An Example 4-Geon

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It has been shown by Hadley that the logic of quantum mechanics is consistent with general relativity when closed time-like curves are permitted. This is done by hypothesising particles as 4-geons: particle-like solutions to Einstein's field equations with closed time-like curves. Hadley provides axioms that need to be satisfied for quantum logic to be possible. Here, a candidate 4-geon and an interpretation of quantum measurement is provided that satisfy all of Hadley's axioms. The candidate solution is based on the extended 'fast' Kerr-Newman singularity and a new interpretation of measurement.

Introduction

This paper is dependent on "The Logic of Quantum Mechanics Derived From Classical General Relativity," *Foundations of Physics Letters*, 1997 by Mark J. Hadley, also available online at the time of writing. [Hadley 15]

It has been shown by Hadley [Hadley 15][Hadley 16] that the logic of quantum mechanics [Ballentine 2][Jauch 24] is consistent with General Relativity [Thorne et al. 35][Hawking et al. 19] when closed time-like curves are permitted. This is done by hypothesising particles as 4-geons: particle-like solutions to Einstein's field equations with closed time-like curves inherent in their structure. Hadley provides axioms that need to be satisfied for such quantum logic to be possible. Here, a candidate 4-geon and an interpretation of quantum measurement is provided that satisfy Hadley's axioms.

'Geons' as particle-like solutions to Einstein's field equations were first suggested by Einstein [Einstein, Rosen 11][Schilpp 32]. Wheeler took-up the idea and developed it further [Wheeler, Misner 28][Wheeler 36]. Neither of the above based the idea on non-time-orientability (or closed time-like curves). Not until Hadley has a way forward been found that could potentially be compatible with quantum logic.

Note: Strictly the existence of closed time-like curves [Friedman et al. 12][Thorne 34] does not require space-time to be non-time-orientable (as evidenced by the Gödel solution to Einstein's field equations that is both time-orientable and admits closed time-like curves [Gödel 14]). However, herein, the distinction will not be needed; no constraint on time-orientability is imposed beyond that required by Hadley: that closed time-like curves are permitted, and that (in general) this implies non-time-orientability. This assumption is reasonable as the Gödel solution and similar impose very restrictive mathematical constraints almost certainly incompatible with observation.

The 4-geon example provided here is based on the 'fast' Kerr-Newman singularity [Hawking et al. 19][Kerr 25][Newman, Janis 29][Newman et al 30] which has a history of suspected particle-like behaviour (usually in comparison with the electron) [Carter

7][Newman, Janis, 29][Lopez 26][Lopez 27][Israelit, Rosen 23][Israel 22]. The concept is Wheeler's "charge without charge" and "mass without mass" [Wheeler 36]. Spin ½ and other properties of the electron have also been found to have possible explanation in geometry and topology consistent with general relativity [Barut, Bracken 4][Barut, Thacker 5][Friedman, Sorkin 13][Hendriks 21][Diemer, Hadley 10][Hadley 17] [Arcos, Pereira 1], although it should be added that alternatives outside of general relativity have also been suggested [Sciama 33] and [Bell et al 6].

Hadley's axioms consist of 1 conjecture and 5 axioms. If these axioms are satisfied quantum logic is shown to be *consistent* with general relativity. This does not however *guarantee* quantum logic. In order to guarantee quantum logic it is further required that, as in a game of dice in Newtonian mechanics, that a probability measure is assumed to exist over the outcomes in question. Hadley [Hadley 15] justifies this by referring to the example of a Newtonian die, that is; probability measures are common place in Newtonian physics via statistical mechanics, likewise the probability measures of quantum mechanics result from similar considerations using the 4-geons of general relativity with closed time-like curves.

Hadley's 4-Geons [Hadley 15][Hadley 16] are hypothetical solutions to Einstein's field equations in a space-time with closed time-like curves (or non-trivial causal structure) that satisfy the following conjecture and axioms:

Hadley's Conjecture 1: (4-Geon) A particle is a semi-Riemannian space-time manifold, M, which is a solution of Einstein's equations of general relativity. The manifold is topologically non-trivial, with a non-trivial causal structure, and is asymptotically flat (see axiom 1) and particle-like (see axiom 2).

Hadley's Axiom 1: (Asymptotic Flatness) Far away from the particle space-time is topologically trivial and asymptotically flat with an approximately Lorentzian metric.

Hadley's Axiom 2: (Particle-Like) In any volume of 3-space an experiment to determine the presence of the particle will yield a true or false value only.

Hadley's Axiom 3: (State Preparation) The state preparation sets boundary conditions for the solutions to the field equations.

Hadley's Axiom 4: (Measurement Process) The measurement process sets boundary conditions for the 4-geon which are not necessarily redundant, in the sense that they contribute to the definition of the 4-manifold.

Hadley's Axiom 5: (Exclusive Experiments) Some pairs of experiments are mutually exclusive in the sense that they cannot be made simultaneously.

None of Hadley's axioms add anything new to general relativity (beyond allowing closed time-like curves/ non-time-orientability). These axioms are constraints that if satisfied enable general relativity to be capable of performing quantum logic in the sense of an orthomodular lattice of propositions, as opposed to a Boolean logic [Hadley 15]. Strictly, Conjecture 1 is not required, only the 5 axioms. In addition other field equations could be substituted for Einstein's field equations with similar results, but as Hadley notes, it is compelling if 4-geons were solutions to Einstein's field equations, hence Conjecture 1.

The 4-geon presented in this paper is shown to satisfy Hadley's Conjecture 1 and Axioms 1 and 2. An interpretation of quantum measurement (called 'open-endedness') is developed that satisfies Hadley's axioms 3, 4 and 5. Compatibility of the 4-geon solution with this interpretation of measurement means that Hadley's 4-geon conjecture is furnished with an example.

A Candidate 4-Geon

One of the problems identified by Hadley with respect to 4-geons is that no possible 4-Geon has yet been identified. In order to clear up this problem, a candidate 4-Geon is presented that satisfies Conjecture 1 and Axioms 1 and 2. It is based on the 'fast' Kerr-Newman singularity [Arcos, Pereira 1] and the extended 'fast' Kerr-Newman interpretation of Hawking and Ellis [Hawking et al 19] [Arcos, Pereira 1]. Arcos and Pereira derive some very interesting properties for their variant of the extended 'fast' Kerr-Newman solution.

From their abstract [Arcos, Pereira 1]: "For $m^2 < a^2 + q^2$, with m, a, and q respectively the source mass, angular momentum per unit mass, and electric charge, the Kerr-Newman (KN) solution of Einstein's equation reduce to a naked singularity of circular shape, enclosing a disk across which the metric components fail to be smooth. By considering the Hawking and Ellis extended interpretation of the KN spacetime, it is shown that, similarly to the electron-positron system, this solution presents four inequivalent classical states. Making use of Wheeler's *charge without charge*, the topological structure of the extended KN spatial section is found to be highly non-trivial, leading thus to the existence of gravitational states with half integral spin..."

As explained in their introduction [Arcos, Pereira 1]: "Due to the absence of an horizon, it does not represent a black hole, but a naked singularity in spacetime. This solution is of particular interest because it describes a massive charged object with spin…"

The KN solution in the Boyer Lindquist coordinates r, θ, φ is given by [Hawking et al 19]:

$$ds^2 = dt^2 - (\rho^2/\Delta)dr^2 - (r^2+a^2)\sin^2\theta d\varphi^2$$
$$-\rho^2 d\theta^2 - (Rr/\rho^2)(dt - a\sin^2\theta d\varphi)^2$$

where,

$$\rho^2 = r^2 + a^2\cos^2\theta,\ \ \Delta = r^2 - Rr + a^2 \text{ and } R = 2m - q^2/r$$

For $m^2 > a^2 + q^2$ this represents the Kerr-Newman blackhole. This metric is invariant under the simultaneous changes (t, a) → (–t, –a), (m, r) → (–m, –r) and separately under q → –q. However, the fast Kerr-Newman singularity is given by $m^2 < a^2 + q^2$ and is a true circular singularity of radius a, enclosing a disk across which the metric components fail to be smooth.

The Hawking and Ellis extended spacetime [Hawking et al. 19] is such that the solution is extended (to overcome lack of smoothness of the metric components, but not to eliminate the singularity itself) so as to form a join (not disimilar to a 'wormhole') with a corresponding singularity in another spacetime, in other words (those of Arcos and Pereira): "the disk surface (with the upper points, considered different from the lower ones) is interpreted as a shared border between our spacetime denoted **M,** and another similar one denoted **M'**…this linking can be seen as solid cylinders going from one 3-manifold to the other."

Observation 1: Such an extended 'fast' Kerr-Newman solution can be used to join two *suitably distant* locations on a single space-time manifold, rather than two different spacetimes. *Far from both singularities* such a spacetime can also be topologically trivial, asymptotically flat and approximately Lorentzian without loss of generality. The two singularities can be placed on the manifold so that a time-like curve connects them. Continuation of the time-like curve through the singularities leads to closed time-like curves; this is possible as a time-like curve entering a singularity does so in finite proper time.

This leaves only Axiom 2 to be demonstrated in order to demonstrate Axiom 1, 2 and conjecture 1:

Observation 2: Any one of the two singularities is particle-like (satisfying Hadley's Axiom 2) since the singularity has well-defined radius a. Further they are defined so as to be 'suitably distant' from each other so that any coupling between the solutions may be modelled as asymptotically particle-like (this is possible due to asymptotic flatness).

Having observed that the extended 'fast' Kerr-Newman singularity can satisfy axioms 1 and 2 and conjecture 1, the feasibility of 4-Geons as solutions to Einstein's field equations has been demonstrated.

Independently of this solution, an interpretation of Hadley's concept of measurement (demonstrating only the last 3 axioms) will now be given. It will be shown in the next section that the 4-geon is compatible with this interpretation of measurement and therefore a complete example of Hadley's 4-Geon is provided by this extended 'fast' Kerr-Newman solution.

As a candidate 4-geon it is interesting to note that Arcos and Pereira [Arcos, Pereira 1] go on to make some assumptions about quantum gravity, which may or may not be warranted, from which they derive the Dirac Equation for their 'electron'.

Open-Ended Space-Times

To provide an interpretation for Hadley's concept of measurement in his hypothetical 4-Geon world, an inherently non-deterministic and unusual definition of space-time will be given by adapting the concept of boundary conditions. The starting point, as with Hadley, is to define space-time manifolds such that they allow closed time-like curves:

Definition 1: A *space-time manifold* is a four-dimensional (+,-,-,-) Lorentz manifold together with its metric (no mention of time-orientability).

Definition 2: An E.F.E. *space-time manifold* is a space-time manifold that additionally satisfies Einstein's field equations.

Definition (Provisional): A *boundary condition on a manifold* is a constraint on a manifold that restricts the metric and/or the topology of the manifold. That is, simply a constraint on the manifold definition.

Definitions (Provisional): A *redundant boundary condition* is a boundary condition that makes no restriction in the situation in question (perhaps because other boundary conditions have already been applied). An *impossible boundary condition* is one that can lead to no possible solution given the other boundary conditions in place. And a *deterministic boundary condition* is one that imposes a single definition on the manifold given the other boundary conditions already in place.

Since these definitions are dependent on other boundary conditions, these definitions fail to have consistent universal meaning. In order to overcome this the following (unusual) definition of *boundary condition* and *space-time* is made:

Definition 3 (Final): A *boundary condition* is defined to be an equivalence class of constraints Λ on a manifold entirely equivalent to stating that the solution must be one of (or none of) a set *M* of well-defined manifolds.

The constraints Λ are no more than a list of included or equivalently excluded solutions, entirely logically equivalent to the set *M*.

Definition 4: A *space-time* S(*M*; Λ) (or S or S(*M*) or S(Λ)) is the set *M* of four-dimensional Lorentz manifolds together with metric that are satisfied by a set Λ of boundary conditions upon that manifold.

Λ can be viewed as a set of sub-boundary conditions or as a single boundary condition as required. These boundary conditions can be 'added' Λ + Λ' or set-unioned equally. There is a 1-to-1 correspondence between S, *M* and Λ.

Definition 5: An *E.F.E. space-time* S(*M*; Λ) (or S or S(*M*) or S(Λ)) can now be defined similarly: as a space-time where each member of *M* satisifies Einstein's field equations, and such that this constraint is implied and not specified in Λ.

We do not need to worry about the exact formulation of Einstein's field equations, or whether there are other factors such as electromagnetism, for the purposes of this definition. But the noninclusion of Einstein's field equations as a boundary condition in Λ makes for simpler definitions later on.

Clearer definitions relating to boundary conditions are now possible:

Definitions 6 (Final): A *redundant boundary condition* on space-time S(*M*; Λ) is a boundary condition on the space-time manifolds *M* of S such that when added to Λ it makes no reduction to the possible solutions (because equivalent boundary conditions are already contained in Λ). An *impossible boundary condition* is a boundary condition that when added to Λ can lead to no possible solution, that is, *M* becomes empty. And a *deterministic boundary condition* is one that imposes a singular definition on the space-time, that is, *M* has one member. A *non-deterministic boundary condition* is one that is neither deterministic nor impossible. We refer to S(*M*; Λ) before a boundary condition *s* has been applied, and S'(*M'*; Λ + *s*) or S'(*M'*; Λ') afterwards. Depending on context S'(*M'*; Λ') could also just refer to an alternative 'space-time' from S(*M*; Λ).

These definitions are motivated by non-determinism in Hadley's hypothetical 4-Geon world, and we can now define a non-deterministic space-time as follows:

Definitions 7: A *non-deterministic space-time* is a space-time whose boundary conditions are non-deterministic. (We can similarly refer to *impossible space-times* and *deterministic space-times*.)

We can now define state preparations (more precisely than [Hadley 15]):

Definition 8: A *state preparation* on a space-time is a non-redundant, non-impossible boundary condition.

Observation 3: State preparations on E.F.E space-times necessarily satisfy Hadley's Axiom 3 via non-redundancy.

Definitions 9: A set of state preparations with respect to space-time S(M; Λ) is called a *state preparation set*. A *non-deterministic state preparation set* consists of non-deterministic state preparations. The *complete state preparation set* C of S(M; Λ) consists of all possible state preparations, and the *empty state preparation set* E contains no state preparations.

Note that E is not a state preparation but an empty set of state preparations. The Λ of S(M; Λ) can also be treated as a state-preparation set with 1 member. In this way we can define S(T; E) where T is the set of all possible manifolds.

Observation 4: Since members of a state preparation set may be mutually impossible with respect to S(M; Λ), a state preparation set may not be considered a state preparation itself. This is an important observation as it leads to the logic of mutually exclusive experiments (i.e. axiom 5).

Definition 10: A state preparation set of S(M; Λ) is *pair-wise non-redundant* if it does not contain two state preparations such that one is redundant with respect to the other with respect to Λ. Similarly, state preparation *s1* in a state preparation set S of S(M; Λ) is said to be *redundant to* another state preparation *s* of S, if *s1* is redundant in S(M'; Λ') where Λ' = Λ + *s*. Note that this property is not commutative.

Definition 11: A state preparation set of S(*M*; Λ) is *pair-wise exclusive* if any two state preparations are mutually impossible with respect to Λ. Any two state preparations can be *pair-wise exclusive*.

Definition 12: A state preparation set of S(*M*; Λ) is *non-trivial* if it has at least 2 members that are pair-wise non-redundant and pair-wise exclusive.

We can equally give these definitions set notation using *M* instead of Λ, for example: Let *M*(Λ) be the set of manifolds associated with state preparation Λ. Non-triviality is such that there are two state preparations *s1* and *s2* in the state preparation set in question such that *M*(Λ+*s1*) is disjoint (exclusive) from *M*(Λ+*s2*) and both are non-empty (non-redundant).

Definitions 13: With respect to space-time S(*M*; Λ). Each member m of *M* can be associated with an element *s* of the complete state preparation set C of S such that *s* is equivalent to specifying that the only member of *M*' is m. Call *s* a *determiner* of m.

We can equally define the *anti-determiner* of m as a state preparation that is equivalent to prohibiting m (that is allowing all others). A determiner and an anti-determiner of m are pair-wise exclusive. Anti-determiners only exist for non-deterministic space-times.

Definition 14: Each member of the complete state preparation set C of S(*M*; Λ) that is pair-wise redundant to state preparation *s* of S(*M*; Λ) is called the *redundancy set* of *s*, or of *M*(s). For example, the anti-determiner of m is trivially in the redundancy sets of the determiners of all other members of *M*.

Lemma 1: For any non-deterministic space-time there exists a non-trivial state preparation set.

Proof: Since the space-time is not uniquely specified (it is non-deterministic) at least two possible solutions will satisfy the boundary conditions, call these A and B. At least two state preparations exist: A state preparation *A* can be applied to the manifold in order to specify A or alternatively a state preparation *B* can be applied to specify the manifold B. These two state preparations are pair-wise non-redundant and pair-wise exclusive by construction.

Lemma 2: Not every non-deterministic space-time has a non-deterministic non-trivial state preparation set.

Proof: By adding to the boundary conditions of the non-deterministic space-time in lemma 1 the constraint 'that only manifolds A or B satisfy the boundary conditions', we have at least two state preparations *A* and *B* that form a non-trivial state preparation set (lemma 1). However, both *A* and *B* are deterministic. Since there are only two possible solutions, all other possible non-redundant state preparations must be equivalent to *A* or *B* – but all state preparations are non-redundant by definition. Therefore all such state preparations are deterministic.

Because of Lemma 2 attempting to equate non-deterministic non-trivial state preparations with quantum measurements/experiments is not sufficient, even though by so doing Hadley's Axioms 3, 4 and 5 could be satisfied. We need to overcome the fact that finite solutions (as above with a choice of A or B) can lead quickly to determinism.

Therefore define open-ended space-times:

Definition 15 (Open-Ended Space-Time): Any non-deterministic space-time that has an infinitely large set M of different possible manifolds is called an *open-ended space-time*.

The existence of such a manifold is trivial to the extent that when no boundary conditions are placed on a space-time (or E.F.E. space-time) an infinite number of solutions are possible.

Observation 5: Similarly the 4-geon example given previously can be part of an open-ended space-time since being particle-like (asymptotically flat) an infinite number of variant solutions are available far from the particle (trivially an infinite number of such particles could be separated by suitably large time-distances in an otherwise flat space-time). Therefore there is an open-ended space-time that satisfies Hadley's conjecture 1 and Axioms 1 and 2.

By considering the determiners of an open-ended space-time M it is clear that there is an infinite set of pair-wise non-redundant and exclusive state preparations. However, determiners are by definition deterministic. An infinite set of non-deterministic, pair-wise non-redundant state preparations exist in the form of the set of anti-determiners, but no two of these are pair-wise exclusive. However, a solution exists.

Lemma 3: Every open-ended space-time S(M; Λ) has a non-deterministic non-trivial state preparation set.

Proof: First divide M into two separate infinite sets $M1$ and $M2$. Now take the anti-determiners for $M1$ and $M2$ and combine them into a single 'anti-determiner' for $M1$ and $M2$ respectively: call these –m1 and –m2. –m1 and –m2 are state preparations. These are pair-wise non-redundant and pair-wise exclusive, but are non-deterministic.

The following extension of Lemma 3 now follows as a matter of course:

An Open-Endedness Theorem: Further to Lemma 3, Every open-ended space-time S(*M*; Λ) has a non-deterministic, non-trivial state preparation set Ψ such that for each member open-endedness of the space-time is invariant.

Proof: Take the redundancy set for *M1* and the redundancy set for *M2*: call these +*M1* and +*M2*, both still infinite sets by construction. –m1 is necessarily in +*M2*, and –m2 is necessarily in +*M1*. Assume Ψ to be the same as in Lemma 3. And without loss of generality choose –m1. The resulting space-time is S'(*M2*; Λ + -m1) which on account of *M2* being infinite by definition is necessarily open-ended.

Definition 16: Call a non-deterministic, non-trivial state preparation set on an open-ended space-time, where every state preparation leaves open-endedness invariant, an *open-ended state preparation set*. It consists of *open-ended state preparations*. Similarly define the *complete open-ended state preparation set* as the set of all possible open-ended state preparations on a space-time.

We can now capitalize on this idea by defining 'experiments' in terms of open-endedness.

Definition 17 (Outcome): An *outcome* of an experiment is an open-ended state preparation.

Hadley's Axiom 3 and Axiom 5 are now guaranteed, Axiom 3 by non-redundancy of state preparations and Axiom 5 by non-triviality of open-ended state preparation sets. Axiom 4 also follows to the extent that non-redundant state preparation is defined to be part of 'measurement':

Definition 18: An *experimental state preparation e* is an open-ended state preparation imposed by the experimental machinery. This can be subsumed without loss of generality into the definition of $S(M; \Lambda)$ as just a re-labelling of $S'(M'; \Lambda + e)$.

The requirement for a definition of experimental state preparations comes from the fact that they are used by Hadley [Hadley 15] in his description of measurement.

Quantum Measurement: is therefore just a space-time, with suitable experimental state preparation, a state preparation set of possible outcomes, and the actual outcome.

Determination of a probability function over the outcomes is conspicuously missing (see Classical and Quantum Measurement below), but axioms 3, 4 and 5 are satisfied as required.

Classical Measurement: becomes the extraction of information about Λ when the underlying manifold is locally causal (see Classical and Quantum Measurement).

Observation 5 plus the interpretation of quantum measurement given here now leads to the main result of this paper:

An Example 4-Geon: A 4-Geon exists that satisfies Hadley's Conjecture 1 and Axioms 1 to 5. Demonstrating the feasibility of Hadley's 4-geons.

Classical and Quantum Measurement

A candidate 4-geon has been presented that satisfies Hadley's Conjecture and Axioms 1 and 2. The candidate 4-geon, a variant of the fast Kerr-Newman singularity, has also been shown to be compatible with the open-ended interpretation of measurement, within the constraints of general relativity (with closed time-like curves); therefore an example 4-geon satisfying all of Hadley's axioms has been provided.

The connection between initial boundary condition Λ (or Λ following experimental state preparation) and 'outcome' is what is here hypothesised to be *quantum measurement*. As such 'open-ended quantum measurement' already satisfies the last 3 of Hadley's Axioms and has been shown to have other necessary properties, such as always allowing for further quantum measurement (and not degenerating into determinism): any measurement under this interpretation leaves the open-endedness of the space time invariant.

That probability functions are missing so far is conspicuous, but all of Hadley's axioms are satisfied, so any probabilistic measure over suitable 'outcomes' must necessarily manifest quantum logic.

Even though open-endedness may be only one possible definition of measurement out of many, whether this definition in fact constitutes 'quantum measurement' (physically) or not in no way diminishes the importance of the interpretation as an example 4-geon. The example remains, mathematically, an example. This is the case because an open-ended interpretation is consistent with classical measurement as follows: all classical measurement assumes a deterministic general relativistic physics (or classical physics), and therefore the causality defined by the existence of a Cauchy surface. Information about such a Cauchy surface or region constitutes classical measurement. But if we remove the causality constraint imposed by the existence of a Cauchy surface we are left with classical measurement as information about a space-like submanifold of co-dimension 1 or a comparable space-like region. A Cauchy surface is just what a space-like submanifold of co-dimension 1 becomes under the usual causal constraints where time-orientability is required. With this in mind, information about such a space-like region of a manifold where closed time-like curves are allowed is the classical limit of measurement. Since such a region imposes little constraint on the 4-manifold of space-time dominated by closed time-like curves far from the region in question, we can conclude that classical measurement is open-ended!

Observation 6: asymptotic flatness of 4-geons allows them to be placed far from a known region of space-time in an infinite number of different locations, hence classical measurement is open-ended.

Therefore open-endedness is a superset of classical measurement that is definable when causality is weakened and that along with the example 4-geon given obeys Hadley's axioms for quantum logic.

Observation 7: This completes the description of the example 4-geon, and the main purpose of this paper. In summary: An example 4-geon has been given with an interpretation of measurement for general relativity where closed time-like curves are allowed. The interpretation of measurement is a superset of classical measurement, where classical measurement is the causal limit.

The Open-Endedness Conjecture: Further questions have perhaps arisen, for example assuming the model for measurement envisaged here to have a physical reality, where are the probability functions of quantum mechanics predicted by Hadley, how can we actually generate them, or impose them? Another question would be, why does causality appear to dominate on the classical scale if space-time is in fact not causal as Hadley's program suggests? Further, there is no reason that open-endedness is the only way to produce example 4-geons, so, what other possible definitions of measurement could produce similar results? Whatever the case, to the extent that these questions go beyond providing a mathematical example to Hadley's axioms they inevitably involve speculation; this is not the purpose of this paper, although some speculation has inevitably happened in passing. The speculation that has occurred in passing is that the model for measurement used here (or one similar) is more than just a mathematical example but actually constitutes physical reality. This can be called the *open-endedness conjecture.*

Conclusion

Hadley [Hadley 15][Hadley 16] has shown that quantum logic is compatible with general relativity when certain axioms are satisfied by non-time-orientable solutions. A candidate 4-geon has been presented here that satisfies Hadley's Conjecture and Axioms 1 and 2. The candidate 4-geon, a variant of the fast Kerr-Newman singularity, has also been shown to be compatible with a new interpretation of measurement called the open-ended interpretation, within the constraints of general relativity, satisfying the rest of Hadley's axioms; therefore an example 4-geon satisfying all of Hadley's axioms and his conjecture has been provided.

The reasonableness of open-endedness as a model for measurement follows from classical measurement being a subset of open-endedness measurements, or their causal limit. Other definitions of measurement producing similar results could no doubt be constructed, whether open-endedness actually corresponds to physical measurement is a conjecture: *the open-endedness conjecture*.

Open-ended space-time is really a collection of possible space-times, defined so as to be consistent with boundary conditions Λ. This is entirely consistent with partial information about a general relativistic manifold. The hypothetical manifold where all information is known could be called the underlying manifold. But whether this is meaningful given that probability measures need to be present is not clear. Actual probability measures over 'outcomes' are conspicuously missing in this construction and are needed for any further description of quantum behaviour. But any probabilistic measure over suitable outcomes necessarily manifests quantum logic as Hadley's axioms have been satisfied.

The underlying premise of open-ended space-times and measurement is that experimentation or observations constitute *state preparation*, that is, a selection of a subset of the possible underlying manifolds. Any deeper interpretation as to why certain interpretations of measurement in general relativity with weakened causality should lead to an example 4-geon and many other questions raised by their construction are not answered in this paper. Stability issues of the solution were also not discussed:

The aim has been to provide an example 4-geon that satisfies Hadley's axioms, and that has been done.

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