component Dirac wavefunction.

An important out-come which lead to the ideas laid down here is the revision carried out of what is a frame of reference and a system of coordinates has lead us to the idea that it is erroneous to treat time much the same as we do when dealing with frames of reference. It has been concluded that the way in which we have treated time and space when it comes to coordinate transformation since Minkowski's 1908 pronouncement in his now famous lecture that:

> *"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein*

> *lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."*

is partly at fault because we have treated transformations between reference frames and systems of coordinates in a manner that makes no physical distinction between the two. If this is the case, that space and time be treated on an equal footing irrespective of whether we are dealing with space and time coordinates or frames of reference, it could mean that the labeling of points in spacetime has a dynamic physical meaning – a clearly visible and serious desideratum. In so doing, time has been identified as a coordinate scalar and this realization enabled us to introduce a vector field into the metric which without much deliberation has been identified with the magnetic vector potential. This prompted me to think of the metric components as representing vector fields of the Weak and the Strong force. By re-defining the metric, it is possible to show that $4D$ Yang-Mills theory is attainable. If this is the case, then the train is set forth to probe the foundations and origins of Yang-Mills Theory.

The resulting field equations 42 and 54 have not been explored fully mainly because there is a need to establish a link between Quantum and Classical Physics by giving the material function J_μ a real physical meaning that will automatically link both Quantum & Classical Physics. This task has been left for Paper II. In this paper we merely establish this link and do briefly explore the resulting equations. One of the interesting results from these equations is that Dark Matter/Energy can be explained from a time varying Gravitational Constant. It is shown in this paper that the present theory allows for the variation of the Fundamental Physical Constants thus the train is set forth to explore this field.

In Paper III, having established the link between Quantum & Classical Physics in Paper II, the theory is applied to the Universe where it is suggested that a rotating Universe explains the origins of mysterious Cosmological Magnetic field. The origins of the Magnetic field has remained a mystery and its origin sort. Further, it seen in this cosmology that negative mass will exist and that the quantisation of Quasar Redshift is a result of the spatial variation of the speed of light and the structure of the Universe that has been proposed in this paper so as to explain the Quasar Redshift quantisation. The non-appearance of negative mass or energy in the part of the Universe that we reside can be understood from the asymmetry between positive mass/energy as partially explained in my earlier paper ([24]).

In Paper IV a new foundation of QM is laid down based on the RHS. It is seen that this geometry implies QM exhibits a random probabilistic nature and at the same it is non-local.

Paper VI will mainly focus on making contact with experiments, that is, give the predictions of the theory, especially on the Super force since this force is predicated by the theory and has never be observed. In closing I would like to say that if the present theory is a true description of natural reality or anything to go by as I believe it to be, then, it is without a doubt that the train and ground for a grander understanding of the natural world from a unified perspective has been set forth. Papers II, III, IV and VI will be submitted in due course for publication to the present Journal.

Acknowledgments:

I would like to thank, in alphabetic order of the surname, my friends for their unremitting support and encouragement that they have given me in writing this paper, Rita Augustinho, Jotham Dondo, Christina Eddington, Eugene Engelbrecht, Daniel Moeketsi, Anna Neff, Donald Ngobeni and Jasper Snyman. I dedicate this paper to my Professor, D. J. van der Walt for his far reaching contributions to my-career and my-life. It is a pleasure to thank C. Roy Keys for valuable comments and the anonymous referee(s) for going through this paper.

References

[1] Brans C. & Dicke R. H., 1961, *Phys. Rev.*, **124**, 925.

[2] Einstein A., 1905, Annalen der Physik, **17**, 981.

[3] Einstein A., 1907, Translation by Schwartz H. M., 1977, Am. Journal of Phys., **45**, 10.

[4] Einstein A., 1915, Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin: *Die Feldgleichungun der Gravitation*, p844-847.

[5] Einstein A., 1917, *Sitz Preuss Akad. d. Wiss Phys.-Math,* **142**.

[6] Einstein A., 1919, *Spielen Gravitationsfelder in Aufbau der materiellen Elementarteilchen eine wesentliche Rolle?*, Sitzungersber. Preuss. Akad. Wiss.(20), 349-356.

[7] Einstein A., 1920, *Antowort auf vorstehende Betrachtung*, Die Naturwissenschaften, **8**, 1010-1011.

[8] Einstein A., 1921, *Geometrie und Erfahrung*, Sitzungersber. Preuss. Akad. Wiss.(12-14), 123-130.

[9] Einstein A., 1921, *Über eine naheliegende Erg ”anzung des Fundamentes der allgemeinen Relativitätstheorie*, Sitzungersber. Preuss. Akad. Wiss.(12, 13, 14), 261-264.

[10] Einstein A., 1923, *Bemerkungen zu meiner Arbeit “Zur allgemeinen Relativitätstheorie*, Sitzungersber. Preuss. Akad. Wiss.(12-14), 76-77.

[11] Einstein A., 1945., Ann. Math., A Generalisation of The Relativistic Theory of Gravitation, **46**, 578-584.

[12] Eddington A., 1921, *Proc. R. Soc. London, Ser. A*, A Generalisation of Weyl’s Theory of the Electromagnetic & Gravitational Fields, **99**, 104-122.

[13] Gödel K., 1949, *An Example of a New Type of Cosmological Solution of Einstein’s Field Equations of Gravitation*, Phys. Mod. Rev. **21**, 447-450.

[14] Kaluza T., 1921, *Zum Unitätsproblem in der Physsik*, *Sitzungsber. Preuss. Akad. Wiss.*, 966-972.

[15] Klein O., 1962, *Quanten-Theorie und 5-dimensionale Relativitätsheorie*, Z. Phys., **37**, 895-906.

[16] Kragh H. S., 1990, Cambridge Univ. Press, *Dirac: A Scientifc Biography*, p 275-292.

[17] Lawden D. F., 1962, *Tensor Calculus & Relativity*, Spottiswoode Ballantyne & Co. Ltd. London & Colchester.

[18] Lee Y. Y., 1965, Chinese Journal of Physics, **3**, No 1, 45-68.

[19] Lorentz H. A., *Versuch einer Thoerie electrischen und optischen Erescheinungen in between Kr̈pen*, Brill, Leyden.

[20] Mach E., 1893, *The Science. La Salle, Ill.: Open Court Publishers*, 6th Eddition of the English Translation, 1960.

[21] Michelson A. A. *The Relative Motion of the Earth and the Luminiferous Aether."*, Amer. J. Sci. 22, 120-129, 1881.

[22] Michelson A. A. & Morley, E. W. *On the Relative Motion of the Earth and the Luminiferous Ether.*, Amer. J. Sci. 34, 333-345, 1887.; also see Philos. Mag. 24, 449-463, 1887.

[23] Maxwell J. C., 1873, Oxford: Clarendon Press. (Reprint of 3th Ed., 1998, Oxford Classic Series), Trease on Electricity and Magnetism, **1**, IX.

[24] Nyambuya G. G., 2007a, *Dirac Equation in Curved Spacetime – On the Anomalous Gyromagnetc Ratio*, EJTP, **Vol. 4 No 14**, 1-11; Visit: www.ejtp.com.

[25] Nyambuya G. G., 2007b, On a Unified Field Theory – Gravitational, Electromagnetic, Weak & the Strong Force, Apeiron Journal, Vol. 14, Issue 3, pages 320-361. http://redshift.vif.com/JournalFiles/V14NO4PDF/V14N4GAD.pdf

[26] Salam A., 1981, *Physics News*, *Einstein's Last Dream: The Space-Time Unification of the Fundamental Forces*, Vol**12**, No 2: Or Visit http://www.iisc.ernet.in/academy/resonance /Dec2005/pdf/Dec2005p246-253.pdf

[27] Schrödinger E., 1948., *Proc. R. Irish Acad. A*, The final Affine Law II, **51**, 205-216.

[28] Smolin L., 2006, Houghton-Mifflin (Sept. 2006)/Penguin (UK, Feb 2007), *The Trouble With Physics*.

[29] Stephani H., 2004, *Cambridge University Press, Relativity*: An Introduction to Special and General Relativity, 3^{th} Edition, p 304.

[30] Thomas J. M., 1991, Adam Higler imprint by IOP Publishing ISBN 0-7503-0145-7, *Michael Faraday and the Royay Institution*, p74-78.

[31] Weyl H., 1918, *Sitzungsber. Preuss. Akad. Wiss(26)*, Gravitation und Elektrizität, 465-478.

[32] Weyl H., 1927a, *Z. Phys.*, Elektron und Gravitation I, **56**, 330-352.

[33] Weyl H., 1927b, *Proc. Nat. Acad. Sci.*, Gravitation and the Electron, **15**, 323-334.

[34] Witten E., 2005, Nature, *Unvelling String Theory*, 435, 1085.

[35] Yang C. N. & Mills R., 1954, Phys. Rev. **96**, 191.

[36] Zee A., 2003, *Quantum Field Theory in a Nutshell*, Princeton University Press, 375.