Unified Field Theory – Paper I

Gravitational, Electromagnetic, Weak & the Strong Force.

Golden Gadzirayi Nyambuya
North-West University -Potchefstroom Campus,
School of Physics - Unit for Space Research,
Potchefstroom, Republic of South Africa.
Email: gadzirai@gmail.com

I propose a Unified Field Theory of the Gravitational, Electromagnetic, Weak & the Strong force. A new force that obeys $SU(4)$ symmetry emerges and this new force constitutes the testable prediction of the theory. The theory rests on a new spacetime geometry with tensorial affine connections and this geometry is a fussion of Reimann and Hilbert spacetimes thus making it possible to bring quantum and classical physics and the task of doing this is left for Paper II. Further work to put the theory on a strong footing will be presented in Papers II, III, IV and VI and other works that may emerge from these investigations.

Keywords: Principle of Equivalence, Law of Equivalence, Frame of Reference, System of Coordinates, Coordinate Transformation, Frame Transformation, Force, Field

1 Introduction

"I am not interested in the spectrum of this and that atom ... I want to know whether God had a choice in building the Universe ..."
– **Albert Einstein** (1879-1955)

FROM a philosophical level, unification of all the forces of Nature imply beauty and simplicity. The dream of unification of all the forces of nature probably began in 1849 in the Royal Academy of Sciences in London with Michael Faraday (1791-1867) soon-after his great works in Electrodynamics when he tried to experimentally find a relationship between the Electromagnetic and Gravitational force - for obvious reasons he failed (see e.g [29]). However despite the failure, Michael Faraday believed that all the forces of nature were but manifestations of a single **Universal Force** and ought therefore to be inter-convertible into one another in much the same way as Electricity and Magnetism. Inspired by Albert Einstein, the pursuit to achieve this seemingly elusive dream of unification of the forces of Nature remains much alive to the present day and is the theme of the present reading. If what is presented in this reading is a correct description of natural reality as I would like to believe, then this paper is a significant contribution toward the attainment of this dream.

Regarding the forces of Nature as described above, a Unified Field Theory (UFT) in the physics literature is a theory that proposes to bring any of the four interactions or forces into one coherent and consistent theoretical framework that conforms with experience. A Grand Unified Theory (GUT) is a theory that proposes to bring all the forces with the exception of the Gravitational force into one coherent and consistent theoretical framework and a Theory Of Every-

thing (TOE) is a theory that proposes to bring all the four forces into one "giant", coherent and consistent theoretical framework which is consistent with natural reality. The present attempt is the ambitious attempt on the so-called "TOE". The title of the paper clearly suggests that this paper is about a UFT instead of a TOE. I have chosen this modest title for philosophical reasons that are not necessary to clarify here. I thus persuade the reader to accept this rather modest title.

Since the renaissance of the dream of a unified theory of the all the the known forces of nature was setforth in 1925 by Albert Einstein (1879-1955) after the emergence of his General Theory of Relativity (GTR) and this being a result of Herman Weyl's failed attempt, which was the first such, on a unification of Electromagnetism and Gravitation ([30]), great progress has been made in the effort of achieving a better understanding of the natural World on this footing. Herman Weyl embarked on his 1918 work after inspiration from Einstein's great works in GTR – an elegant and beautiful but incomplete unification theory of spacetime and matter. This he achieved by pure mathematical reasoning which brought into being the powerful Gauge Concept without which the current efforts of unification could not be. To this day the two forces theoretically stand side-by-side independent of each other and the attempts to bring them together has since been abandoned if not forgotten as a historical footnote.

The GTR is one of the pillars of Modern Physics and has not only revolutionalized our way of viewing space, time and matter but has also greatly advanced our knowledge insofar as unity of Nature is concerned. The search for a unified theory of all the forces of nature has largely continued on a theoretical front and as already mentioned, beginning with Herman Weyl ([30], [31], [32]) and thereafter

followed by Theodore Kaluza ([14]), Albert Einstein ([6], [7], [8], [9], [10], [11]), Oscar Klein ([15]), Erwin Schrödinger ([26]), Author S. Eddington ([12]) and many others. These authors sought a unified theory of the Gravitational and Electromagnetic force because Gravitation and Electromagnetism, then, were the only forces known to humankind. Latter, with the discovery of the nuclear and sub-nuclear forces, the attempts to unify Gravitation with Electromagnetism were abandoned by the mainstream physicists with the simple remark that this was a fruitless adventure for the reason that the subatomic forces needed to be taken into account.

The emergence of, or the discovery of the existence of sub-atomic forces marked a new era in the history of Physics bringing forth another pillar of Modern Physics – Quantum Field Theory. The effort of unification now largely depended on both observations and theoretical insight because the Quantum phenomena must be taken into account and this requires counter-intuitive pondering & delicate observations of the Quantum phenomena since it is alien to our everyday experience in that it defies common sense. Despite the fact that we don't understand the deeper meaning of the Quantum phenomena 80 years after the emergence of Quantum Theory, unremitting and unwavering attempts on the unification of all the known forces of Nature has proceeded undaunted and unabated. This is despite the fact that most if not all efforts to apply the rules the Quantum phenomena to the Gravitational phenomena that apply well to the other forces has brought nothing frustration to the physicist.

In the effort of unification, it is believed or supposed that the two key pillars of Modern Physics – Quantum and the Relativity – behold the secrets to the "final unification program" and these must fuse into one consistent theory but much to the chagrin of the esoteric and curious practitioners in this field, these two bodies of knowl-

edge appear to be fragmently disjoint in that they seem little adapted to fussion into one harmonious, coherent and consistent unified theoretical system. They do not directly contradict though they have taken physics to the terrains of philosophy because of their adamant refusal to come to the alter and marry. Their marriage is thought to be absolutely essential because it is generally agreed that a complete, unified, and deeper understanding of the Natural World lies in bringing the two theoretical systems together into one coherent and consistent unified structure since each describe a different world – for there to be unity, it is logical that there must be one world. It is thus the **dream** of every practicing "Theoretical Physicist" to find such a system, if it is exist to begin with. The belief and faith is that such a system ought to exist in order to preserve beauty, simplicity, an independent reality and harmony in the Natural World.

The first ever successful UFT was that by the Scottish Physicist James Clerk Maxwell (1831-1879) – he brought the Electric and Magnetic forces into one theoretical framework ([23]). Amongst others, Maxwell's theory showed that light is part and parcel of Electricity and Magnetism. Maxwell's theory was however not consistent with Newtonian Mechanics – a very successful theory at that time. The inconsistency between Maxwellian and Newtonian world views lead Einstein to ponder deeper into the intimate relationship between space and time and by so doing he [Einstein] arrived at a new theory now known as the Special Theory of Relativity (STR) ([2]). Preserving the Maxwellian world view, the STR asymptotically overturned the Newtonian doctrine of absolute space and time by proposing that time and space were not absolute as Newton had wanted or postulated, but relative – different observers measure different time lapses and length depending on their relative states of motion. I will elaborate further on the section on the STR. The STR

applies to inertial observers and Einstein did not stop there but proceeded to generalize the STR to include non-inertial observers thus arriving at the simple, elegant and all-time beautiful GTR which as presently understood is essentially is a theory of the Gravitational phenomena.

Naturally, after the achievement of the GTR, the next task is to bring the other forces within the framework of the GTR or the GTR into the framework of the other forces, which is to bring the GTR into Quantum Theory or Quantum Theory into the GTR. To achieving this, the main thrust amongst the majority of present day physicist is to seek a GUT, where upon it is thought that ideas to finding a "TOE" will dawn and shade light on the way forward (see e.g [25]). Currently, the only successful unification of forces in the micro-world is the 1967-68 theory by Sheldon Lee Glashow, Steven Weinberg & Abdus Salam. They succeeded in showing that the Weak & Electromagnetic force can be brought together into one theoretical framework. Since then, no satisfactory attempts (that is, experience and theory are in harmony) have come forth. The promising Standard Model (SM) of Particle Physics is also a good unification of the Weak, the Strong and the Electromagnetic force but many questions, largely theoretical ones, remain unanswered.

According to the popular science media, the most promising theoretical attempts made to date that bring the sub-nuclear forces together including the Gravitational force are the theories that embrace the notion of extra dimensions beyond the known four of space and time such as String Theory. It is said by String Theory's foremost proponents that this theory offers the best yet clues about a unified theory that en-campuses all the forces of nature and at the sametime it is not understood (e.g [33]). It is my view and the view shared by many that the draw-back of theories that employ extra-dimensions is

that they do not submit themselves to experience hence there is little room if any at all, to know whether these theories conform with natural reality. I am of the opinion that no matter how beautiful, elegant and appealing or seductive a theory may be or may appear to be, it ought only to be accepted as a truly physical theory if and only if it successfully submits itself to experience, otherwise it remains but an elegant piece of mathematics probably best left to be admired by mathematicians and mathematically minded Poets and Philosophers. From the physical view-point, there is not much one can say if anything at all about ideas based on the notion of higher dimensions since they do not naturally submit themselves to experience the reason of which is that "our technology has not reached that level where we can submit these theories to experience" or that "the conditions of experience to test these ideas are only found at the unique moment of birth of space and time." As someone that wishes to fathom the mysteries of the natural world, I so much would love that String Theory be the right theory given its exquisite beauty, elegance and far reaching imagination but at the sametime, I find it hard to forever keep my heard stuck in the sands thereof knowing that there is no way to verifying the theory.

Adding further to highlight the discontentness and or frustration with String Theory, Smolin ([27]) a leading theoretical physicist, who is a founding member and researcher at the Perimeter Institute for Theoretical Physics is of the opinion that String Theory is at a dead end and openly encourages young physicists to investigate new alternatives because there is not much chance that String Theory will be verified in the foreseeable future. In fact, he and others argue convincingly that String Theory is not even a fully formed theory in the true sense and spirit of a scientific theory but is just but a conjecture because the theory has not been able to prove any of the exotic ideas

posited by it. The discovery of "Dark Energy/Matter" he says is not even explained by String Theory and is proving troublesome for the theory's foremost advocates. Further, Smolin ([27]) writes in his book "*The Trouble with Physics*", that he believes that physicists are making the mistake of searching for a theory that is "beautiful" and "elegant" like String Theory but instead they should seek falsifyable theories that can be backed up by experiments. Seeking beauty and elegance in a theory is a philosophy developed by Paul Dirac (see e.g [16]) – this is a philosophy which I follow with the important difference that I believe that all ideas that purport to describe the true Physical World, no matter how elegant and beautiful they may appear, they must naturally submit themself to experience.

In the spirit of or on the advice of Smolin ([27]), I seek a new avenue of thought through a probably overlooked inconsistence in physics and I hope this paves the way for the attainment of the much sort and long awaited unified theory of all the forces of Nature. Through the aforementioned inconsistency which is our "flawed" treatment of inertial references frames and system of coordinates – something that I will elucidate in section IV, I bring the Electromagnetic force into the metric of spacetime whereafter the other forces are explained by the metric by required that the field energy be invariant under different space and time rotations. Before this is so, I demand of all the Laws of Physics to **absolutely** remain invariant and/or covariant under both the change of the system of coordinates and frame of reference and more importantly that the physics under a change of the system of coordinates remains **absolutely** invariant. In this way, I seek to realize fully the *Principle of Equivalence* by extending it to include the physical description of events in any given system of coordinates and frame of reference. In the deliverance of ideas in this reading, I will take the snail pace – developing and nur-

turing these ideas as we go from the section on STR, to the section the GTR till about section V where these ideas blossom to the full light of the day.

Keynote Because of the length of the original paper, this paper has been cut into two parts and the second part of this paper will appear issue succeeding the present.

2 Special & General Relativity

The STR was developed by Einstein in an effort to iron-out the inconsistencies between Newtonian Mechanics and Maxwellian Electrodynamics. The problem at hand was as follows;

(1) After a careful study of the great works of Galileo Galilee (1542-1642), Isaac Newton (1642-1727) founded a body of knowledge that beheld that in moving from one inertial system of reference to another time preserved its nature absolutely. That is to say, given the three space dimensions and also that of time – suppose we have two inertial observers (the primed and unprimed) whose space-time coordinates are (x, y, z, t) and (x', y', z', t') respectively, with one moving along the $x-axis$ relative to the other at a speed v, then, the two observers' coordinates intervals are related

$$\begin{aligned} \triangle x' &= \triangle x + v \triangle t \\ \triangle y' &= \triangle y \\ \triangle z' &= \triangle z \\ \triangle t' &= \triangle t \end{aligned} \quad , \tag{1}$$

declared Galileo in his great-works. Essentially this is the entire con-

ceptual constitution of Newtonian spacetime and the above transformation laws are known as the Galilean Transformation Law (GTL). What the law implies is that (assuming that the law is fundamentally true) all objects in the Universe move relative to one another – there is no such thing as an absolute motion. On the other hand, the GTL predicts that, like time ($t' = t$), acceleration ($a = d^2x/dt^2 : a' = a$) is an absolute quantity. This means that motion is both absolute and relative. This apparent contradiction bothered Newton and lead to many philosophical debates between him and some of his contemporaries – How can motion be relative while acceleration is absolute, is acceleration not some kind of motion or is it a special kind of motion? Newton proposed that accelerations be measured relative to the immovable absolute space which he identified with the background of the "fixed" stars. We shall not go into this difficult analysis.

(2) Maxwell's theory however predicted that light was a wave and its speed was universally constant in fragment contradiction with the Newtonian doctrine and at the sametime solving another problem, that of the existence of absolute space/frame of reference. That is, if the speed of light were absolute it [light] ought to move relative to some universal frame of reference that is at absolute rest. Also light being a wave means it ought to move through some medium – this medium would then naturally explain Newton's doctrine of absolute space and time, so it was thought. This hypothetical medium was then postulated to exist and it was coined the *Aether*. Attempts to detect this Aether by measuring the speed of the Earth through its passage suggested that there is no such thing as an Aether. With the Aether having escaped detection by one the finest and most beautiful experiment ever carried out by humankind – the Michelson-Mosley Experiment (MM Experiment) ([21], [22]), theoretical attempts to save the Aether paradigm were made by notable figures such as Hen-

drick Lorentz (1853-1928) amongst others. Lorentz's theory ([19]) preserved the Aether hypothesis by proposing that the lengths of objects underwent physical length contraction relative to the stationery Aether (Lorentz-Fitzgerald contraction) and a change in the temporal rate (time dilation). At that time, this appeared to reconcile Electrodynamics and Newtonian Physics by replacing the GTL with a new set of transformation laws which came to be known as the Lorentz Transformation Law (LTL). If $\triangle t$, $\triangle x$, $\triangle y$, $\triangle z$ are the time and space separation relative to the Aether and $\triangle t'$, $\triangle x'$, $\triangle y'$, $\triangle z'$ the time and space separations in the moving frame (speed v), then

$$\begin{aligned} \triangle t' &= \Gamma\left(\triangle t + v \triangle x/c^2\right) \\ \triangle x' &= \Gamma\left(\triangle x + v \triangle t\right) \\ \triangle y' &= \triangle y \\ \triangle z' &= \triangle z \end{aligned} \tag{2}$$

where

$$\Gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{3}$$

are the LTL. Indirectly, after much careful pondering on the negative result of the MM Experiment by considering the apparent contradictions between Newtonian and Maxwellian Electrodynamics, with a leap of **faith** and **boldness**, Einstein cut the Gordian knot and then untied it thereafter by the following reasoning; If we accept the Laws of Electromagnetism as fundamental and also Newtonian Laws of motion as fundamental, then there ought not to be a contradiction when Newtonian Laws of motion are applied to inertia frames of reference in which the Electrodynamic Laws hold good and these Laws are transformed to an equivalent reference frame within the

framework of Newtonian Mechanics. Either of the two must be at fault or both. Newtonian Mechanics, then had stood the test of time – for nearly 250 years it passed all the experimental tests to which it was submitted and was almost taken for granted as a self evident truth. The celebrated physicist and philosopher Lord Kelvin was amongst other prominent and highly esteemed thinkers of his time, so confident of Newtonian Mechanics that he proclaimed before the turn of the past century that "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." – we know now, he was not right, Einstein was to soon show this.

On the other hand Electrodynamics was a new field where more elaborate experiments to confirm it where yet to be carried out. It is here that Einstein boldly & faithfully cut sharply though the thick dark clouds hovering over the horizon of science, chopping and untieing the Gordian knot by upholding Electrodynamics as more fundamental than Newtonian Mechanics and thus went on to replace it with a new Mechanics by putting forward the following two postulates:

1. The Laws of Physics are the same for all inertial frames of reference in uniform relative motion.

2. The speed of light in free space is the same for all inertial observers.

The first postulate, known also as the **Principle of Relativity**, dispels the notion that there is such a thing as a preferred or absolute frame of reference. The Laws of Physics must be the same in equivalent frames of reference. Inertial frames of reference have the same status of motion in that Newton's first Law holds good in them. If the first postulate were true and Maxwells theory were a fundamental theory of nature, then the second postulate follows immediately

since Maxwell's theory predicts explicitly that the speed of light has a definite numerical value. The constancy of the speed of light predicted here lead us via Einstein's great insight to rethink our view of space and time. Time for different frames of reference runs at different rates and lengths are not absolute but depend on the observers state of motion. The LTL follow immediately from these two postulates but with the important difference that the Aether hypothesis is not any longer necessary.

This is the entire conceptual content of the STR. Einstein was not satisfied with the STR because it only dealt with observers in uniform relative motion and he wanted to know how the Laws of Nature manifest themselves in the case of non-inertial observers and the quest for answer to this question culminated in the GTR ([4]). The problem with non-inertial observers is that Gravitation becomes a problem since it is an all pervading "non-vanishing force". By analyzing the motion of a body in free-fall in a Gravitational field, Einstein was able to overcome the problem of Gravitation by noting that if Gravitational mass (m_g) and inertia mass (m_i) were equal or equivalent, then Gravitation and acceleration are equivalent too ([3]). Because of the importance of this, it came to be known as the *Principle of Equivalence*. This meant that the effect(s) of acceleration and Gravitation are the same – one can introduce or get rid of the Gravitational force by introducing acceleration. The deep rooted meaning of the Principle of Equivalence is that Physical Laws should remain the same in a local frame of reference in the presence of a Gravitational field as they do in an inertial frame of reference in the absence of Gravitation. In Einstein's own words:

Principle of Equivalence: "We shall therefore assume the complete physical equivalence of a Gravitational field and the corresponding acceleration of the reference frame. This assumption extends the Principle of Relativity to the case of uniformly accelerated motion of the reference frame."

A consequence of this is that no mechanical or optical experiment can locally distinguish between a uniform Gravitational field and uniform acceleration. It is here that I would like to point out that the Principle of Equivalence as used in the formulation of the GTR does not demand that the physics must remain invariant. By "***the physics***" I mean that the description of a physical event ought to remain invariant unlike for example in black-hole physics – depending on the system of coordinates employed (and not the frame of reference – this is important), a particle can be seen to pass or not pass through the Schwarzschild sphere for the same observer supposedly under the same conditions of motion. Also the chronological ordering of events is violated – that is, the Law of Causality is not upheld. For example, in a rotating Universe as first pointed-out by Kant Gödel ([13]) it is possible to travel back in time meaning to say it is possible to violate the Second Law of Thermodynamics. Though the idea of time travel is very fascinating and appealing to the mind, it is difficult to visualize by means of binary logical reasoning how it can work in the Physical World as we know it. The Laws of Nature must somehow have it deeply embedded in them the non-permissibility of time travel.

Therefore, we must demand that the physics, that to say, the physical state and chronological ordering of events, must remain invariant that is, extend the Principle of Equivalence to include the physical state/physical description of events and the Law of Causality. Because this must be universal and important, let us call the

extended Principle of Equivalence the Law of Equivalence:

Law of Equivalence: Physical Laws are the same in all equivalent frames of reference independently of the system of coordinates used to express them and the complete physical state or physical description of an event emerging from these laws in the respective frames of reference must remain absolutely and independently unaltered or invariant by the transition to a new or change of the system of coordinates.

This forms the basic guiding principle of the present theory. The deeper meaning of the Law of Equivalence is that it should not be permissable to transform a singularity by employing a different set of coordinates. If the singularity exists, it exists independently of the system of coordinates and frame of reference used – it is permanant. Therefore if we are to have no singularities, the theory itself must be free of these. If a particle is seen not to pass through the event horizon, it will not be seen to pass the event horizon no matter the system of coordinates employed and the frame of reference to which the current situation is transformed into.

Back to the main vein, the Principle of Equivalence is in the context of Riemann geometry, mathematically embodied in the mathematical expression

$$g_{\mu\nu;\sigma} = g_{\mu\nu,\sigma} + \Gamma^{\lambda}_{\sigma\mu} g_{\lambda\nu} + \Gamma^{\lambda}_{\sigma\nu} g_{\mu\lambda} = 0, \tag{4}$$

where $g_{\mu\nu}$ is the metric tensor describing the geometry of space-time and

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} \left\{ g_{\alpha\mu,\nu} + g_{\nu\alpha,\mu} - g_{\mu\nu,\alpha} \right\}, \tag{5}$$

are the affine connections or the Christoffel symbols. The affine con-

nections play an important role in that they relate tensors between different frames of reference and systems of coordinates. Its draw back insofar as Physical Laws are concerned is that it is not a tensor. It transforms as:

$$\Gamma^{\lambda'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \Gamma^{\lambda}_{\mu\nu} + \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \frac{\partial^2 x^{\lambda}}{\partial x^{\mu'} \partial x^{\nu'}}. \tag{6}$$

The extra term on the right makes it a non-tensor. Most of the problems facing the GTR can be traced back to the non-tensorial nature of the affine connections – some of the problems will be highlighted in the section following.

The invariance of Physical Laws under a change of the system of coordinates is in Riemann geometry encoded and expressed through the invariance of the line element

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}. \tag{7}$$

The line element is a measure of the distance between points in space-time and remains invariant under any kind of transformation of the frame of reference and or the system of coordinates. This is the essence of the GTR. From this Einstein was able to deduce that Gravitation is and or can be described by the metric tensor, thus, according to the Einstein doctrine of Gravitation, it [Gravitation] manifests itself as the curvature of space-time. Through his [Einstein] own intuition & imagination, he was able to deduce that the curvature of space-time ought to be proportional to the amount of matter-energy present - a fact that has been verified by numerous experiments. The resulting law emerging from Einstein's thesis is

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu}, \tag{8}$$

which is the well known Einstein's field equation of Gravitation where

$$R_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\lambda\mu}, \tag{9}$$

is the contracted Riemann curvature tensor and

$$T_{\mu\nu} = \rho_0 v_\mu v_\nu + p g_{\mu\nu}, \tag{10}$$

is the stress and energy tensor where ρ_0 is the density of matter, p is the pressure and v_μ the four velocity, $\kappa = 8\pi G/c^4$ is the Einstein constant of Gravitation with G being Newton's Universal Constant of Gravitation, c the speed of light and Λ is the controversial and so-called Cosmological Constant term added by Einstein so as to stop the Universe from expanding ([5]). Einstein was motivated to include the Cosmological Constant because of the strong influence from the astronomical wisdom of his day that the Universe appeared to be static and thus was assumed to be so. Besides this, the Cosmological Constant fullfiled Mach's Principle ([20]), a principle that had inspired Einstein to search for the GTR and thus thought that the GTR will have this naturally embedded in it. Mach's principle forbids the existence of a truly empty space and at the sametime supposes that the inertia of an object is due to the induction effect(s) of the totality of all-matter in the Universe.

3 Problem & Quest

In my view, the major problem that the GTR faces is that it is based on pure Riemann geometry – a geometry that is well known to violate the Principle of Equivalence at the affine level because the affine

connections are not tensors. If pure Riemannian geometry is to be the true geometry to describe the Natural World, then, no Laws of Physics should exist at the affine level of Riemann geometry. However, this is not so, since the Geodesic Law

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\nu}\frac{dx^\alpha}{ds}\frac{dx^\nu}{ds} = 0, \tag{11}$$

that describes the path and motion of particles in spacetime emerges at the affine level. Thus accepting Riemann geometry as a true geometry of nature means we must accept contrary to the Principle of Relativity that there exists in Nature preferred frames of reference and systems of coordinates because the above Geodesic Law leads us to formulating the equation of motion in prefaced frames of reference and system of coordinates, namely, geodesic frames or system of coordinates. It can be shown for example that given a flat spacetime in which say the rectangular system of coordinates are used to begin with where the affine vanish identically in this system and changing the system of coordinates to spherical, the affine do not vanish. This is a serious desideratum, akin to the Newton-Maxwell conundrum prior to Einstein's STR.

Given for example, that the affinities represent forces as is the case in the GTR, this means a particle could be made to pass from existence into non-existence (or vise-versa) by simply changing the system of coordinates. This on its own violates the Laws of Logic and the need for Nature to preserve an independent reality devoid of magic. For this reason, there is a need to ask what exactly do we mean by a system of coordinates and frame of reference and what relationship should these have to Physical Laws so that the Law of Equivalence is upheld? This shall constitute the subject of the next section.

The only way out of this conundrum is to seek a theory in which the affinities have a tensor form hence in the present approach, the first and most important guide is to seek tensorial affinities. Einstein, Schrödinger etc have made attempts along these lines only to fail. The reason for their failure may stem from the fact that theirs was an exercise to try to find a set of tensorial affinities from within the framework of the classical spacetime of Riemannian geometry.

4 Nature of Time

"Absolute, true, and mathematical time, of itself, and from its own nature, flows equable without relation to anything external ..."
– **Sir Isaac Newton** (1642-1727)

We already know from the STR that time does not transform absolutely when dealing with different frames of reference. The question I wish to answer in this section is whether or not the time coordinate is invariant under a change of the system of coordinates – that is $t' = t$. In the process we shall answer the question paused in the previous section. In conclusion, I establish that time – vis-a-vis when transforming between different systems of coordinates – is a scalar quantity and this is a "self-evident-truth" provided the Law of Equivalence holds true. In order to do this, it is necessary that we define succinctly what we mean by frame of reference and system of coordinates. These two are used interchangeably in most text books of physics.

For example Stephani ([28]) in his effort of trying to describe events near and at the event horizon of in blackhole - starting with the Schwarzschild metric, goes on to say "We seek coordinate systems which are better adapted to the description of physical pro-

cesses ...". This is nothing more than an admission that physics in different coordinate systems will be different – there exist systems of coordinates that are unsuitable for the the description of physical events. Why should this be so? Physics and or physical processes should never be dependent on the choice of coordinates. Let us devote some little time to understanding what is a coordinate system and a reference frame/system.

System of Coordinates: When thinking about space, it is extremely useful to think of it as constituting of points, each labeled so that one can distinguish one point from another – each point is and must be unique. These labels are called coordinates. One must choose these labels in such a way that it is easy to manipulate. In practice, numbers are used because we understand and can manipulate them. To manipulate these labels, a universal and well defined rule must be set out so as to label and manipulate the labels and this is what is called the *system of coordinates*. One ought to be free to choose any coordinate system of their choice provided the labeling scheme makes each point to be unique because any space exists independent of the system of coordinates used. Examples of system of coordinates are the spherical coordinates (r, θ, ϕ), rectangular (x, y, z), cylindrical (r, θ, z) and curvilinear (x_1, x_2, x_3) to mention but a few. The coordinate itself has no physical significance but only its relative distance from other coordinates is what is of physical significance.

Frame of Reference: After having chosen a system of coordinates of our liking, suppose we station an observer at every-point of space. For any given system of coordinates there exists a point that one can call the point of origin, this point can be any-point, there ought not to be a preferred point. In the usual three dimensions of space,

this point is the point (0, 0, 0) – this choice gives the easiest way to manipulate the coordinates. Once the observer has set the $(0, 0, 0)$ point, they will set up about this point $(0, 0, 0)$, their axis and the set of axis then constitutes the frame of reference. The observer that has declared their point of origin and has set their frame of reference "sees" every other point relative to the (0, 0, 0) point thus this point is their point of reference which together with the set of axis is in the usual language of STR is the *Frame of Reference.* The frame of reference thus provides one with a reference point $(0, 0, 0)$ and a set of axes relative to which the observer can measure the position and motion of all other points in spacetime as seen in other frames of reference.

The above defines a frame of reference and I hope the reader is able to make a clear distinction between the two – that is a system of coordinates and frame of reference. It follows that the STR is concerned with nature of Physical Laws under a change of the frame of reference, that is, from one-point of spacetime to another depending on these points's state of motion while the GTR is concerned with nature of Physical Laws under both a change of the system of coordinates and frame of reference. The STR posits that the Laws of Physics remain the same for observers in uniform relative motion with the GTR positing through the Principle of Equivalence that even for observers in uniform relative acceleration the Laws of Physics remain the same and these are the same as for those observers in uniform relative motion. The GTR goes further and extends this to encamps different system of coordinates by maintaining that the Laws of Physics remain invariant under a change of system of coordinates. I will point out here a logical flew in the GTR in its endeavors. This is deeply rooted in its treatment of time under a change of the system of coordinates. The logical flew lies in the

equal-footing treatment of the space and time coordinates applicable to the STR or transformation between different but equivalent frames of reference being unconsciously extended to describe natural processes under a change of the system of coordinates. Let us look closely at the coordinate transformation law

$$x^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} x^{\mu}. \tag{12}$$

Lets pluck out the time coordinate, that is $\mu' = \mu = 0$. It follows that a time difference of $\triangle t'$ in the primed system of coordinates is related to the time lapse $\triangle t$ in the unprimed system of coordinates by

$$\triangle t' = \frac{\partial x^{0'}}{\partial x^{0}} \triangle t. \tag{13}$$

In a nutshell, if $\partial x^{0'}/\partial x^{0} \neq 1$ (identically not equal to zero) or is a function of position or anything for that matter that has a numerical value other than unity, then this means that for different system of coordinates time moves at different rates – time dilation. Herein lies the problem – what have we just said! **This means a photon can be blue or red-shifted by just changing the system of coordinates!** Red or blue shifting is a physical process but changing of the system of coordinates is not a physical process at all! Here we have it - this is the source of our problems in our endeavors to completely understand nature from the current GTR view-point. The only way in which a photon's physical state will remain invariant is if time preserved its nature under a change of the system of coordinates. This could mean time is not a vector but a scalar when it comes to coordinate transformations. If time behaved as predicted by equation 13 with $\partial x^{0'}/\partial x^{0} \neq 1$, it could mean all physical events in space and

time are affected by a change of the system of coordinates and as already stated it means the way in which we label points does has a physical significance?! This own its own makes no physical or logical sense at all and constitute a serious desideratum. A priori to this analysis, is that, it is absolutely necessary that we put forward the following **Protection Postulate** so as to uphold the Law of Equivalence:

Postulate I: In order to preserve the physical state and the chronological evolution of a physical system in the transition from one system of coordinates to another, of itself and from its own nature time must flow equable without relation to anything external – It must remain invariant under any kind of transformation of the system of coordinates.

It is not difficult to show that if the spatial coordinate transformations where to be non-linear with respect to the corresponding coordinate, events and or points in spacetime will cease to be unique and also the physics is altered just by changing the system of coordinates! Inorder to strictly preserve the physics and second to preserve the uniqueness of events when a transition to a new system of coordinates is made, it is necessary to put forward another protection postulate:

Postulate II: In order to preserve the physics when a transition to a new coordinate system is made $\mathbf{e}'_\mu \cdot \mathbf{e}'_\nu = \mathbf{e}_\mu \cdot \mathbf{e}_\nu$ and for the same transition to preserve the uniqueness of physical events in spacetime, the points in the new coordinate system for a non-periodic coordinate system, must be linear and have a one-to-one relation with the old one and in the case of a periodic coordinate system the periodicity must be ignored.

Linearity has a two-fold meaning here: (1) Suppose in a transformation of the coordinate system from A to B a point in the coordinate system A has more than one corresponding coordinate for a non periodic coordinate system like spherical coordinate system (this periodicity can be ignored because it does not physically place the point to another point in the same space), then in such a coordinate transformation, events cease to be unique. (2) The geometry of the new coordinate system upon transformation must remain unaltered otherwise this changes the physics – by the geometry it is meant that $\mathbf{e}'_\mu \cdot \mathbf{e}'_\nu = \mathbf{e}_\mu \cdot \mathbf{e}_\nu$. Simple said, the angles between the unit vectors must remain unchanged otherwise we could have a situation as in current blackhole physics where one can alter the system of coordinates to obtain a completely different kind of physics. If one comes to think of it really seriously, it is not difficult to come to the conclusion that changing the system of coordinates in such a manner that the new system of coordinates has different angles for the corresponding angles for the unit vectors, changes the physics altogether. Changing the angles between the unit vectors is without doubt an introduction of some curvature or a new geometry altogether and this by itself is a physical process. As already said before, changing the system of coordinate is nothing more than a change of the labeling of the points akin to renaming a street. A name change of a street does not in any way physically change the street just as changing one's name does not change the physical person that they are. So, to preserve the physics upon a change of the system of coordinates, we should not temper or change the angles between the unit vectors.

Mathematically speaking, the first postulate means that time is a scalar quantity and thus

$$\frac{\partial x^{0'}}{\partial x^0} = 1. \tag{14}$$

We have established here that time must behave as a scalar when transforming from one system of spacetime coordinates to another and this is not so when transforming from one frame of reference to another. Because of this, let us adopt the terminology **coordinate scalar** or **coordinate vector** to mean a quantity behaves as a scalar under a coordinate transformation and likewise we will have a **frame scalar** and **frame vector** to mean a quantity that transforms as a scalar or vector when transforming from one frame of reference to the other.

Now, from equation 14, an Abelian coordinate vector field emerges naturally from the metric tensor, that is to say, given that the metric transforms as

$$g_{\mu\nu} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} g_{\mu\nu}, \tag{15}$$

and from the first postulate, it follows that

$$g_{i0} = \frac{\partial x^{i'}}{\partial x^{i}} g_{i0}, \tag{16}$$

which is an Abelian vector field. If the metric is thought of as representing force fields and given that there is only one known Abelian vector field, the Magnetic force, we shall without loss identify this vector with the Magnetic vector potential. The g_{00} component is according to the postulate a coordinate scalar. It is clear that all 00 components in spacetime according to the current new understanding will all be coordinate scalars and all $0i$ components will be coordinate vectors. Putting the Gravitational force aside for the time

being, the only scalar force known is the Electric force, likewise, we shall identify the g_{00} component with the Electric force hence $g_{\mu 0}$ represents the Electromagnetic four vector potential. At this present moment, the natural question to ask is what force fields do the other components (g_{ij}) represent? This object transforms as

$$g_{i'j'} = \frac{\partial x^i}{\partial x^{i'}}\left(\frac{\partial x^j}{\partial x^{j'}} g_{ij}\right), \tag{17}$$

which clearly shows that this quantity is a product of a three and two component vector for the case $\mu = 1, \nu = 1, 2, 3$ and $\mu = 2, \nu = 2, 3$, and some one vector component and for the case $\nu = 3$. It is tempting (as I initially did) to think of these components as the vector fields of the Strong and the Weak force since these are represented by three and two component vectors respectively. When one comes to think of it in the simplistic manner, the metric must be symmetric and it is a product of the same vector, that is for example if e_μ are the unit vectors, then the metric of this spacetime is $g_{\mu\nu} = e_\mu e_\nu$ – this is the present understanding of the metric tensor that it is a product of the unit vectors. With the new proposal that the $g_{0\mu}$ be a the Electromagnetic four vector potential, the suggestion to me is that we must write

$$g_{\mu\nu} = A_\mu A_\nu, \tag{18}$$

where for now, the vector A_μ is to be understood as the Electromagnetic vector potential but as shall be seen in due course, this vector will be able to represent the other forces and this depending on the choice we make for the quantity $\partial_\mu A_\nu$ and the rotational state of spacetime. A suitable choice of this quantity, $\partial_\mu A_\nu$, will allow us to recover not only the Electromagnetic force but the Strong and Weak

force fields as-well.

5 Theory

We have seen that it is possible to incooperate the Electromagnetic force into the metric and also it has been suggested that the other forces of nature can be brought into the metric as well – this is the subject of the sections following the present. Now what I shall do here is to seek a geometry that gives tensorial affinities in such a way that one can obtain both the respective geometries on which Quantum and Classical Physics are founded. Quantum Physics is defined on a Hilbert space or Hilbert geometry while Classical Physics is defined on the classical spacetime of Riemannian geometry. The main idea is to find a geometry that fuses these two geometries in a natural way. Let us begin by defining these two geometries and fuse them in such a manner as described above – that is, the resulting affine must be tensors.

Hilbert Space: Every inner product $\langle , \rangle$ on a real or complex vector space H gives rise to a norm

$$ds^2 = \langle x, x \rangle \,, \tag{19}$$

and the space H is said to be a Hilbert space if it is complete with respect to this norm. Completeness in this context means that any cauchy sequence of elements of the space converges to an element in the space, in the sense that the norm of differences approaches zero. On the other hand we define a Riemannian space:

Riemann Spacetime: A space is said to be Riemannian if the norm is invariant under a coordinate transformation such that the metric of the space satisfies the fundamental theorem of Reimann geometry, that is the covariant derivative equation (4) resulting in the definition of the affine connection as given by equation 5.

From these spaces as defined above, one can by a closer inspection of the Riemann Geometry imagine a union of both the Riemann and Hilbert space. Let us coin this space the Riemann-Hilbert Space (RHS). This space is some-kind of a Reimann Space in its formulation with it embedded the Hilbert objects that gives the space the necessary machinery to overcome the criticism leveled earlier against pure Riemann geometry that of the affinities being non tensorial.

Riemann-Hilbert Spacetime: In ordinary spacetime geometry the unit vectors are represented by the four objects:

$$\mathbf{e}_0 = \begin{pmatrix} i \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{20}$$

Notice that the components or length of the axis unit vectors are all constants – why is this so? Is it really necessary that they be-

come constants and at the sametime is it really necessary that the significant component of these unit vectors be equal? Suppose we set up a $3D$ system of coordinates in the usual space that we inhabit with three orthogonal axis. Let each of these axes have an observer, say X monitors the $x-axis$ and Y monitors the $y-axis$ and like wise Z monitors the $z-axis$. Along each of these axis the observer can define a unit length and it need not be equal to that of the others. Having defined their unit length to compare it with that of the others, they will have to measure the resultant vector which is the magnitude of the vector sum of the three "unit" vectors along their respective axis. This setting does not affect anything in the physical world for as long as one commits to mind that the unit vectors along each of the axis are different and they have in mind the length of the resultant unit vector. This little picture tells us we can have variable unit vectors along each of the axis that is,

$$\mathbf{e}_0 = \begin{pmatrix} \psi_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_1 = \begin{pmatrix} 0 \\ \psi_1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_2 = \begin{pmatrix} 0 \\ 0 \\ \psi_2 \\ 0 \end{pmatrix}, \quad \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \psi_3 \end{pmatrix}, \tag{21}$$

where $\psi_\alpha = \psi_\alpha(x^\mu)$ for $\alpha = 0, 1, 2, 3$ variable. If as usual the position vector in this space is given by $\mathbf{X}_\mu = x_\mu \mathbf{e}_\mu$ where x_μ is the usual spacetime coordinate in Riemann geometry, then, it is not difficult for one to see that the resulting metric from the above set of unit vectors will be diagonal, meaning that all the off-diagonal terms will

equal zero. We must in general be able to obtain a metric with non-zero components and not only diagonal as is the case if the unit vectors are as given in equation 21. For this to be so, that is, obtain a metric with non-zero components, we will need to have

$$\mathbf{e}_\mu = \bar{\gamma}_\mu \begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \tag{22}$$

where

$$\bar{\gamma}_0 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix}, \quad \bar{\gamma}_i = \begin{pmatrix} \mathbf{I} & i\sqrt{2}\sigma_i \\ i\sqrt{2}\sigma_i & -\mathbf{I} \end{pmatrix}, \tag{23}$$

are what I shall call the gamma-bar matrices where $\mathbf{I}$ is the 2×2 identity matrix and σ_i are the usual Pauli matrices. The term gamma-bar matrix for these matrices has been coined in my earlier paper ([24]). From this we can write the line element of the space in the Dirac bra-ket notion

$$ds^2_{RH} = \langle d\mathbf{X}_\mu || d\mathbf{X}^\mu \rangle = \rho g_{\mu\nu} dx^\mu dx^\nu, \tag{24}$$

where $\rho = \psi^\dagger_\alpha \psi^\alpha$, $\alpha = 0, 1, 2, 3$. This line element is similar to that for the Scalar-Tensor theories of gravity in which ρ is a pure scalar quantity ([1]). Scalar-Tensor theories are an alternative theory to Einstein's GTR whose endeavor is similar to the present, that is, incorporate or unify Quantum phenomena with the Gravitational phenomena.

Unlike Scalar-Tensor theories, the object ρ shall here be chosen such that it is not a scalar as in Brans-Dicke Theory. This choice of ρ affords us the opportunity and the economy to un-chain ourself from

the bondage of non-tensorial affinities as will be seen shortly because we can forcefully choose this object in such a way that the resultant affine connections are tensors. Comparing this with Reimann geometry and demanding that in the limiting case, that is $\rho = 1$, RHS reduces to the well known Reimann space would require that we make the substitution $g_{\mu\nu} \longrightarrow \rho g_{\mu\nu}$ into equation 4, that is

$$\left\langle g_{\mu\nu} \right\rangle_{;\sigma} = \rho \left(g_{\mu\nu,\sigma} + \bar{\Gamma}^{\lambda}_{\sigma\mu} g_{\lambda\nu} + \bar{\Gamma}^{\lambda}_{\sigma\nu} g_{\mu\lambda} + J_{\sigma} g_{\mu\nu} \right) = 0, \tag{25}$$

where

$$\bar{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + M^{\lambda}_{\mu\nu}, \tag{26}$$

and

$$M^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} \left\{ g_{\alpha\mu} J_{\nu} + g_{\nu\alpha} J_{\mu} - g_{\mu\nu} J_{\alpha} \right\}, \tag{27}$$

which for reasons that will become clear in due course, we shall call this the matter-connection where

$$J_{\mu} = \frac{1}{\rho} \frac{\partial \rho}{\partial x^{\mu}} = \frac{\partial \ln \rho}{\partial x^{\mu}}. \tag{28}$$

Now, taking advantage of the fact that the liberty is ours to make a proper choice of ψ, let us demand as set out in section 3 that the affine $\bar{\Gamma}^{\lambda}_{\mu\nu}$, be a tensor by making a suitable or proper choice of ρ. Also our choice would be to have the objects ψ_{α} defined on the Hilbert space, the subtle aim being to identify this object with the material field that in QM is the wavefunction. In light of the aforesaid, it is clear that if we envisage the material field to be defined by the Dirac wavefunction, then ρ can not be a scalar. It is clear that

making this object a scalar reduces the theory to a theory much akin to Weyl's un-successful unified theory ([30], [31], [32]).

Now supposing we have this suitable choice of ρ as given and knowing that for any affine connection that is symmetric in the lower indices, there is always a frame of reference and system of coordinates in which they vanish identically (see for example [17]), it follows that the affine connection will vanish identically in all system of coordinates and frame of reference, that is

$$\bar{\Gamma}^{\lambda}_{\mu\nu} = 0. \tag{29}$$

This is an important equation from which the definition of J_μ in-terms of the metric follows hence that of ψ as-well. The space just described is what we shall define as the Riemann-Hilbert spacetime and the condition equation 29 leads to a linear and exactly soluble theory. This spacetime is the normal spacetime with the exception that it is endowed with variable unit vectors that will later in Paper II be clearly identified with the normal wavefunction of QM. The meaning therefore of the existence of ponderable matter according to the foregoing is to ensure that the Laws of Nature/Physics in spacetime remain invariant under a change of the system of coordinates at the affine level.

Proceeding further, from equation 29 it follows that if this condition is to hold identically, then

$$g_{\mu\nu,\sigma} + J_\sigma g_{\mu\nu} = 0, \tag{30}$$

which in a more natural way defines J_μ in terms of the metric, that is : $J_\mu = -\partial g_{\mu\lambda}/\partial x_\lambda$. This leads to the transformation

$$J_{\mu'} = \frac{\partial x^\mu}{\partial x^{\mu'}} J_\mu - 2\frac{\partial^2 x^\lambda}{\partial x^\lambda \partial x^{\mu'}} \tag{31}$$

and in-turn leading to the matter-connection transforming as

$$M^{\lambda'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\nu}}{\partial x^{\nu'}}\frac{\partial x^{\lambda'}}{\partial x^{\lambda}}M^{\lambda}_{\mu\nu} - \frac{\partial x^{\lambda'}}{\partial x^{\lambda}}\frac{\partial^2 x^{\lambda}}{\partial x^{\mu'}\partial x^{\nu'}}. \tag{32}$$

The above transformation law clearly and immediately verifies the fact that the affine connection, $\overline{\Gamma}^{\lambda}_{\mu\nu}$, is indeed a tensor. At this point, we have achieved with relative ease to obtain tensorial affinities and thus the task now is to obtain physically meaningful field equations that conform with natural reality. Equation 30 can be written

$$\nabla_{\sigma} g_{\mu\nu} = 0, \tag{33}$$

where $\nabla_{\mu} = \partial_{\mu} + J_{\mu}$ which in comparison to Quantum Mechanics is the Gauge Covariant Derivative. Before leaving this section, it is for latter purposes a good exercise to define the transformation properties of the object ρ. If $\psi' = \mathbf{S}'\psi$ where $\mathbf{S}'$ is some 4×4 transformation matrix, then transformation equation 31 requires that $\rho' = \chi'\rho$ where $\chi' = \chi'(x^{\mu})$. For $\rho' = \chi'\rho$ would require that

$$\mathbf{S}'^{\dagger}\mathbf{S}' = \phi'\mathbf{I} \tag{34}$$

where here $\mathbf{I}$ is the 4×4 identity matrix. As will be seen in Paper IV, the above transformation law will prove vital. It is not difficult to check or see that

$$\chi' = \exp\left(-2\int\left(\frac{\partial^2 x^{\lambda}}{\partial x^{\lambda}\partial x^{\mu'}}\right)dx^{\mu}\right). \tag{35}$$

6 Klein-Gordon & the Higgs Field

We obtain the Klein-Gordon equation and introduce a scalar field

into the theory. To derive the Klein-Gordon equation, let us begin by taking the derivative of equation 30 with respect to x_σ, that is

$$\Box g_{\mu\nu} - V(\rho)\, g_{\mu\nu} = 0, \tag{36}$$

where $V(\rho) = \partial^\lambda J_\lambda - J^\lambda J_\lambda$ and $\Box = \partial^\lambda \partial_\lambda$. This equation has the same form as the Klein-Gordon equation and thus shall be taken as such, the meaning of which is that we have to set $V(\rho) = m_0^2 c^2/\hbar^2$ where m_0 is the rest mass of the particle and $\hbar$ is Planck's normalized constant. Now if we raise and contract the indices of the metric, that is g^μ_μ, it is clear that $V(\rho) \equiv 0$ because $g^\mu_\mu = -4$, thus this means particles will have to a zero mass contrary to the fact that particles do have mass. To avert this problem and thus endow particles with mass, it is necessary to introduce a scalar field into the theory, that is $g_{\mu\nu} \longmapsto \bar{g}_{\mu\nu}$ where

$$\bar{g}_{\mu\nu} = \Phi g_{\mu\nu}, \tag{37}$$

this means $\bar{g}^\mu_\mu = -4\Phi$. The introduction of this scalar field is consistent with the Reimann-Hilbert geometry. Since this field endows particles with mass and we know that in the Standard Model of Particle Physics, particles acquire their mass via a scalar field known as the Higgs field or Higgs Particle, it is most natural here to identify the scalar field Φ with the Higgs field and this field satisfies the Klein-Gordon equation

$$\Box\Phi - V(\rho)\,\Phi = 0. \tag{38}$$

With the metric replacement equation 37 into equation 36 can be now written as $\Box g_{\mu\nu} - V(\rho,\Phi) g_{\mu\nu} = 0$ where $V(\rho,\Phi) = (\Box\Phi)/\Phi - 2\partial_\lambda\Phi\partial^\lambda g_{\mu\nu} - V(\rho)$ and now this means $V(\rho,\Phi) = 0$, and if the setting

$\partial_\lambda \Phi \partial^\lambda g_{\mu\nu} = 0$ is made, this means $V(\rho) = (\Box\Phi)/\Phi = m_0^2 c^2/\hbar^2$ and hence m_0 is not identically equal to zero. I will not explore further the meaning of the above equation vis-a-vis its meaning regarding particle solutions but leave this for Paper VI where the full meaning field is sought.

7 Source Coupled Field Equations

Just a reminder, the introduction of the Higgs field or the scalar field Φ now requires us to write the metric as $\bar{g}_{\mu\nu}$ and not as $g_{\mu\nu}$, thus the Reimann tensor on the RHS is defined

$$\bar{R}_{\sigma\mu\alpha\nu} = \frac{1}{2}\left(\nabla_\mu\nabla_\alpha\bar{g}_{\sigma\nu} - \nabla_\mu\nabla_\nu\bar{g}_{\sigma\alpha} + \nabla_\sigma\nabla_\nu\bar{g}_{\mu\alpha} - \nabla_\sigma\nabla_\alpha\bar{g}_{\mu\nu}\right), \qquad (39)$$

and this can be split into two, that is $\bar{R}_{\sigma\mu\alpha\nu} = R_{\sigma\mu\alpha\nu} + T_{\sigma\mu\alpha\nu}$ where

$$R_{\sigma\mu\alpha\nu} = \frac{1}{2}\left(\partial_\mu\partial_\alpha\bar{g}_{\sigma\nu} - \partial_\mu\partial_\nu\bar{g}_{\sigma\alpha} + \partial_\sigma\partial_\nu\bar{g}_{\mu\alpha} - \partial_\sigma\partial_\alpha\bar{g}_{\mu\nu}\right), \qquad (40)$$

and

$$\begin{aligned} T_{\sigma\mu\alpha\nu} = {} & \tfrac{1}{2}\left(\partial_\mu J_\alpha g_{\sigma\nu} - \partial_\mu J_\nu g_{\sigma\alpha} + \partial_\sigma J_\nu g_{\mu\alpha} - \partial_\sigma J_\alpha g_{\mu\nu}\right) \\ & -\tfrac{1}{2}\left(J_\mu J_\alpha g_{\sigma\nu} - J_\mu J_\nu g_{\sigma\alpha} + J_\sigma J_\nu g_{\mu\alpha} - J_\sigma J_\alpha g_{\mu\nu}\right) \end{aligned} . \qquad (41)$$

This tensor must identically equal zero since $\bar{\Gamma}^\lambda_{\mu\nu} = 0$ and $\nabla_\sigma\bar{g}_{\mu\nu} = 0$ thus this means the Ricci tensor on this spacetime must equal zero, that is $\bar{R}_{\mu\nu} = 0$. Using equation 30 to substitute for the derivatives, it

is easy to show that $\bar{R}_{\mu\nu} = -\Box\bar{g}_{\mu\nu}/2 + 2\Phi J_\mu J_\nu$ and further remembering equation 37 and using equation 36 and 38 this reduces to give

$$\Box g_{\mu\nu} + V(\rho)\, g_{\mu\nu} - 4J_\mu J_\nu = 0. \tag{42}$$

The above equation 42 constitutes the source coupled field equation in much the same-way as Maxwells source coupled equation of motion. Even if $\bar{R}_{\sigma\mu\alpha\nu} \neq 0$ because $\bar{\Gamma}^{\lambda}_{\mu\nu} \neq 0$, one would use the second Bianchi identity $\nabla_\nu \bar{R}_{\alpha\beta\mu\rho} + \nabla_\rho \bar{R}_{\alpha\beta\nu\mu} + \nabla_\mu \bar{R}_{\alpha\beta\rho\nu} = 0$ and derive the Einstein tensor equation for this spacetime, that is $\bar{G}_{\mu\nu} = \bar{R}_{\mu\nu} - R\bar{g}_{\mu\nu}/2 = 0$ where R is the Ricci scalar for this spacetime, to arrive at the same equation. Equation 42 can be re-written as

$$\Box A_\mu + V(\rho)\, A_\mu - \epsilon J_\mu = 0, \tag{43}$$

where $\epsilon = 4A^\mu J_\mu$. This equation is the well known Maxwells-Proca source field equations and its is clear that the gauge condition $\partial^\mu A_\mu$ is in-built. Raising and contracting the indices of equation 42, we obtain

$$J^\mu J_\mu = -V(\rho) = -\left(\frac{m_0 c}{\hbar}\right)^2, \tag{44}$$

and from this equation, one is able to obtain the equation governing the field ψ and to deduce the conservation equation $\partial^\mu J_\mu = 0$.

8 Source Free Field Equations

Without much loss let us proceed to derive the source free field equations from the first Bianchi identity. Since $\bar{R}_{\sigma\mu\rho\nu} = 0$, it follows that $R_{\sigma\mu\rho\nu} = T_{\sigma\mu\rho\nu}$ and the first Bianchi identity gives

$$R_{\sigma\mu\rho\nu} + R_{\sigma\nu\mu\rho} + R_{\sigma\rho\nu\mu} = 0, \tag{45}$$

hence $T_{\sigma\mu\rho\nu} + T_{\sigma\nu\mu\rho} + T_{\sigma\rho\nu\mu} = 0$. Now from this equation, we will be able to generate the second group of Maxwell's equations. We will obtain the source free field equation from the Bianchi identity equation 45. Given the definition equation 48 we have

$$R_{\sigma\mu\rho\nu} = \partial_\nu \left(F_{\mu\rho}A_\sigma\right) + \partial_\sigma \left(F_{\nu\rho}A_\mu\right), \tag{46}$$

where $F_{\mu\rho}A_\sigma = \left[\partial_\mu, \Phi A_\rho\right] A_\sigma + \left[A_\mu, \partial_\rho\right](\Phi A_\sigma) = \partial_\mu \overline{g}_{\sigma\rho} - \partial_\rho \overline{g}_{\sigma\mu}$ and where [,] is the usual commutator bracket and the commutation here and hereafter is in the indices (μ, ν). The quantity $F_{\mu\nu}$ is a operator acting on A_σ and when this operator acts on A_σ, we shall write the resultant equation as $F_{\mu\nu}A_\sigma = \Phi \mathrm{F}_{\mu\nu}A_\sigma$ where

$$\mathrm{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g^*_{\mu\nu} A_\mu A_\nu, \tag{47}$$

is the well known Yang-Mills Tensor ([34]) where $g^*_{\mu\nu} = g^*_\mu - g^*_\nu$ and this tensor equation 47 is only possible if the choice

$$\partial_\mu A_\nu = g^*_\mu A_\mu A_\nu, \tag{48}$$

is made where g^*_μ are generally non-zero constants is necessary. This choice leads directly to the Proca equation. To obtain the Proca equation, we take the partial derivative ∂^μ of equation 48 and this results in the Proca equation

$$\Box A_\mu - \left(\frac{1}{\ell_c}\right)^2 A_\mu = 0, \tag{49}$$

where $1/\ell_c^2 = g^*_\mu \partial^\mu A_\mu + (g^*_\mu A_\mu)(g^{*\mu} A^\mu)$ can be a constant or a variable. Equation 49 is exactly soluble and this means the dreams of exactly

solubility of Yang-Mills Theory as outlined by Antony Zee ([35]) are realizable. Before going to the next section, for latter purposes (in Paper II), it is important to note the following, that the four vector quantity $g^*_\mu A_\mu$ is a tanget to A_μ, thus

$$A^\mu g^*_\mu A_\mu \equiv 0, \tag{50}$$

and this condition automatically imposes the Lorentz gauge condition

$$\partial^\mu A_\mu \equiv 0, \tag{51}$$

and further, if $A_\mu = 1/A^\mu$, this means $\sum_0^3 g^*_\mu \equiv 0$. In Paper IV, where the Reimann-Hilbert geometry is clearly laid down, the condition $A_\mu = 1/A^\mu$ will be laid down as one of the axioms of the geometry. Thanks to the imposition equation 48 without which there is no forseeable way to obtain the Yang-Mills Theory. **Note** that because $\partial_\mu \Phi = A_\nu \partial^\nu \left(\Phi A_\mu\right) \Longrightarrow [\partial_\mu \Phi, A_\nu] \equiv 0$. Since $F_{\mu\nu} A_\sigma = \partial_\mu g_{\sigma\nu} - \partial_\nu g_{\sigma\mu}$ it is easy to check that

$$F_{\mu\nu} A_\sigma + F_{\sigma\mu} A_\nu + F_{\nu\sigma} A_\mu \equiv 0, \tag{52}$$

and this implies $A_\sigma \mathrm{F}_{\mu\nu} + A_\nu \mathrm{F}_{\sigma\mu} + A_\mu \mathrm{F}_{\nu\sigma} \equiv 0$. It can also be shown that $J_\sigma \mathrm{F}_{\mu\nu} + J_\nu \mathrm{F}_{\sigma\mu} + J_\mu \mathrm{F}_{\nu\sigma} \equiv 0$ and with this we will have $R_{\sigma\mu\rho\nu} = \Phi A_\sigma \mathrm{D}_\nu \mathrm{F}_{\mu\rho} - \Phi A_\sigma J_\nu \mathrm{F}_{\mu\rho} + \partial_\sigma \left(F_{\nu\rho} A_\mu\right)$ where $\mathrm{D}_\nu = \partial_\nu + g^*_\nu A_\nu$ and now substituting all the above into the Bianchi identity equation 45, we will have after some cleaning-up

$$\mathrm{D}_\rho \mathrm{F}_{\mu\nu} + \mathrm{D}_\nu \mathrm{F}_{\rho\mu} + \mathrm{D}_\mu \mathrm{F}_{\nu\rho} = 0. \tag{53}$$

There we have it again! In summary, we have arrived at the desired field equations, that is, equations 42 and 53. As for equation 53, not

only does it give us the second group of Maxwell's equations but also the source free non-Abelian Gauge field equations that describe nuclear forces and this depending on the values of $g^*_{\mu\nu}$ – basically the nuclear forces are governed by the same Maxwell's equations as will be seen soon!

In the following sections I will proceed to show that the components of the metric have the capability to explain the known natural forces, that is, the Electromagnetic force, the Weak and the Strong and this depends on the values that g^*_μ takes and as will be seen, the already rich library of the worked out mathematics of the these forces makes the task of showing this a relatively easy task.

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References

[1] Brans C. & Dicke R. H., 1961, *Phys. Rev.*, **124**, 925.

[2] Einstein A., 1905, Annalen der Physik, **17**, 981.

[3] Einstein A., 1907, Translation by Schwartz H. M., 1977, Am. Journal of Phys., **45**, 10.

[4] Einstein A., 1915, Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin: *Die Feldgleichungun der Gravitation*, p844-847.

[5] Einstein A., 1917, *Sitz Preuss Akad. d. Wiss Phys.-Math,* **142**.

[6] Einstein A., 1919, *Spielen Gravitationsfelder in Aufbau der materiellen Elementarteilchen eine wesentliche Rolle?*, Sitzungersber. Preuss. Akad. Wiss.(20), 349-356.

[7] Einstein A., 1920, *Antowort auf vorstehende Betrachtung*, Die Naturwissenschaften, **8**, 1010-1011.

[8] Einstein A., 1921, *Geometrie und Erfahrung*, Sitzungersber. Preuss. Akad. Wiss.(12-14), 123-130.

[9] Einstein A., 1921, *Über eine naheliegende Erg "anzung des Fundamentes der allgemeinen Relativitätstheorie*, Sitzungersber. Preuss. Akad. Wiss.(12, 13, 14), 261-264.

[10] Einstein A., 1923, *Bemerkungen zu meiner Arbeit "Zur allgemeinen Relativitätstheorie*, Sitzungersber. Preuss. Akad. Wiss.(12-14), 76-77.

[11] Einstein A., 1945., Ann. Math., A Generalisation of The Relativistic Theory of Gravitation, **46**, 578-584.

[12] Eddington A., 1921, *Proc. R. Soc. London, Ser. A*, A Generalisation of Weyl's Theory of the Electromagnetic & Gravitational Fields, **99**, 104-122.

[13] Gödel K., 1949, *An Example of a New Type of Cosmological Solution of Einstein's Field Equations of Gravitation*, Phys. Mod. Rev. **21**, 447-450.

[14] Kaluza T., 1921, *Zum Unitätsproblem in der Physsik*, *Sitzungsber. Preuss. Akad. Wiss.*, 966-972.

[15] Klein O., 1962, *Quanten-Theorie und 5-dimensionale Relativitätsheorie*, Z. Phys., **37**, 895-906.

[16] Kragh H. S., 1990, Cambridge Univ. Press, *Dirac: A Scientifc Biography*, p 275-292.

[17] Lawden D. F., 1962, *Tensor Calculus & Relativity*, Spottiswoode Ballantyne & Co. Ltd. London & Colchester.

[18] Lee Y. Y., 1965, Chinese Journal of Physics, **3**, No 1, 45-68.

[19] Lorentz H. A., *Versuch einer Thoerie electrischen und optischen Erescheinungen in between Kŕper*, Brill, Leyden.

[20] Mach E., 1893, *The Science. La Salle, Ill.: Open Court Publishers*, 6th Eddition of the English Translation, 1960.

[21] Michelson A. A. *The Relative Motion of the Earth and the Luminiferous Aether."*, Amer. J. Sci. 22, 120-129, 1881.

[22] Michelson A. A. & Morley, E. W. *On the Relative Motion of the Earth and the Luminiferous Ether.*, Amer. J. Sci. 34, 333-345, 1887.; also see Philos. Mag. 24, 449-463, 1887.

[23] Maxwell J. C., 1873, Oxford: Clarendon Press. (Reprint of 3th Ed., 1998, Oxford Classic Series), Trease on Electricity and Magnetism, **1**, IX.

[24] Nyambuya G. G., 2007, *Dirac Equation in Curved Spacetime – On the Anomalous Gyromagnetc Ratio*, Electronic Journal of Theoretical Physics, **Vol. 4 No 14**, 1-11; Visit:http://arxiv.org/abs/0709.0936

[25] Salam A., 1981, *Physics News*, *Einstein's Last Dream: The Space-Time Unification of the Fundamental Forces*, Vol**12**, No 2: Or Visit http://www.iisc.ernet.in/academy/resonance /Dec2005/pdf/Dec2005p246-253.pdf

[26] Schrödinger E., 1948., *Proc. R. Irish Acad. A*, The final Affine Law II, **51**, 205-216.

[27] Smolin L., 2006, Houghton-Mifflin (Sept. 2006)/Penguin (UK, Feb 2007), *The Trouble With Physics*.

[28] Stephani H., 2004, *Cambridge University Press, Relativity*: An Introduction to Special and General Relativity, 3^{th} Edition, p 304.

[29] Thomas J. M., 1991, Adam Higler imprint by IOP Publishing ISBN 0-7503-0145-7, *Michael Faraday and the Royay Institution*, p74-78.

[30] Weyl H., 1918, *Sitzungsber. Preuss. Akad. Wiss(26)*, Gravitation und Elektrizität, 465-478.

[31] Weyl H., 1927a, *Z. Phys.*, Elektron und Gravitation I, **56**, 330-352.

[32] Weyl H., 1927b, *Proc. Nat. Acad. Sci*., Gravitation and the Electron, **15**, 323-334.

[33] Witten E., 2005, Nature, *Unvelling String Theory*, 435, 1085.

[34] Yang C. N. & Mills R., 1954, Phys. Rev. **96**, 191.

[35] Zee A., 2003, *Quantum Field Theory in a Nutshell*, Princeton University Press, 375.