

Testing Relativity Theory for One-way Light Propagation

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Classical or Bradley stellar aberration is correctly described by special relativity theory, which predicts also a second-order departure that has never been verified. We point out that the Very Long-Based Interferometry system appears now to offer sufficient resolution to allow confirmation of this truly “relativistic” aspect of starlight. The one-way nature of starlight propagation, in conjunction with the fact that most existing verifications of the special theory rest implicitly on two-way light-speed averaging, suggests the desirability of such measurements as a further independent verification of the theory.

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I. Introduction

Special relativity theory (SRT) describes a plane wave of starlight by means of a four-vector $k_\mu = (\mathbf{k}, i\omega/c)$, its wave normal vector \mathbf{k} being of magnitude $k = 2\pi/\lambda = \omega/c$, where $f = \omega/2\pi$ is its frequency. If the angle between this \mathbf{k} -vector and the direction of motion of inertial system S' relative to S is α , as measured in S , and is α' , as measured in S' , then application of the Lorentz

transformation between S and S' to the four-vectors k_μ and k'_μ leads to the well-known formula [1,2]

$$\text{ctn } \alpha' = \gamma \left(\text{ctn } \alpha - (v/c) \text{csc } \alpha \right), \quad (1)$$

where v is the magnitude of the velocity \mathbf{v} of S' relative to S , and γ is $(1 - v^2/c^2)^{-1/2}$. Applying the identity [3]

$$\text{ctn } \alpha - \text{ctn } \alpha' = \frac{\sin(\alpha' - \alpha)}{\sin \alpha \cdot \sin \alpha'}, \quad (2)$$

as well as $\sin \alpha' = (1 + \text{ctn}^2 \alpha')^{-1/2}$, we deduce from Eq. (1) that

$$\Delta \alpha = \sin^{-1} \left(\frac{\cos \alpha - \gamma (\cos \alpha - v/c)}{\left(1 + \gamma^2 [\text{ctn } \alpha - (v/c) \text{csc } \alpha]^2\right)^{1/2}} \right), \quad (3)$$

where $\Delta \alpha = \alpha' - \alpha$ is the radian angle of change of light propagation direction due to the Lorentz transformation. If v is interpreted as the speed of the earth's motion in its orbit, $v = v_{orb}$, then this formula gives the exact SRT prediction of the angle of stellar aberration associated with passage from S to S' , in agreement with the observed Bradley aberration. Eq. (3) can be expanded (most easily by a computer algebra program) into

$$\Delta \alpha = \sin \alpha \left(\frac{v}{c} \right) + \left(\frac{1}{2} \right) \sin \alpha \cos \alpha \left(\frac{v^2}{c^2} \right) + O \left(\frac{v^3}{c^3} \right). \quad (4)$$

The first-order term on the right is the one that describes Bradley aberration. Considered over the course of a year, it describes for each star a “figure of aberration,” which is the projection onto the celestial sphere of the earth's orbit, an ellipse of semi-major axis

$(v_{orb}/c) \approx 10^{-4}$ radian, or about 20.5 arc-seconds, the “constant of aberration.” The second-order term gives the relativistic correction. Third and higher orders are entirely unobservable. Textbooks convey the impression that the second-order term is also beyond reach. Thus, Bergmann [1] says, “The observed effect is the first-order effect, while the relativistic second-order effect is far below the attainable accuracy of observation.” It is the purpose of the present paper to call attention to the fact that this is no longer necessarily the case, given the impressive astrometric advances that have taken place during the past half-century. It is now possible, employing interferometric techniques, to contemplate quantitative measurements that would test the genuinely relativistic prediction of SRT by interrogating the second-order term in Eq. (4). We shall also give some attention to motivating such measurements by mentioning a few alternative theoretical possibilities.

II. VLBI to the rescue?

From the above, we see that as far as stellar aberration is concerned SRT has so far merely confirmed what was known historically to Bradley. The theory has not had its own unique and specific predictions verified. With the advent of Very Long-based Interferometry (VLBI) techniques, this situation could change completely. The current resolution of that system at the shorter microwave lengths (around 1 cm) is said to be of the order of 10^{-4} arc-second, or about 5×10^{-10} radian. For comparison, the maximum magnitude of the second-order term in Eq. (4) is about 2×10^{-9} radian. The important question for testing SRT is therefore whether VLBI system accuracy is comparable with system resolution.

To achieve the needed absolute accuracy of measurement, it would be necessary to bridge across a wide arc of the sky, perhaps linking measurements from star to star, proceeding from a test star (whose absolute angular position is required) at some substantial elevation (say, around 45°) above the plane of the ecliptic to a fiducial reference star with approximately zero aberration, lying on or near the circular projection of that plane upon the celestial sphere. Such a procedure would be required because, although high-precision *absolute* angular measurements are not routinely done or needed, they are needed here in view of the “conformal” nature [4] of stellar aberration. That is, for stars of negligible parallax within a given small region of the night sky (all having approximately the same value of the angle α) the same aberration is always observed. In effect, they all move annually in concert, with identical figures of aberration. Consequently the fiducial reference star cannot be located in the same part of the sky as the test star.

In what follows it will be assumed that the astrometric problems associated with absolute angular accuracy comparable with the angular resolution of the VLBI system can be solved. In that case a test of SRT’s predicted second-order departure of observable stellar aberration from Bradley aberration becomes possible. There is no wide margin for error, and the effect to be sought is so small that it is unlikely to have been observed fortuitously. It is also unlikely that analysis of existing data would settle the matter unambiguously. Thus a research project dedicated to the objective would be required ... and it may be asked whether the cost and effort would be worthwhile. Let us address that question next.

III. Alternative theoretical possibilities

It may seem that SRT lacks competitors, so that observations in disagreement with that theory are literally inconceivable. However, the presence or absence of competitive theories is not a time-invariant feature of any science. In any era this is more a question of sociology than of physics. If the observational evidence for SRT is examined with detachment, it will be noted that the theory and most of its supporting experiments share the feature of involving two-way light propagation. There thus arises the formal possibility of logical circularity, a vicious circle that can be broken only by introducing empirical evidence of radically different character. Such evidence might be that of stellar aberration, which involves a strictly one-way propagation of light.

Before considering theories alternative to SRT we might mention one apparent reason to question SRT's description of starlight. This is that any four-vector treatment of starlight encounters a problem of parameterization. We know that the simplest Lorentz transformation or rectilinear boost gives rise to a one-parameter group, the group parameter being a velocity v . We see by consideration of $k_\mu = (\mathbf{k}, i\omega/c)$ that in order to agree with Bradley's observations the spacelike part of any starlight four-vector must employ the velocity parameter $v = v_{orb}$ in describing stellar aberration; whereas its timelike part must employ a velocity parameter $v = v_{ss}$, where v_{ss} is the relative velocity of light source and sink (detector or absorber). That is clear from the fact that we use the timelike part to describe the stellar Doppler effect (cosmic red shift, etc.). Now it seldom happens that $v_{orb} = v_{ss}$, so the one-parameter nature of the group fails in the case of starlight ... or else one could say that the group property fails. This failure is entirely different from, and more serious than, the well-

known failure of group properties of non-collinear Lorentz boosts in more than one spatial dimension (that being correctable via the Thomas rotation [5] of “inertial” frames, which restores the group properties). Here we can confine our analysis entirely to one spatial dimension and still have trouble making a single group parameter do the work of two. Thus it is far from a foregone conclusion that SRT is capable of giving a four-vector description of starlight that is consistent with the known phenomenology of both stellar aberration and Doppler effect. The decisive way to deal for all time with such an issue is not through theoretical ratiocinations but through empirical observations testing SRT’s prediction, Eq. (4).

We now turn briefly to alternative theorizings. Since the Einstein clock synchronization method foundational to SRT renders the theory “blind” to one-way light propagation effects, a number of theories have been proposed over the years that hypothesize various non-standard one-way propagation behaviors of light. It must be emphasized that such proposals are not phenomena, but are speculations concerning what happens to the photon during its presumed “journey,” when it is in a quantum pure state and thus by definition inaccessible to direct observational confirmation or disconfirmation. In the case of starlight it is a very long journey indeed ... but that only opens the door wider to speculation. Moon and Spencer [6], for example, suggested in 1956 a modification of the Ritzian idea. They proposed that the photon, lacking degrees of freedom independent of its source, could be thought of as “rigidly” linked to (or convected by) that source; so the expanding light sphere classically descriptive of wave propagation is pictured as remaining centered on its source after emission, for arbitrary source motions. This can be shown to lead to a predicted second-order term of the same form as that in Eq. (4), but of twice the magnitude. The present writer has more recently proposed [7] a complementary alternative –

suggested by the Wheeler-Feynman idea of the absorber as the “mechanism of radiation” – that the photon might be thought of as lacking degrees of freedom independent of its absorber; i.e., as rigidly convected by the absorber. This leads to a predicted angle of stellar aberration

$$\Delta\alpha = \tan^{-1} \left(\frac{(v/c) \sin \alpha}{\left(1 - (v/c)^2\right)^{1/2}} \right), \quad (5)$$

$$= \sin \alpha \left(\frac{v}{c} \right) + \left(\frac{1}{6} \right) \sin \alpha \left(1 + 2 \cos^2 \alpha \right) \left(\frac{v^3}{c^3} \right) + O \left(\frac{v^5}{c^5} \right)$$

from which the second-order term is seen to be missing, so that no observable departure from classical aberration is predicted. There are in fact numerous other proposals that can be found in the less-known literature. All such alternatives (as well as the Maxwell-Einstein model itself) must be understood to be in a sense physically illegitimate, since they force upon the mind classical “pictures” of what is happening in quantum pure states – states which are known by their nature to resist pictorialization and also to possess acausal attributes. Still, these alternatives have the virtue of providing distinctly different predictions of the observable $\Delta\alpha$. Thus they furnish motivation for experimental testing and for that reason cannot without risk be ignored or condemned wholesale.

IV. Conclusion

Even in the absence of doubts regarding the description of one-way light propagation given (or not given) by SRT, it would be desirable to test SRT’s second-order prediction of stellar aberration, as given by Eq. (4). This has been true since the earliest advent of the theory. The

new element to which we call attention here is that the VLBI system, at least in terms of resolution capability, should now be capable of verifying the small effect in question. The observations required are probably near the limit of practically attainable absolute accuracy. But other tests of special relativity – which it has always passed with flying colors – have also in many cases been technically difficult; and have had an aspect of redundancy, in that most of them have implicitly concerned two-way light propagation. A quantitative measurement of the departure from Bradley aberration would greatly strengthen the empirical evidence bearing on the one-way case.

References

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