Stellar Aberration: the Contradiction between Einstein and Bradley

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Abstract: The classical and relativistic explanations of the stellar aberration are compared, on the basis of the physical models implied by the two interpretations. Our analysis shows that the physical model required by the Special Relativity theory is inconsistent with the observed effect.

Keywords: (Bradley, Einstein, Special Relativity, Stellar Aberration, Shrödinger).

Introduction

As is well known, stellar aberration is an apparent change in the direction of the starlight viewed by a terrestrial observer, because of the Earth's orbital motion around the Sun. Discovered by Bradley[1] in 1727, and at first explained in the ambit of the Newtonian corpuscular light model, this effect then became part of the experimental basis supporting the idea of a stationary ether (relative to the Sun). Subsequent observations by Arago[2], carried out utilizing the Earth's longitudinal motion (approaching and separating) relative to the starlight, instead of the Earth's transversal motion, evidenced no change in the velocity of this light. This result appeared in contradiction with Bradley's model, and suggested the hypothesis of an ether dragged by the Earth instead, inside of which the speed of light would have possessed a sort of "local" constancy (Stokes).[3]

The consequent *querelle* related to the elusive properties of a light medium, fuelled by the contradictory results of Fizeau's[4] and Airy's[5] experiments, and particularly by the controversial "null result" of the Michelson-Morley test on light isotropy,[6] was only put an end to by the advent of the Special Relativity theory (hereafter referred to as SRT). However, this last theory did not solve the problem of the "local" constancy of c by means of a casual explanation, but by a postulate which simply imposed what seemed so difficult to explain, stating "a priori" that such apparent speed constancy was a special property of light. (It is to be reminded that a Pre-Newtonian postulate stated that the circular motion of planets, impossible to explain without the idea of a gravitational force, was a special property of the celestial bodies.)

1. Stellar aberration according to Bradley

According to Bradley, the phenomenon of the stellar aberration is quite simple. Because of the enormous distance between any star and our solar system, the starlight reaches the Earth with a practically parallel irradiation. According to an ideal observer at rest with respect to the Sun, a terrestrial observer and a starlight pulse meet after traveling two different paths in the same time. According to an observer co-moving with the Earth around the Sun, the same light pulse reaches him with a velocity $\vec{c}\,'$ which is the vector sum of the starlight's velocity vector $\vec{c}$ and of a vector $\vec{v}$ with equal magnitude and opposite direction of the Earth's velocity (both representations in Fig.1 are equally valid). Consequently, the direction of this light pulse appears to this observer to have changed, too. (The analogy often used in many popular texts with the falling raindrops and the moving observer is quite perfect.)

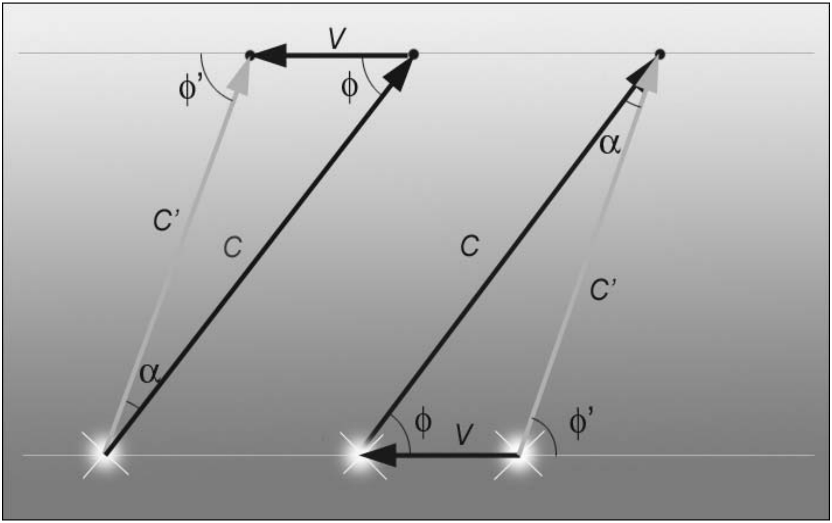

Fig.1 Bradley's stellar aberration, considered as a vector addition of $\vec{v}$ and $\vec{c}$.

Understanding by ϕ the angle between the velocity of the moving observer and the unaberrated starlight ray, by ϕ' the angle between the same above velocity and the aberrated ray viewed by the moving observer, and by α the angle between the aberrated ray and the unaberrated one, on the basis of fundamental trigonometric identities the following relations are obtained:

$$\sin\alpha = \frac{v\sin\phi}{c'} \qquad (1),$$

$$\cos\alpha = \frac{c - v\cos\phi}{c'} \qquad (2),$$

from which: $$\tan\alpha = \frac{\sin\phi}{c/v - \cos\phi} \qquad (3);$$

$$\sin\phi' = \frac{c\sin\phi}{c'} \qquad (4),$$

$$\cos\phi' = \frac{c\cos\phi - v}{c'} \qquad (5),$$

from which: $$\tan\phi' = \frac{\sin\phi}{\cos\phi - v/c} \qquad (6),$$

where $$c' = \sqrt{c^2 + v^2 - 2cv\cos\phi}\,.$$

For the simplest case the unaberrated starlight ray makes a right angle with the observer's velocity, (Bradley himself chose as a first object of his research γ Draconis, a star observable in the zenith direction at the latitude of London), vectors $\vec{v}$ and $\vec{c}$ become the catheti of an ideal right triangle, whose hypotenuse $\vec{c'}$ is the starlight's velocity reaching the terrestrial observer (Fig.2). The magnitude of this last velocity becomes $\sqrt{c^2 + v^2}$, and relations (3) and (6) reduce to:

$$\tan\alpha = \frac{v}{c} \qquad (7),$$

$$\tan\phi' = -\frac{c}{v} \qquad (8).$$

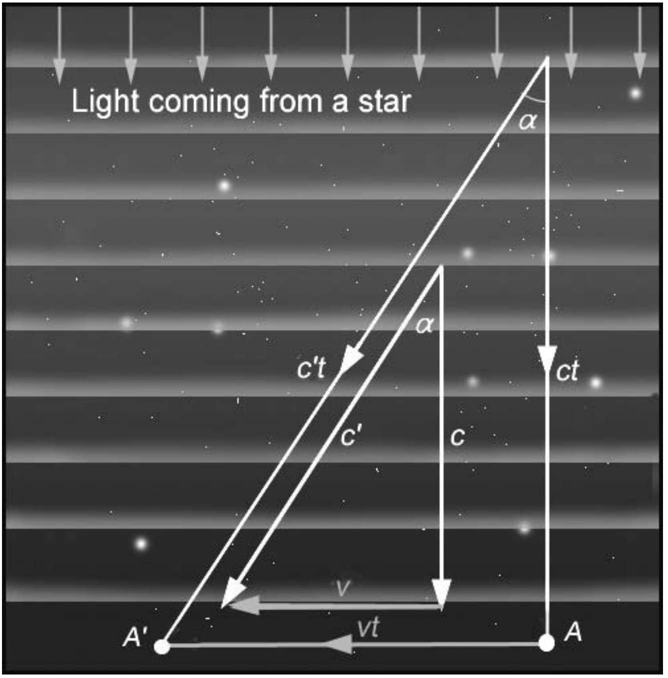

Fig. 2 Stellar aberration according to Bradley.

2. Stellar aberration according to Einstein

Because of the postulate of the constancy of *c*, the stellar aberration model of the SRT requires the vector addition of starlight's and observer's velocities be always *c*. Thus, what changes in this case is not the speed of light, but the rate of flowing of absolute time (a proper time and an improper time are therefore considered).

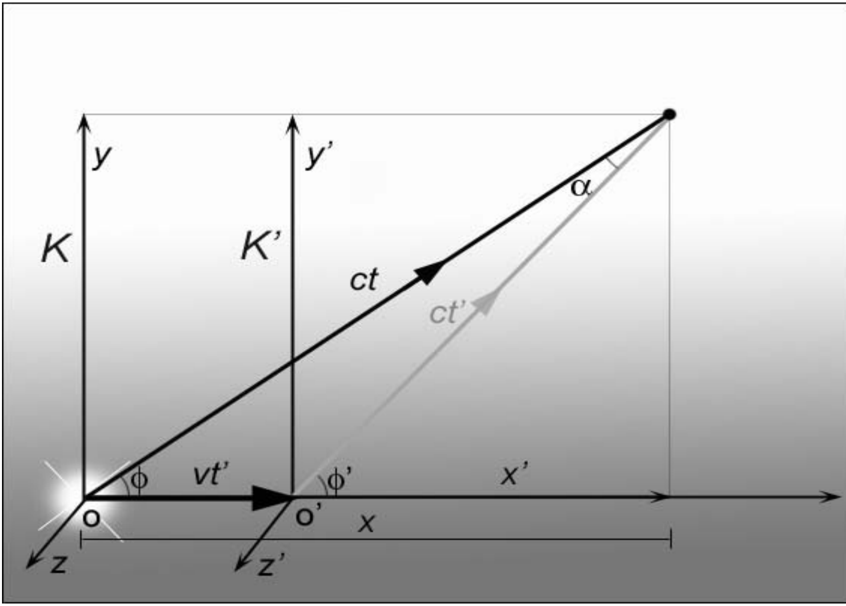

Fig. 3 Stellar aberration according to Einstein.

The relativistic stellar aberration angle is usually obtained as follows. Let us consider two systems of coordinates, *K* of coordinates *x*, *y*, *z*, *t*, and *K'* of coordinates *x'*, *y'*, *z'*, *t'*, moving relative to one another along the X axis with velocity *v*, and assume that, at the time $t = t' = 0$ the two systems are coincident, a light pulse is emitted from the origin of the system *K*, making the angle ϕ with the X axis. The relativistic stellar aberration angle ϕ' is then determined by the motion of this light pulse with respect to the system *K'*, obtained by applying the SRT transformation (Fig.3). As a consequence of the light postulate, which imposes light speed in both paths to be *c*, it follows that *t* cannot equal *t'*, and consequently:

$$x = ct\cos\phi,\ y = ct\sin\phi,\ x' = ct'\cos\phi',\ y' = ct'\sin\phi'.$$

Substituting the above trigonometric relations into the reverse SRT transformation,

$$t=\frac{t'+vx'/c^2}{\sqrt{1-v^2/c^2}},\ x=\frac{x'+vt'}{\sqrt{1-v^2/c^2}},\ y=y',\ z=z' \quad (9),$$

(this reverse form is used here to obtain the SRT stellar aberration formulas in their common form), we obtain:

$$\cos\phi'=\frac{\cos\phi-v/c}{1-\cos\phi\, v/c} \quad (10),$$

$$\sin\phi'=\frac{\sin\phi\sqrt{1-v^2/c^2}}{1-\cos\phi\, v/c} \quad (11),$$

from which:

$$\tan\phi'=\frac{\sin\phi\sqrt{1-v^2/c^2}}{\cos\phi-v/c} \quad (12),$$

and

$$\sin\alpha=\sin\phi\, v/c \quad (13).$$

For $\phi=\pi/2$, equations (10), (11), (12) and (13) reduce to

$$\cos\phi'=-v/c \quad (14),$$

$$\sin\phi'=\sqrt{1-v^2/c^2} \quad (15),$$

$$\tan\phi'=-(c/v)\sqrt{1-v^2/c^2} \quad (16),$$

$$\sin\alpha=v/c \quad (17).$$

Formulas (10) and (14) appear in the Einstein's 1905 first paper on the SRT.[7] Formula (17) is the relativistic equivalent of the famous Bradley's relation (7). It is now to be pointed out that the kind of process here exposed, and commonly used in many texts, does not render the real physical meaning of the relativistic stellar aberration.

This meaning is instead better represented considering all four points of view expressed by the SRT transformation. Algebraically, four viewpoints arise from the fact that, differently from the Gailean and Lorentz transformations (this last one understood in the ambit of the Lorentz's ether model), which are based on two sets of variables only, each of the two SRT transformation sets of variables can be regarded as proper or improper, because of the required reciprocity of the relativistic effects, in fact doubling the points of view - see Russo.[8]

Physically, four viewpoints come from the fact that, on the basis of the relativity postulate, each of two observers in *K* and *K'* views the other reference frame moving away, with velocity *v* and *–v*, respectively, but, on the basis of the light postulate, both observers view the same light spherical wave propagating from the source, satisfying relations

$$x^2 + y^2 + z^2 = c^2t^2 \qquad \text{and} \qquad x'^2 + y'^2 + z'^2 = c^2t'^2 .$$

Thus, the SRT transformation considers the viewpoint of an observer in *K'* who views a light pulse propagating relative to *K'* and that of an observer in *K* who views the same light pulse relative to *K'*; the reverse SRT transformation instead considers the viewpoint of an observer in *K* who views a light pulse propagating relative to *K*, and that of an observer in *K'* who views the same light pulse moving relative to *K* (Fig.4). (Galilean and Lorentz transformations instead consider the viewpoints of two observers in *K* and *K'* respectively, relative to the propagation of the same light pulse. Thus, one of the two observers views a not spherical light propagation wave front.)

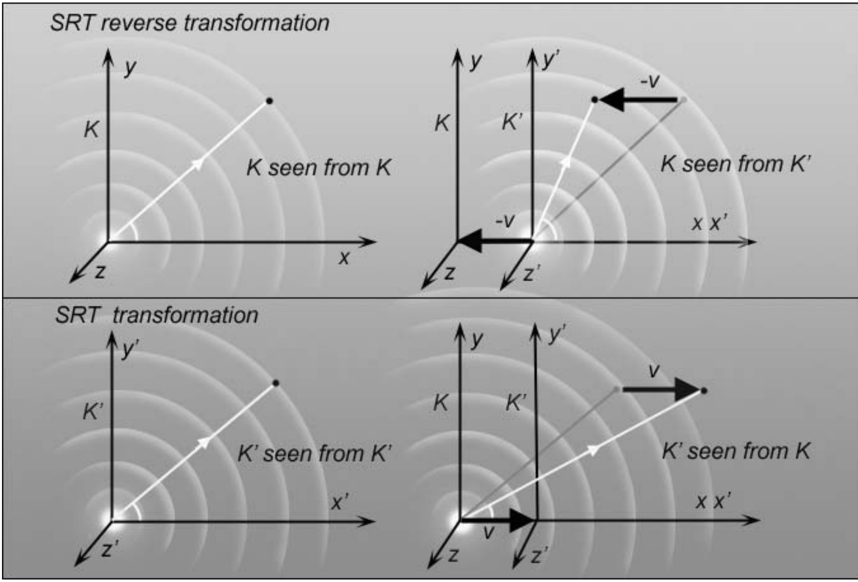

Fig.4

In this view, it is just the motion of reference frames, relative to the absolute motion of light, that gives rise to the SRT light aberration. The most relevant consequence is that the deflected ray and the not deflected one are in fact two different rays, though emitted by the same light source and belonging to the same spherical wave (this consequence is not clear from the usual derivation of the relativistic effect expounded at the beginning of this paragraph).

This physical model resembles that of the so called relativistic "light clock". (The scheme of this clock is often used to obtain in a simple but rigorous way the time dilatation factor predicted by the SRT). Substantially, it is an ideal clock that measures time by means of a back and forth light travel (obtained by reflection) along an oscillation axis.

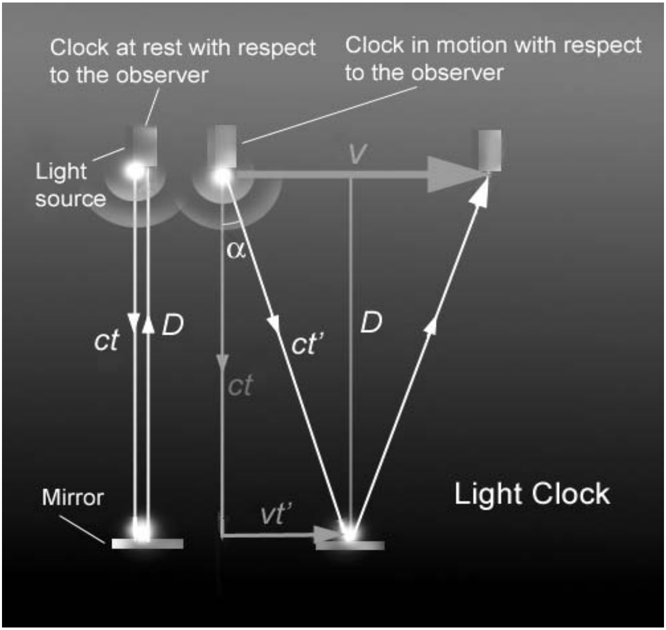

Fig.5 Behaviour of the light clock according to the SRT.

On the basis of the light postulate, an observer at rest with respect to this clock sees a light ray travel back and forth the oscillation axis D (major cathetus of an ideal right triangle) in the time $2D/c$. An observer in perpendicular motion with velocity v relative to the oscillation axis instead sees another light ray (which is part of the same spherical wave) travel back and forth a longer path $D\big/\sqrt{1-v^2/c^2}$ (hypotenuse), in a longer time, $2D\big/c\sqrt{1-v^2/c^2}$ * (Fig.5). If we assume that what changes, according to this last observer, is not the distance covered by light, but the flowing itself of

* The Lorentz length contraction is here not considered, since the motion of the observer is perpendicular to the oscillation axis of the clock.

time, from the ratio between the times of the two light travels we obtain the relativistic factor of time dilatation, $1/\sqrt{1-v^2/c^2}$. But from this model it is also possible to obtain the relativistic stellar aberration angle for $\phi = \pi/2$. In fact, if we assume the clock's light source is a star, then the major cathetus *ct* of our right triangle becomes the path of the unaberrated starlight, and the hypotenuse *ct'* becomes the path of the aberrated starlight (Fig.6), from which we immediately obtain $\sin\alpha = v/c$, and more generally, all the SRT formulas for $\phi = \pi/2$.

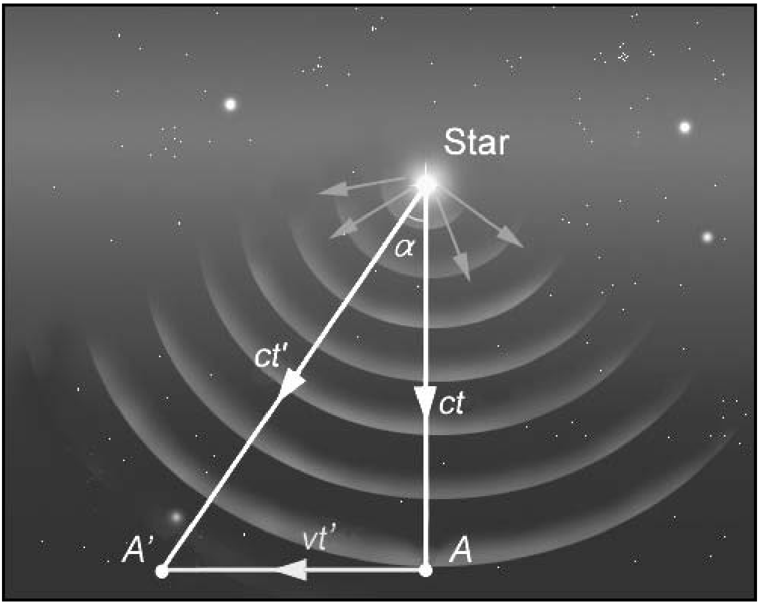

Fig.6 Stellar aberration according to Einstein.

The great advantage of the light clock model is that it allows us to obtain these last SRT stellar aberration formulas directly from a physical model based on Einstein's two postulates, and not from a purely algebraic route (application of the SRT transformation).

Conversely, what said demonstrates that applying the SRT formulas for the stellar aberration means in fact applying the physical model of the relativistic light clock.

3. Discussion of the two models

Let us consider again the case in which the unaberrated starlight ray makes a right angle with the observer's velocity. In this case, in the Bradley's model the aberration angle is obtained from the ratio between the catheti of a right triangle: the starlight covers the major cathetus in the same time the observer covers a distance equal to the minor cathetus.

In the Einstein's model the aberration angle is instead obtained from the ratio between the major cathetus and the hypotenuse of a right triangle: the starlight covers the hypotenuse in the same time the observer covers a distance equal to the minor cathetus. However, the most relevant difference concerns the kind of light irradiation. The model by Bradley requires a parallel light irradiation (rays emitted by a very distant source - plane wave front). The relativistic model, at least according to Einstein, also lays on the assumption of a parallel light irradiation. But, as previously seen, it is actually based on the model of the light clock, which in its turn requires a radial irradiation (rays emitted by a relatively nearby source - spherical wave front).

Figures 2 and 6 highlight the fundamental difference between the two models. According to Bradley (Fig.2), it is the same light ray which simultaneously reaches points *A* and *A'*, which in fact are the same point viewed by two different observers (we can imagine this light ray as the path of a single photon). On the contrary, according to Einstein (Fig.6), two different light rays reach points *A* and *A'* at different times (we can imagine these two rays as the paths of two distinct photons), or, the same way, a light spherical wave front reaches first *A* and then *A'*, meaning that, in this context, *A* and *A'* are two different points. The main implication is that the Einstein's physical model, differently from the Bradley's one, must necessarily include the light source. In fact, while the "classical" aberration for $\phi = \pi/2$ depends on the ratios of a right triangle whose sides lengths are not comparable with the distance star-observer, the relativistic aberration for the same case can be only obtained from a right triangle whose corner between the major cathetus *ct* and the hypotenuse *ct'* coincide with the position of the star.

But, as to the STR model, a problem arises. In fact, since the axis of the Earth's orbit around the Sun is absolutely insignificant if compared to the distance between our solar system and any star, the minor cathetus of our right triangle turns out to be actually null, meaning that *ct* coincides with *ct'*. This means that also the aberration angle between *ct* and *ct'* is null, and that therefore, according to a correct physical interpretation of the SRT, a terrestrial observer who views a star at his zenith cannot see any stellar aberration, a consequence evidently contradicting the simple observed phenomenon. From what has been said, it is clear that the relativistic model is inapplicable to the stellar aberration effect.

Conclusions

In classical Physics, a mathematical description of a phenomenon always lays on a physical model. In the case of the SRT, in spite of the simple mathematical model involved, the contrary to experience consequences of the light postulate make it often difficult to conceive adequate physical models for the various relativistic effects. Probably because of this reason, most expositions of the SRT are based on algebraic demonstrations, but lack adequate "physical" explanations, that should instead be the basis of every physical theory about the macrocosmic world, as well as an indispensable element to any possible analysis or confutation. The case of the stellar aberration is emblematic. The algebraic route, consisting in the application of the SRT transformation to the system of the star and to that of the observer, does not apparently lead to contradictions. But the underlying physical model, based on a radial light radiation (light clock model), turns out to be incompatible with the parallel starlight irradiation actually reaching the Earth. The fact that the stellar aberration can instead be easily explained by assuming an addition/subtraction of c and v, or more generally, a not constant velocity of light (discussed by Marmet[9], Selleri & Puccini[10], Schulz Poquet[11] and others), seems to be a strong proof against the postulate of the c constancy.

The SRT stellar aberration is also contradicted by the apparent lack of symmetry in the observed effects (Phipps[12]), and particularly by the lack of any aberration effect in the observation of binary stars, in spite of the very high orbital speeds involved (Ives[13], Eisner[14] and Hayden[15]). It is finally to be added that, up to today, no direct experimental evidence of the small relativistic correction for the Bradley's stellar aberration exists (maximum predicted correction for a terrestrial observer: ≈0.0006 arcseconds). Relativistic corrections have indeed been only applied to - but not obtained from - astrometric observed data, such as those collected by the ESA astrometric space mission Hipparcos (astrometric resolution: ≈0.002 arcseconds).

This criticism towards the SRT does not obviously answer all open questions about an alternative light propagation model based on a light medium, questions also left open because of the few, not decisive and often contradictory experimental data nowadays available on the subject - it is sufficient to mention all tests on light isotropy (*in primis* the Michelson-Morley experiment), that never gave a real null fringe-shift, giving rise to alternative interpretations (Miller[16], Vigier[17], Lévy,[18] Cahill-Kitto[19] and others).

In the author's view, Beckman's hypothesis of identifying some properties of the light medium in those of the gravitational field seems to be the most interesting one.[20] However, only new *ad hoc* experiments, such as a repetition in space of the M.&M. experiment, as auspicated by Hayden, or space tests on the ether properties by means of electromagnetic transmissions, for example between the ISS (International Space Station) and an artificial satellite (Russo 2006), could cast some light upon this controversial matter. These experiments, simply, have not yet been made.

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Appendix I: A relativistic Shrödinger's cat

The spherical light propagation for all inertial observers imposed by the light postulate, besides being incompatible with the observed stellar aberration effect (as showed in this article), gives rise to the following paradox. Let us imagine a slightly modified version of the relativistic light clock, in which the wave source is a laser, and thus capable of emitting light not in a radial way, but in one single direction.

Furthermore, imagine that along this direction, at a distance D from the source, there is a detector capable of releasing, if hit by a light pulse, a lethal gas in a box which contains a cat (Fig.7).

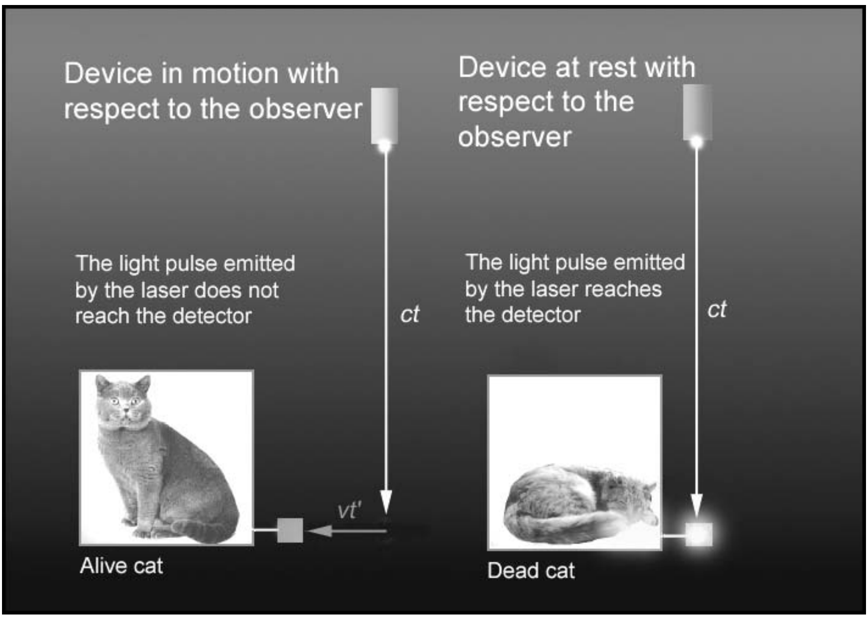

Fig.7

At a given instant, the laser emits a light pulse towards the detector. According to an observer at rest with respect to this device, the light pulse reaches the detector after a time D/c, and the cat dies.

But according to an observer in perpendicular motion relative to the velocity of the light pulse, on the basis of the light postulate the light pulse does not reach the detector, because in the time this pulse travels the distance D, the detector has changed its place, travelling a distance vt, and, in absence of a radial emission, no spherical wave front can reach it. Therefore, according to the observer in motion, the cat does not die. We are therefore now facing a similar result to that obtained in the famous thought experiment conceived by Shrödinger to disprove Quantum Mechanics.† In fact, on the basis of the principles of the SRT, two observers do not view the same event at two different times (relativity of simultaneity), but view two different events, that is, two different realities! The Shrödinger paradox is usually solved by appealing to the inapplicability of Quantum laws to macrocosm systems, instead ruled by the entropy law. Our relativistic paradox takes instead place entirely in the macrocosmic world, and therefore a superposition of contradictory macroscopic events cannot be avoided. But simply because of this reason, it turns out to be unacceptable.

† Erwin Shrödinger conceived the following thought experiment in 1935 in order to evidence contradictions in the Quantum Theory: A detector, if hit by a particle emitted by a radioactive element, is capable of releasing a poison in a box, in which there is a cat. On the basis of the quantum principle of superposition of possible states, till when it is not observed whether the detector has been hit or not by a particle, the two quantic states of “alive cat” and “died cat” coexist.