The Massive Neutrino-like Particle of the Non-linear Electromagnetic Field Theory

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A special non-linear electromagnetic field theory—the curvilinear wave electrodynamics (CWED), whose equation in the matrix form is similar to the equation of the Dirac lepton theory, is presented. A solution of this equation is shown to represent a circularly polarized electromagnetic wave on a circular trajectory. It is also shown, that such a wave can be considered to be a massive neutral particle with a spin of one half, like that of the neutrino. This theory is mathematically similar to the neutrino theory of the Standard Model, but with the advantage that it avoids the contradictions of the Standard Model with the latest data from neutrino experiments.

Keywords: non-linear electrodynamics, QED.

1.0. The Neutrino in the Standard Model Theory

As a prelude to the non-linear electromagnetic theory of a massive neutrino-like particle, the presently accepted (Standard Model) theory of the neutrino is reviewed; the prevailing status of neutrino theory is summarized in [1,2]. This theory is that of electroweak interactions (including neutrinos) combined with Quantum Chromo-Dynamics (QCD); it is called the Standard Model (SM).

1.1. Features of Neutrinos in SM theory

In the Standard Model neutrinos are strictly massless, $m = 0$, all neutrinos are left-handed, helicity = –1, and all antineutrinos are right-handed, helicity = +1, and the lepton family number is strictly conserved. However, the latest experimental evidence indicates that these three simple properties are not exhibited by real neutrinos [1].

In the Standard Model, the neutrino and the antineutrino have opposite helicity. It is mathematically possible that this is in fact the *only* difference between neutrinos and antineutrinos, *i.e.,* a right-handed "neutrino" would be an antineutrino. Particles of this sort are called Majorana particles.

As long as the neutrino is massless, its helicity is completely defined, and a Majorana neutrino would be a different particle from its antineutrino. But if neutrinos have mass, and therefore do not travel at exactly the speed of light, it is possible to define a reference frame in which the helicity would be flipped. This means that there is effectively a mixing between the neutrino and the antineutrino (thus violating lepton number conservation).

1.1.1. Helicity and Chirality

In the Dirac theory there are two bispinor conjugate equations [3]:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\cdot\hat{\vec{p}}\right) + \hat{\beta}\, mc^2\right]\psi = 0\,, \tag{1.1}$$

$$\psi^{+}\left[\left(\hat{\alpha}_{o}\hat{\varepsilon}-c\hat{\vec{\alpha}}\cdot\hat{\vec{p}}\right)-\hat{\beta}\,mc^{2}\right]=0, \tag{1.2}$$

where $\hat{\varepsilon}=i\hbar\dfrac{\partial}{\partial t}, \hat{\vec{p}}=-i\hbar\vec{\nabla}$ are the operators for the electron energy and momentum, c is the velocity of light, m is the electron mass, and $\hat{\alpha}_o=\hat{1}$, $\hat{\vec{\alpha}}$, $\hat{\alpha}_4\equiv\hat{\beta}$ are the original set of Dirac matrices:

$$\hat{\alpha}_0=\begin{pmatrix}\hat{\sigma}_0 & 0\\ 0 & \hat{\sigma}_0\end{pmatrix},\ \hat{\vec{\alpha}}=\begin{pmatrix}0 & \hat{\vec{\sigma}}\\ \hat{\vec{\sigma}} & 0\end{pmatrix},\ \hat{\beta}\equiv\hat{\alpha}_4=\begin{pmatrix}\hat{\sigma}_0 & 0\\ 0 & -\hat{\sigma}_0\end{pmatrix},\ \hat{\alpha}_5=i\begin{pmatrix}0 & -\hat{\sigma}_0\\ \hat{\sigma}_0 & 0\end{pmatrix}, \tag{1.3}$$

where $\hat{\vec{\sigma}}$ are Pauli spin matrices: $\hat{\sigma}_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \hat{\sigma}_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$,

$\hat{\sigma}_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$, $\hat{\sigma}_0=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$; $\psi=\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix}$ is the wave function called a bispinor, and ψ^{+} is the Hermitian-conjugate wave function.

Helicity refers to the relation between a particle's spin and direction of motion. A particle in motion has an axis defined by its momentum; its helicity is defined by the projection of the particle's spin $\vec{s}$ onto this axis: the helicity is $h=\dfrac{\vec{s}\vec{p}}{|\vec{s}||\vec{p}|}$, *i.e.,* the component of angular momentum along the direction of linear momentum.

The helicity operator thus projects out two physical states: one with the spin along the direction of motion, and the other with the spin opposite to this direction; this is true regardless of whether the particle is massive or not. If the spin is along the direction of motion, the particle is said to have right helicity, and if the spin is opposite to the direction of motion, the particle is said to have left helicity.

Something is *chiral* when it cannot be superimposed on its mirror image, like for example our hands. Like our hands, chiral objects are classified into left-chiral and right-chiral objects.

For a massless fermion, the Dirac equation reads

$$\alpha^{\mu}\partial_{\mu}\psi = 0, \;\; \mu = 1,2,3,4, \tag{1.4}$$

which is also satisfied by $\alpha_5\psi$:

$$\alpha^{\mu}\partial_{\mu}(\alpha_5\psi) = 0, \tag{1.5}$$

where the combination of the α matrices, $\alpha_5 = \alpha_0\alpha_1\alpha_2\alpha_3$ has the property $\alpha_5^2 = 1$ with the commutators $\left\{\alpha_5, \alpha_\mu\right\} = 0$. This allows us to define the *chirality operators* which project out left-handed and right-handed states:

$$\psi_L = \frac{1}{2}\left(1-\alpha_5\right)\;\psi \text{ and } \psi_R = \frac{1}{2}\left(1+\alpha_5\right)\;\psi, \tag{1.6}$$

where ψ_L and ψ_R satisfy the equations $\alpha_5\psi_L = -\psi_L$ and $\alpha_5\psi_R = \psi_R$: *i.e.,* the chiral fields are eigenfields of α_5, regardless of their mass.

We can express any fermion as $\psi = \psi_L + \psi_R$, so that a massive particle always has a *L*-handed as well as a *R*-handed component. In the massless case however, ψ separates into distinct helicity states: the Dirac equation splits into two independent parts, reformulated as the Weyl equations:

$$\frac{\hat{\vec{\sigma}}\hat{\vec{p}}}{\left|\hat{\vec{\sigma}}\right|\left|\hat{\vec{p}}\right|}\left[\frac{1}{2}\left(1\pm\alpha_5\right)\;\psi\right] = \pm\frac{1}{2}\left(1\pm\alpha_5\right)\;\psi, \tag{1.7}$$

A massless particle, which is in perpetual motion, thus has an unchangeable helicity. The reason is that its momentum cannot be altered, and its spin of course remains unchanged.

For a massive particle however, we can perform a Lorentz transformation along the direction of the particle's momentum with a velocity larger than the particle's, changing the direction of the momentum. Since the spin direction remains the same, the helicity of the particle changes.

1.1.2. Electromagnetic Characteristics of the Neutrino

In spite of being electrically neutral (no electric charge), the neutrino possesses electromagnetic properties. Analysis of these properties has implications for the nature of neutrino mass [4]. The electromagnetic properties of Dirac's and Majorano's neutrino appear to be essentially different. Dirac's massive neutrino receives its magnetic moment by interaction with the vacuum: the neutrino's magnetic moment is in the same direction as its spin, while the antineutrino's magnetic moment is directed opposite to its spin. Thus the particle and the antiparticle differ by the direction of their magnetic moments. The massive Majorano neutrino, which is identical to its antiparticle, cannot have either a magnetic moment or an electric moment.

The mass and the magnetic moment of a neutrino are complex nonlinear functions of the external field strength and the energy of the particle.

When moving in an external field, the magnetic moment of the Dirac neutrino becomes partly an electric dipole moment d_ν; this is in part, a well-known classical effect of a Lorentz transformation producing an inter-conversion of electric and magnetic fields. Quantitatively, the electric moment of a massive Dirac neutrino, moving in a constant external, electromagnetic field, is proportional to a pseudo-scalar $\left(\vec{E}\cdot\vec{H}\right)$, which changes sign under time reversal. Thus, the electric moment is induced by an external field, if the pseudo-scalar of this field is not zero: i.e. $\left(\vec{E}\cdot\vec{H}\right)\neq 0$, and if its existence conforms with the T-invariancy of the Standard Model. In this Dirac model both the electric dipole and magnetic moments of the neutrino have a dynamic nature.

The electromagnetic characteristic known as the anapole (or toroidal dipole) moment, arises in both the Dirac and Majorana models of the neutrino.

In the framework of the non-linear electromagnetic theory (below) the massive neutrino has a conserved inner (poloidal) helicity, which gives rise to the above features; it is fully described by the Dirac-like lepton equation.

2.0. Neutrino-like Particles in the Electrodynamics of Curvilinear Waves

In previous papers [5,6] based on non-linear electromagnetic theory we have considered the electromagnetic representation of the quantum theory of electron and obtained a Dirac-like equation for electron-like particles. In the framework of this theory, named curvilinear wave electrodynamics (CWED), the electron-like particles are the twirling *plane-polarized* semi-photon-like particles.

In the present paper we will show that the solution of this Dirac-like equation describes also the motion of the *circular-polarized* semi-photon-like particle on a circular trajectory. It is shown that such a particle can be regarded as a neutral particle with a spin of one half, similar to the neutrino.

3.0. Plane and circularly polarized electromagnetic waves

Electromagnetic waves emitted by charged particles are in general circularly (or elliptically) polarized; these electromagnetic waves are transverse in the sense that the associated electric and magnetic field vectors are both perpendicular to the direction of wave propagation.

Circularly polarized waves carry energy ε and momentum $\vec{p}$ as well as angular momentum $\vec{J}$, which are defined by energy density $U = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right)$, momentum density $\vec{g} = \frac{1}{c^2}\vec{S}_P$ and angular momentum flux density, which is given by

$$\vec{s} = \vec{r} \times \vec{g} = \frac{1}{4\pi\, c} r \times \vec{E} \times \vec{H}\,, \tag{3.1}$$

where $\vec{S}_P = \frac{c}{4\pi}\vec{E} \times \vec{H}$ is the Poynting vector, which represents not only the magnitude of the energy flux density, but also the direction of energy flow. For simple electromagnetic waves, the Poynting vector is in the same direction as the wavevector.

The angular momentum flux density is related to the circular motion of the electron in the circularly polarized wave field [7].

A plane electromagnetic wave (generally, elliptically polarized waves) can be regarded as a vector combination of two or more circularly polarized waves rotating in opposite directions. Figure 1 (below) shows the propagation of the electric field associated with a circularly polarized wave with positive (right) and negative (left) helicity.

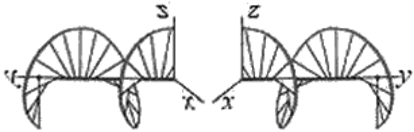

Fig. 1

Positive helicity is the case such that a right screw would move in the direction of wave propagation if rotated with the electric field (in optics, this is called "left hand" circular polarization). Negative helicity (right hand polarization in optics) refers to rotation in the opposite direction. The direction of the end of the helix indicates the head of the electric field vector, which is rotating about the *y* axis.

Since it is impossible by any transformation (excepting spatial reflection) to transfer the right (left) spiral to the left (right) spiral, the *circular polarization of photons is their integral characteristic invariant under all transformations, except that of mirror reflection.*

Since photon helicity is related to field rotation in classical electrodynamics, this is sometimes referred to 'rotation of a photon', the photon rotational characteristic being the angular momentum or spin of the photon. In quantum mechanics the spin of a photon is regarded differently: as the internal angular momentum of a particle: *i.e.,* its angular momentum when its linear momentum is zero. Therefore in case of a photon, whose speed can not be other than the speed of light, it is more correct to talk more about photon helicity than about photon spin [8]. In this case it is possible to define the helicity by the vector [7]

$$\vec{h}_{ph} \equiv \vec{s}_{ph} = \pm \frac{\varepsilon_{ph}}{\omega} \vec{p}^{\,0} \ , \tag{3.2}$$

where $\vec{p}^{\,0}$ is the unit Pointing vector, ε_{ph} and ω are the photon energy and circular frequency correspondingly. Apparently the angular momentum magnitude of this vector is equal to $\left|\vec{h}_{ph}\right| = 1\hbar$.

Thus according to our hypothesis of the p-helicity vector of neutrino being a twirled semi-photon, it should have a direction tangential to the curvilinear trajectory of twirled wave motion, and its angular momentum should be equal to half of the above value $\left|\vec{h}_{\nu}\right| = \frac{1}{2}\hbar$.

4.0. Quantum Form of the Circularly Polarized Electromagnetic Wave Equations

The twirling of the circularly polarized photon in a ring does not cause its helicity to disappear; it becomes poloidal (toroidal) helicity (p-helicity). Also, the movement of the fields of a photon along a circular trajectory forms other characteristics of an elementary particle—namely the angular momentum of the particle, or spin. Apparently, the spin of a massive particle and its poloidal angular momentum (p-helicity) are different characteristics. Since these characteristics are internal characteristics of a photon, *in the non-linear electromagnetic theory the spin and the poloidal helicity of a particle are independent and have conserved values*.

We will show that there is a Dirac-like equation, which can be considered as the equation of twirled, circularly polarized waves.

Let us consider the plane electromagnetic wave moving, for example, along y - axis:

$$\begin{cases} \vec{E} = \vec{E}_o e^{-i(\omega t \pm ky)}, \\ \vec{H} = \vec{H}_o e^{-i(\omega t \pm ky)}, \end{cases} \tag{4.1}$$

In general, the electromagnetic wave, moving along y - axis, has two polarizations and contains the following four field vectors:

$$(E_x, E_z, H_x, H_z), \tag{4.2}$$

As in this case $E_y = H_y = 0$ for all transformations, the set (4.2) can be compared with the Dirac wave function.

Consider the electromagnetic wave equation:

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2 \right) \vec{F} = 0, \tag{4.3}$$

where $\vec{F}$ is any of the field components (4.2); this equation represents four equations for total electromagnetic field.

In case of the wave, moving along the y - axis, this equation can be written in the following form:

$$\left(\hat{\varepsilon}^2 - c^2 \hat{\vec{p}}^2 \right) \vec{F}(y) = 0, \tag{4.4}$$

The equation (4.4) can also be represented in the form of the Klein-Gordon equation [3] without mass:

$$\left[\left(\hat{\alpha}_o \hat{\varepsilon} \right)^2 - c^2 \left(\hat{\vec{\alpha}} \ \hat{\vec{p}} \right)^2 \right] \Psi = 0, \tag{4.5}$$

where Ψ is a matrix consisting of the fields (4.2), and taking into account that $\left(\hat{\alpha}_o \hat{\varepsilon} \right)^2 = \hat{\varepsilon}^2$, $\left(\hat{\vec{\alpha}} \ \hat{\vec{p}} \right)^2 = \hat{\vec{p}}^2$, it is apparent that equations (4.4) and (4.5) are equivalent.

Taking into account that in case of photon: $\omega = \varepsilon/\hbar$ and $k = p/\hbar$, and using (4.1), then from (4.5) we obtain $\varepsilon = cp$, as for a photon. Therefore we can regard the Ψ - wave as the photon-like particle.

Factorizing (4.5) and multiplying it from the left by the Hermitian-conjugate function ψ^{+} we get:

$$\Psi^{+}\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)\Psi = 0, \tag{4.6}$$

Equation (4.6) may be separated into two Dirac equations without mass:

$$\Psi^{+}\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) = 0, \tag{4.7}$$

$$\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)\Psi = 0, \tag{4.8}$$

We will regard these as representing semi-photon particles.

If we choose the wave function as (and only as)

$$\Psi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \ \Psi^{+} = \begin{pmatrix} E_x & E_z & -iH_x & -iH_z \end{pmatrix}, \tag{4.9}$$

then *equations (4.7) and (4.8) are the correct Maxwell equations for the circularly polarized electromagnetic wave.* (For all other directions of the electromagnetic waves the matrices can be obtained by permutations of indexes [5]).

Putting (4.4) in (4.7) and (4.8) we derive the Maxwell equations in the case of electromagnetic waves:

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0 \end{cases}, \quad (4.10') \qquad \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = 0 \end{cases}, \quad (4.10'')$$

or in the vector form [9,10]:

$$\begin{cases} \frac{1}{c}\frac{\partial \vec{H}}{\partial t} + rot\ \vec{E} = 0 \\ \frac{1}{c}\frac{\partial \vec{E}}{\partial t} - rot\ \vec{H} = 0 \end{cases}, \qquad (4.11)$$

The electromagnetic wave equation has the harmonic solution:

$$\psi_\mu = A_\mu e^{-\frac{i}{\hbar}(\varepsilon t - \vec{p}\vec{r} + \delta)}, \qquad (4.12)$$

where $\mu = 1,2,3,4$, A_j are the amplitudes and δ is the constant phase.

Putting here $A_\mu = A_0$, $\delta = 0$, we obtain the following the trigonometric form of the solutions of these equations:

$$\begin{cases} E_x = A_0 \cos\left(\omega t - ky\right) \\ H_z = -A_0 \cos\left(\omega t - ky\right) \\ E_z = -A_0 \sin\left(\omega t - ky\right) \\ H_x = -A_0 \sin\left(\omega t - ky\right) \end{cases}, (4.13') \quad \begin{cases} E_x = A_0 \cos\left(\omega t - ky\right) \\ H_z = A_0 \cos\left(\omega t - ky\right) \\ E_z = -A_0 \sin\left(\omega t - ky\right) \\ H_x = A_0 \sin\left(\omega t - ky\right) \end{cases}, (4.13'')$$

It is pertinent to show that the vectors $\vec{E}$ and $\vec{H}$ rotate in the XOZ plain. Actually, putting $y = 0$ we obtain:

$$\vec{E} = E_x\vec{i} + E_z\vec{k} = A_0\left(\vec{i}\cos\omega t - \vec{k}\sin\omega t\right), \qquad (4.14')$$

$$\vec{H} = H_x\vec{i} + H_z\vec{k} = A_0\left(-\vec{i}\sin\omega t - \vec{k}\cos\omega t\right), \qquad (4.14'')$$

and

$$\vec{E} = E_x\vec{i} + E_z\vec{k} = A_0\left(\vec{i}\cos\omega t - \vec{k}\sin\omega t\right), \qquad (4.15')$$

$$\vec{H} = H_x\vec{i} + H_z\vec{k} = A_0\left(\vec{i}\sin\omega t + \vec{k}\cos\omega t\right), \qquad (4.15'')$$

where $\vec{i}$, $\vec{k}$ are the unit vectors of the *OX* and *OZ* axes. It is not difficult to show by known algebraic analysis [10] that we have obtained the cyclic polarized wave, but we will analyse these relations from a geometrical point of view.

The Poynting vector defines the direction of the wave motion:

$$\vec{S}_P = \frac{c}{4\pi}\vec{E}\times\vec{H} = -\vec{j}\frac{c}{4\pi}\left(E_xH_z - E_zH_x\right), \qquad (4.16)$$

where $\vec{j}$ is the unit vectors of the *OY* axis. Calculating the above we have for (4.13') and (4.13''):

$$\vec{S}_P = \frac{c}{4\pi}A_0^2\vec{j}\ , \qquad (4.17)$$

and

$$\vec{S}_P = -\frac{c}{4\pi} A_0^2 \vec{j}\,, \tag{4.18}$$

respectively. Thus, the photons of the right and left systems (4.13') and (4.13") move in opposite directions.

Fixing the the positions of the vectors $\vec{E}, \vec{H}$ at two successive time instants (in the initial instant $t = 0$ and after a small time period $\Delta\, t$), we can define the direction of rotation. The results are shown in figures 2 and 3:

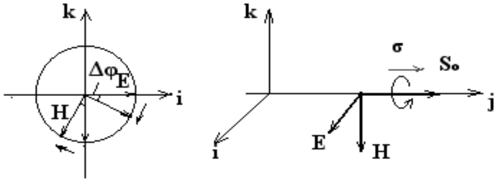

Fig. 2

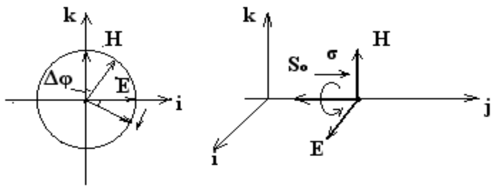

Fig. 3

It is apparent that the equation sets (4.10') and (4.10'') describe the waves with right and left circular polarization respectively.

However, the neutrino-like particle (according to the latest experimental data) must have mass. Therefore, it must be described not by the equations (4.7) or (4.8), but by the Dirac-like equation with a mass term.

This raises the question: which mathematical transformation can turn the equations (4.7) and (4.8) (without mass term) into the Dirac equations (1.1) and (1.2) (which have the mass term)?

We have shown [6] that this can be done in at least two ways: either by using curvilinear metrics, or by using differential geometry. Here we use only the second way.

5.0. Appearance of the Mass Term

Let us show that mass term appears when the initial electromagnetic wave changes its trajectory from being linear to being curvilinear. Since the Pauli matrices are the generators of the rotation transformations in 2D-space, we can suppose that this curvilinear trajectory is plane.

Let the circularly-polarized wave (4.1), which has the field vectors (E_x, E_z, H_x, H_z), be twirled with some radius r_K in the plane (X', O', Y') of a fixed co-ordinate system (X', Y', Z', O') so that E_x, H_x are parallel to the plane (X', O', Y') and E_z, H_z are perpendicular to it (see fig. 4)

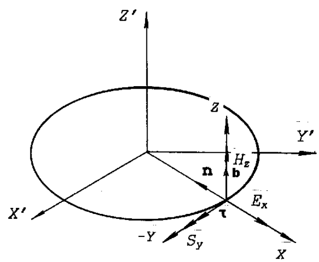

Fig. 4

Consider the expressions

$$\vec{j}^{\,e} = \frac{1}{4\pi}\frac{\partial \vec{E}}{\partial t}, \tag{5.1'}$$

$$\vec{j}^{\,m} = \frac{1}{4\pi}\frac{\partial \vec{H}}{\partial t}, \tag{5.1''}$$

(according to Maxwell [9,10] the expression (5.1') is the displacement electric current; correspondingly the expression (5.1'') can be regarded as the displacement magnetic current).

The above electric field vector $\vec{E}$ (which moves along the curvilinear trajectory) can be written in the form:

$$\vec{E}_x = E_x \cdot \vec{n}, \tag{5.2}$$

where $\vec{n}$ is the normal unit-vector of the curve (directed towards the center) and $E_x = \vec{E} \cdot \vec{n}$. The derivative of $\vec{E}_x$ can be represented as:

$$\frac{\partial \vec{E}_x}{\partial t} = \frac{\partial E_x}{\partial t}\vec{n} + E_x \frac{\partial \vec{n}}{\partial t}, \tag{5.3}$$

Here the first term has the same direction as $\vec{E}_x$. The existence of the second term shows that at the twirling of the wave the additional displacement current appears. It is not difficult to show that it has a direction tangential to the ring:

$$\frac{\partial \vec{n}}{\partial t} = -\upsilon_p \mathrm{K} \vec{\tau}\,, \tag{5.4}$$

where $\vec{\tau}$ is the tangential unit-vector, $\upsilon_p \equiv c$ is the electromagnetic wave velocity, $\mathrm{K} = \dfrac{1}{r_{\mathrm{K}}}$ is the curvature of the trajectory and r_{K} is the radius of curvature. Thus, the displacement current of the plane wave, moving along the ring, can be written in the form:

$$\vec{j}^{e} = \frac{1}{4\pi}\frac{\partial E_x}{\partial t}\vec{n} - \frac{1}{4\pi}\omega_K E_x \cdot \vec{\tau}\ , \tag{5.5}$$

where $\omega_K = \dfrac{\upsilon_p}{r_K} \equiv c\mathrm{K}$ is the curvilinear angular velocity, and $\vec{j}^{e}_{n} = \dfrac{1}{4\pi}\dfrac{\partial E_x}{\partial t}\vec{n}$ and $\vec{j}^{e}_{\tau} = -\dfrac{\omega_K}{4\pi}E_x \cdot \vec{\tau}$ are the normal and tangent components of the electric current of the twirled electromagnetic wave, respectively.

Thus:

$$\vec{j}^{e} = \vec{j}^{e}_{n} + \vec{j}^{e}_{\tau}\ , \tag{5.6}$$

The currents $\vec{j}_n$ and $\vec{j}_\tau$ are always mutually perpendicular, so that we can write them in the complex form:

$$j^{e} = j^{e}_{n} + i j^{e}_{\tau}\ , \tag{5.7}$$

By the same kind of algebraic derivation it is easy to show that the magnetic current appears as:

$$j^{m} = j^{m}_{n} + i j^{m}_{\tau}\ , \tag{5.8}$$

where $\vec{j}^{m}_{n} = \dfrac{1}{4\pi}\dfrac{\partial H_x}{\partial t}\vec{n}$ and $\vec{j}^{m}_{\tau} = -\dfrac{\omega_K}{4\pi}H_x \cdot \vec{\tau}$ are the normal and tangent components of the magnetic current.

For the circularly polarized wave,

$$\begin{cases} E_x = E_{xo}e^{-i(\omega t \pm ky)} \\ H_x = H_{xo}e^{-i(\omega t \pm ky)} \end{cases}, \tag{5.9}$$

the tangential currents are alternate.

5.1. The Dirac-like Equation of the Curvilinear Electromagnetic Theory

Taking into account the previous section results from (4.10) we obtain the twirled semi-photon equations:

$$c\frac{\partial \psi}{\partial t} - c\hat{\vec{\alpha}}\,\vec{\nabla}\psi \; - i\hat{\beta}\frac{c}{r_C}\psi = 0, \tag{5.10'}$$

$$\frac{\partial \psi}{\partial t} + c\hat{\vec{\alpha}}\,\vec{\nabla}\psi \; + i\hat{\beta}\frac{c}{r_C}\psi = 0, \tag{5.10''}$$

where ψ - function is not the Ψ-function, but is related to it by a transformation, as it will be shown below.

Since [3] ψ - function is mathematically the same as the function of Dirac's equations (1.1) and (1.2), we will name these equations the Dirac electron-like equation.

Using (4.10), from (1.1) and (1.2) we obtain the electromagnetic form of these equations:

$$\begin{cases} \frac{1}{c}\frac{\partial E'_x}{\partial t} - \frac{\partial H'_z}{\partial y} = -ij_x^e \\ \frac{1}{c}\frac{\partial H'_z}{\partial t} - \frac{\partial E'_x}{\partial y} = ij_z^m \\ \frac{1}{c}\frac{\partial E'_z}{\partial t} + \frac{\partial H'_x}{\partial y} = -ij_z^e \\ \frac{1}{c}\frac{\partial H'_x}{\partial t} + \frac{\partial E'_z}{\partial y} = ij_x^m \end{cases}, (5.12') \begin{cases} \frac{1}{c}\frac{\partial E'_x}{\partial t} + \frac{\partial H'_z}{\partial y} = -ij_x^e \\ \frac{1}{c}\frac{\partial H'_z}{\partial t} + \frac{\partial E'_x}{\partial y} = ij_z^m \\ \frac{1}{c}\frac{\partial E'_z}{\partial t} - \frac{\partial H'_x}{\partial y} = -ij_z^e \\ \frac{1}{c}\frac{\partial H'_x}{\partial t} - \frac{\partial E'_z}{\partial y} = ij_x^m \end{cases}, \quad (5.12'')$$

where in our case the *z*-components of the currents are equal to zero.

(Note also that the electromagnetic fields (E'_x, E'_z, H'_x, H'_z), which define the ψ- function, are different than fields (E_x, E_z, H_x, H_z), which define the Ψ-function).

Thus, the equations (5.12’) and (5.12’’) are *Maxwell equations with imaginary tangential alternative currents* and simultaneously they are the Dirac-like equation with mass.

We can schematically represent the motion of particles in this field as described by these equations, in the following way (fig. 5):

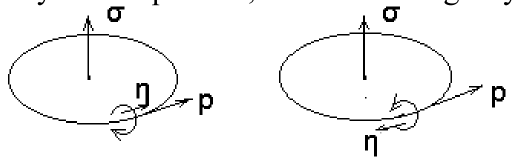

Fig. 5

According to figs. 2 and 3 the semi-photons (fig. 5) have contrary p-helicities. In the first case the helicity vector and the Poynting vector have the same direction; in the second case they are contrary. Therefore in the non-linear theory we can define the *inner or p-helicity* as the projection of the poloidal rotationly momentum on the momentum of the ring field motion.

It is not difficult to show [11] that actually the helicity is described in CWED by the matrix $\hat{\alpha}_5$. In the general case of massive particles, multiplying the Dirac equation (1.1) and (1.2) on $i\hat{\alpha}_5\hat{\beta}$ and taking in account that $i\hat{\alpha}_5\hat{\beta}\,\vec{\alpha} = \hat{\vec{\sigma}}$, where $\hat{\vec{\sigma}}' = \begin{pmatrix} \hat{\vec{\sigma}} & 0 \\ 0 & \hat{\vec{\sigma}} \end{pmatrix}$ are the spin matrix, and $\hat{\beta}\hat{\alpha}_5 = -\hat{\alpha}_5\hat{\beta}$, $\hat{\beta}^2 = 1$, we obtain:

$$\left(i\hat{\beta}\hat{\alpha}_5\hat{\varepsilon}+c\hat{\vec{\sigma}}'\hat{\vec{\rho}}-imc^2\hat{\alpha}_5\right)\ \psi=0, \tag{5.13'}$$

$$\left(i\hat{\beta}\hat{\alpha}_5\hat{\varepsilon}-c\hat{\vec{\sigma}}'\hat{\vec{\rho}}+imc^2\hat{\alpha}_5\right)\ \psi=0, \tag{5.13''}$$

Then we can extract from the right and left helicity matrix the following expressions:

$$\hat{\alpha}_5=\frac{c\hat{\vec{\sigma}}'\vec{p}}{i\left(\hat{\beta}\hat{\varepsilon}+mc^2\right)}, \tag{5.14'}$$

$$\hat{\alpha}_5=\frac{-c\hat{\vec{\sigma}}'\vec{p}}{i\left(\hat{\beta}\hat{\varepsilon}-mc^2\right)}, \tag{5.14''}$$

which in the case $m=0$ connect the $\hat{\alpha}_5$ matrix with helicity.

From the above it follows that according to our theory, inside the particle the operator $\hat{\alpha}_5$ describes the poloidal rotation of the fields (fig.5), and remembering that according to the electromagnetic interpretation [1] the value $\psi^+\hat{\alpha}_5\psi$ is the pseudoscalar of electromagnetic theory $\psi^+\hat{\alpha}_5\psi=\vec{E}\cdot\vec{H}$, we can affirm, that in the CWED the p-helicity is the Lorentz-invariant value for a massive particle, and it is also the origin of the non-conservation of parity observed for massive particles.

Let's show now, that the electromagnetic field of matrix (4.9), which satisfies the Dirac-like equation, is not, the field of a photon.

As is known [12], the field of a photon is a vector that will transform according to the elements of group (O3). The spinor fields of the Dirac equation will transform as elements of the group (SU2). As shown by L.H. Ryder [12], two spinor transformations correspond to one transformation of a vector. For this reason the spinors are also named "semi-vectors" or " tensors of half rank''.

Division and twirling of the photon-like waves corresponds to transition from the usual linear Maxwell equations to the Maxwell equations for the curvilinear wave with an imaginary tangential current (*i.e.,* to the Dirac-like equation). Obviously, the transformation properties of electromagnetic fields at this transition change. As wave functions of the Dirac equation (*i.e.,* spinors) transform according to elements of the group (SU2), the semi-photon fields will also transform in the same way.

The question arises: what do semi-photon fields represent and how do they differ from the photon fields (*i.e.,* an electromagnetic wave). In the following section we will try to answer this question.

6.0. Specification of Neutrino-Like Particle Structure

Let us suppose that a neutrino-like particle of the above Dirac-like equation is the twirled half-period of a photon-like particle.

In this case the neutrino is a twirled helicoid represented by the Möbius's strip: its field vector (electric or magnetic) at the end of one rotation through a complete circle, has a direction opposite to that of the initial vector, and only after two complete rotations, does the vector come back to its starting position (see fig. 6)

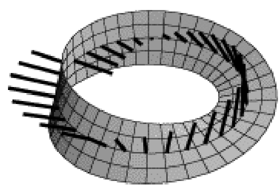

Fig. 6

(see also the figure of the Möbius's strip from [13], where the animation shows a series of gears arranged along a Möbius strip as the electric and magnetic field vectors motion)

Verification of the above conclusion about the neutrino-like particle structure is obtained by analysis of the transformation properties of the twirled semi-photon wave function. In view of them having the same mathematical equations, we affirm that these transformation features coincide with those of the spinor [8,12].

The spinor transformation has the form:

$$\psi' = U\psi \,, \tag{6.1}$$

where the operator of transformation is entered as follows:

$$U\left(\vec{n}\theta\right) = \cos\frac{1}{2}\theta - i\vec{n}\cdot\vec{\sigma}'\sin\frac{1}{2}\theta \,, \tag{6.2}$$

where $\vec{n}$ is the unit vector of an axis, θ is a rotation angle around this axis and $\vec{\sigma}' = \left(\sigma_x', \sigma_y', \sigma_z'\right)$ is the spin vector.

The rotation matrix (6.2) possesses a remarkable property. If the rotation occurs by the angle $\theta = 2\pi$ around any axis (thus causing a return to the initial position) we find, that $U = -1$, instead of $U = 1$ as is normally expected; this corresponds to the state vector of a system with spin one half (in three-dimensional space) having an ambiguity, and only returning to itself after a rotation by an angle of 4π; this rotation corresponds to one wave length of the electromagnetic wave.

From the above it follows that a semi-photon can only appear in CWED, and that in classical, linear electrodynamics it does not exist.

In the next section, it is shown that such a particle has mass, but is electrically neutral (*i.e.,* it has no electric charge).

7.0. Particle Charge and Mass in CWED

The particle mass is defined as the integral of the energy density over the space wherein the field is not zero. This integral is proportional to the field strength squared, and hence the integral is always positive if the field is different from zero in part of the space.

Similarly, the charge is defined by the integral over space of the current density; this is proportional to the first power of the field strength, and hence it is possible for the integral to be zero, even though the field itself (the integrand) is not equal to zero; this result is obtained when the integrand is a harmonic function as in equations [4.13’’].

7.1. Charge Calculation

The charge density of the twirled semi-photon-like particle is:

$$\rho_p = \frac{j_\tau}{c} = \frac{1}{4\pi}\frac{\omega_p}{c}E = \frac{1}{4\pi}\frac{1}{r_p}E\,, \tag{7.1}$$

The charge of the twirled semi-photon-like is defined by:

$$q = \int\limits_{\Delta\tau_t} \rho_p d\tau\,, \tag{7.2}$$

where $\Delta\tau_t$ is the volume within which the fields are non-zero.

Using the model (Fig.5) and taking $\vec{E} = \vec{E}(l)$, where l is the length along the axis of propagation, we obtain:

$$q = \int_{S_t} \int_0^{\lambda_p} \frac{1}{4\pi} \frac{\omega_p}{c} E_o \cos k_p l \, dl \, ds = \frac{1}{4\pi} \frac{\omega_p}{c} E_o S_c \int_0^{\lambda_p} \cos k_p l \, dl = 0 \text{ ,(7.3)}$$

E_o is the amplitude of the twirled photon wave field, S_c the cross-sectional area of the torus, ds is the element of the cross-section surface, dl the element of length, and $k_p = \dfrac{\omega_p}{c}$ the wave-vector.

This result arises because the ring current is alternating, and hence the charge is zero.

7.2. Mass Calculation

The energy density of the electromagnetic field is:

$$\rho_\varepsilon = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right), \tag{7.4}$$

In the linear approximation, $\left|\vec{E}\right| = \left|\vec{H}\right|$ in Gaussian units, so that (7.4) can be simplified to:

$$\rho_\varepsilon = \frac{1}{4\pi} E^2 , \tag{7.5}$$

Using (7.5) and the Einstein relationship between mass and energy:

$$\rho_m = \frac{1}{c^2} \rho_\varepsilon , \tag{7.6}$$

we obtain:

$$\rho_m = \frac{1}{4\pi\ c^2} E^2 = \frac{1}{4\pi\ c^2} E_o \cos^2 k_s l\,, \tag{7.7}$$

This yields for the semi-photon mass:

$$m_s = \int\limits_{S_t}\int\limits_{l} \rho_m ds\ dl = \frac{S_c {E_o}^2}{\pi\ c^2}\int\limits_0^{\lambda_s} \cos^2 k_s l\ kl\,, \tag{7.8}$$

which evaluates to:

$$m_s = \frac{E_o S_c}{4\omega_s c} = \frac{\pi\ E_o^2 r_s^2}{4\omega_s c}\,, \tag{7.9}$$

Thus in CWED there are the cases where the particle mass is not zero, but the particle charge is zero.

The properties of the CWED particle, described above are:

1. it is a fermion, it has mass but it does not have any charge.
2. there are particles and antiparticles, which are distinguished only by p-helicity .
3. the CWED neutrino has all the invariant properties required to conform with the theory of the weak interaction.

Thus the particle described by the above theory corresponds to the experimental properties of real neutrinos.

Also in CWED two neutrino-like particles (with left and right poloidal helicity) correspond to one twirled circularly polarized photon-like particle. This accords with the theory of Louis de Broglie about the neutrino nature of light [14], and reveals that a photon of L. de Broglie theory is a twirled (non-linear, curvilinear) particle, rather than a particle whose motion obeys the classical, linear, Maxwell equations.

Conclusion

In the above nonlinear theory (CWED) there is a neutrino-like particle possessing an internal, poloidal rotation of its own fields. This rotation corresponds with the circular-polarization of the semi-photon, which has the properties of a neutrino-like particle.

It was shown that the internal motion of this semi-photon on a circular trajectory is described by a Dirac-like equation for a lepton with mass equal to zero, *i.e.,* by the Weyl equation; in the framework of CWED the Weyl equation is the equation for the internal motion of a neutrino-like particle-field. Solution of this Weyl equation (as shown above) yields an internal poloidal rotation (p-helicity), which confers the massive neutrino with the property of appearing to be massless. On the other hand, when regarded as a "stopped" twirled electromagnetic wave, the massive neutrino-like particle is described by a Dirac-like equation with a mass term.

If we identify the CWED neutrino with the neutrino of the Standard Model, we eliminate the difficulties of the Standard Model with only a small alteration of the theory: the recognition that the neutrino has an internal motion described by the Weyl equation.

References

1. S.M. Bilenky, **C.** Giunti and C.W. Kim. Finally neutrino has mass. *Journal: Int. J. Mod. Phys*. A **15**, 625 (2000). Preprint: KIAS-P99011, DFTT 08/99, hep- ph/9902462.
2. S.L. Glashow, Nucl. Phys. **22**, 597 (1961); S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); A. Salam, Proc. of the 8th Nobel Symposium on *Elementary particle theory, relativistic groups and analyticity*, edited by N. Svartholm, 1969.
3. L.T. Schiff. *Quantum Mechanics*. 2nd edition, McGraw-Hill Book Company, Jnc, New York, 1955.
4. A. I. Ternov, *Electromagnetic features of massive neutrino*. SOJ, No. 10, 2000 (in Russian).

5. A.G. Kyriakos, The electromagnetic form of the Dirac electron theory. *Apeiron*, Vol. 11, No. 2, April 2004: http://redshift.vif.com/JournalFiles/V11NO2PDF/V11N2KYR.pdf
6. A.G. Kyriakos, The Dirac electron theory as the approximation of the nonlinear electrodynamics. *Apeiron*, Vol. 11, No. 3, July 2004: http://redshift.vif.com/JournalFiles/V11NO3PDF/V1N3KYR.pdf
7. F.S. Grawford. *Waves*. Berkeley physics course. 1970.
8. K.Gottfried, V.F.Weisskopf. *Concepts of Particle Physics*. Oxford, 1984.
9. M.-A. Tonnelat. *Les Principes de la Theorie Electromagnetique et de la Relativite.* Masson et C. Editeurs, Paris, 1959.
10. J. D. Jackson, *Classical Electrodynamics*, 3rd ed., 808 pp. Wiley, New York, 1999.
11. A.S.Davidov. *Quantum Mechanics* (in Russian). Moscow, 1963.
12. L.H. Ryder. *Quantum field theory*. Cambridge University Press, 1985.
13. MathWord, Möbius strip: http://mathworld.wolfram.com/MoebiusStrip.html
14. L.de Broglie, *C.R.Acad.Sci.* (Paris) **195** (1932) 536, 577, 862 ; *ibid*. **1973**(1932) 1337; *ibid*. **198** (1934) 135; *etc.*