The Dirac Electron Theory as an Approximation of Nonlinear Electrodynamics

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It is shown that the Dirac electron theory is an approximation of a nonlinear electromagnetic field theory.
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1.0. Introduction

In the previous paper, [1] we showed that the Dirac electron theory can be presented one-to-one as a complex form of the Maxwell theory. The question arises: why should this be so? Is this representation sheer coincidence, or is it a result of a deep relation between these theories. This question can also be formulated differently: can the vectorial Maxwell theory produce the spinor Dirac theory?

To see how this question can be solved, we will consider a brief review of the development of field theory.

After the discovery of the Dirac electron equation, the main task of physicists was to find the equations that describe all other particles and fields. Attempts were made, mainly, on the basis of a generalization of the already known equations: the equations of the Maxwell electromagnetic theory, the equation of the Dirac electron theory and of the Einstein equation of gravitation.

On one hand, various generalizations of the Maxwell equation (see the review in [2]) have been suggested. The most interesting of them were the nonlinear generalization suggested by G. Mie [3], and then concretized by M. Born and L. Infeld [3], the generalization on the high order derivatives of F. Bopp and B. Podolsky [5], the electromagnetic equations with a mass term of A. Proca [6], and some other.

On the basis of the Einstein equation of gravitation, attempts were made within the framework of the unified field theory to create generalizations of the electromagnetic theory on a curvilinear space (see earlier works in [7], and more recently [8, 9] and others).

On the other hand, various generalizations of the Dirac theory have been suggested (see the review in [10]). The most interesting was the nonlinear generalization of the Dirac equation postulated by Heisenberg [11] and analyzed by him and his collaborators [12]. Some promising results were obtained.

One generalization of the Dirac electron equation is the Yang-Mills equation, which is used for the description of the electroweak and the strong interactions. Also it has been shown that the classical electrodynamics equations constructed on the rotation group O(3), practically coincide with the Yang - Mills equation [13]

Nevertheless, many studies have been devoted to various mathematical representations of the Maxwell and Dirac theories, and also to the connections between these theories. The initial work here is [14] (for the Schroedinger equation). It has also been shown that the Maxwell equation can be written down in the form of the Dirac equation without a free term (see e.g. [15,16]). It was shown later that many features of the Dirac theory can be presented in an electromagnetic form [17,18,19] (here see also the detailed bibliography).

Along the same lines, the following exclusion has been found: it appeared that field vectors of the Maxwell theory could not be represented in spinor form (see [20]).

In the present work it is shown that this exclusion is removed only in the case when, instead of the linear Maxwell theory, we consider a nonlinear theory of a special kind, in which the fields of an electromagnetic wave are considered as vectors of an electromagnetic field, and non-linearity arises due to the curvature of a movement trajectory of this wave. In this case, the electromagnetic field forms the objects described as spinors.

The question about what physical sense these "electromagnetic spinors" have, demands further discussion.

2.0. The electrodynamic form of the Dirac equation without mass

Let us recall the usual quantum form of the Dirac electron equation. We will begin the consideration from two typical bispinor Dirac equation forms [15,21-23]:

$$\left[\left(\hat{a}_o\hat{e} + c\hat{\vec{a}}\cdot\hat{\vec{p}}\right) + \hat{b}\; m_e c^2\right] y = 0\,, \tag{2.1}$$

$$y^{+}\left[\left(\hat{a}_o\hat{e} - c\hat{\vec{a}}\cdot\hat{\vec{p}}\right) - \hat{b}\; m_e c^2\right] = 0\,, \tag{2.2}$$

which correspond to the classical relativistic expression of the energy of the electron:

$$e = \pm\sqrt{c^2\vec{p}^{\,2} + m_e^2 c^4}\,, \tag{2.3}$$

where $\hat{e} = i\hbar \frac{¶}{¶\ t}$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ are the operators of the energy and momentum, $e, \vec{p}$ are the electron energy and momentum, c is the light velocity, m_e is the electron mass, and $\hat{a}_o = \hat{1}$, $\hat{\vec{a}}$, $\hat{a}_4 \equiv \hat{b}$ are the Dirac matrices:

$$\hat{a}_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \hat{a}_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{a}_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \vec{a}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

y is the wave function $y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$ called bispinor, y^+ is the Hermitian-conjugate wave function.

Now we will consider the derivation of the electrodynamic form of the Dirac equations without mass term. Let us consider the plane electromagnetic wave moving on the *y*-axis. In the general case it has two polarizations and contains the following field vectors:

$$E_x, E_z, H_x, H_z, \tag{2.4}$$

(As it is known, for all transformations the relation $E_y = H_y = 0$ takes place, so that there are always only four components (2.4) as in the Dirac theory).

Let us enter the electromagnetic wave fields as the Dirac bispinor matrix:

$$y = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, y^+ = \begin{pmatrix} E_x & E_z & -iH_x & -iH_z \end{pmatrix}, \tag{2.5}$$

(For all other directions of the electromagnetic waves the choice of matrices was considered in [1]).

Then the Klein-Gordon equation without mass can be written as:

$$\left(\hat{e}^2 - c^2 \hat{\vec{p}}^2\right) y = 0, \tag{2.6}$$

Using (2.5), we can prove that (2.6) is also the equation of the electromagnetic wave, moving along the *y*-axis. The equation (2.6) can also be written in the following form:

$$\left[\left(\hat{a}_o \hat{e}\right)^2 - c^2 \left(\hat{\vec{a}} \cdot \hat{\vec{p}}\right)^2\right] y = 0, \tag{2.7}$$

In fact, taking into account that

$$\left(\hat{a}_o \hat{e}\right)^2 = \hat{e}^2, \quad \left(\hat{\vec{a}} \cdot \hat{\vec{p}}\right)^2 = \hat{\vec{p}}^2, \tag{2.8}$$

we see that the equations (2.6) and (2.7) are equivalent.

Factorizing (2.7) and multiplying it from the left by the Hermitian-conjugate function y^+ we get:

$$y^+ \left(\hat{a}_o \hat{e} - c\hat{\vec{a}} \cdot \hat{\vec{p}}\right) \left(\hat{a}_o \hat{e} + c\hat{\vec{a}} \cdot \hat{\vec{p}}\right) y = 0, \tag{2.9}$$

The equation (2.9) may be decomposed into two Dirac equations without mass:

$$y^{+}\left(\hat{a}_{o}\hat{e} - c\hat{\vec{a}} \cdot \hat{\vec{p}}\right) = 0\,, \tag{2.10}$$

$$\left(\hat{a}_{o}\hat{e} + c\hat{\vec{a}} \cdot \hat{\vec{p}}\right) y = 0\,, \tag{2.11}$$

It is not difficult to show (using (2.5) that the equations (2.10) and (2.11) are also the Maxwell equations in the case of plane electromagnetic waves [24].

Now we will consider the question of the appearance of mass in the Dirac equation.

3.0. The appearance of the mass term

The decomposition of (2.6) into (2.10) and (2.11) can be compared with the typical transformation of the massless quantum of an electromagnetic wave g into two massive particles (electron-positron) e^{-}, e^{+}:

$$g \rightarrow e^{+} + e^{-}\,, \tag{3.1}$$

Then the question arises: which mathematical transformation can turn the equations (2.10) and (2.11) without mass term into the equations (2.1) and (2.2) with mass term?

We will show that it can be done, at least, in two ways: either by using curvilinear metrics, or by using differential geometry.

3.1. Generalization of the Dirac equation on Riemann geometry

Generalization of the Dirac equation on Riemann geometry is connected with the parallel transport and covariant differentiation of the spinor in curvilinear space. These problems were considered for the first time in the articles [25] and further in the articles [26]. We will use the most important results of this theory below.

For generalization of the Dirac equation on Riemann geometry it is necessary [25,26] to replace the usual derivative $\partial_m \equiv \P / \P x_m$ (where x_m are the co-ordinates in the 4-space) with the covariant derivative:

$$D_m = \P_m + \Gamma_m , \qquad (3.2)$$

where $m = 0, 1, 2, 3$ are the summing indices and Γ_m is the analogue of Christoffel's symbols in the case of the spinor theory (called Ricci connection coefficients or the coefficient of the parallel transport of the spinor). In the theory it is shown [25] that $\hat{a}_m \Gamma_m = \hat{a}_i p_i + i \hat{a}_0 p_0$, where p_i and p_0 are not the operators, but the physical values.

Thus, using (3.2) we obtain from (2.10) and (2.11):

$$a^m D_m y = a^m (\P_m + \Gamma_m)\ y = 0 ,$$

When a spinor moves along a straight line, all of the $\Gamma_m = 0$ and we have a usual derivative. But if a spinor moves along a curvilinear trajectory, then not all of the Γ_m are equal to zero and a supplementary term appears. Typically, the latter is not the derivative, but it is equal to the product of the spinor itself with some coefficient Γ_m. It is not difficult to show that the supplementary term contains a mass.

Since, according to general theory [25,26], the increment in spinor Γ_m has the form and the dimension of the 4-vector of the energy-momentum, it is logical to identify Γ_m with 4-vector of energy-momentum of the photon electromagnetic field:

$$\Gamma_m = \{e_p, c\vec{p}_p\}, \tag{3.3}$$

where e_p and p_p are the electromagnetic wave energy and momentum. (Below we will prove this supposition from another point of view.) *Depending on the direction of wave motion and its spin direction,* Γ_m *can have a different form, corresponding to the equation (3.6). Thus, there are various substitutions and this allows us to obtain in the curvilinear space from (2.10) and (2.11) the following forms:*

$$y^{+}\left[\left(\hat{a}_o\hat{e} - c\hat{\vec{a}}\cdot\hat{\vec{p}}\right) - \left(\hat{a}_o e_p - c\hat{\vec{a}}\cdot\vec{p}_p\right)\right] = 0, \tag{3.4}$$

$$\left[\left(\hat{a}_o\hat{e} + c\hat{\vec{a}}\cdot\hat{\vec{p}}\right) + \left(\hat{a}_o e_p + c\hat{\vec{a}}\cdot\vec{p}_p\right)\right] y = 0, \tag{3.5}$$

The equation (3.1) in the electromagnetic representation corresponds to the transition from one photon wave to two spinning (advanced + retarded) waves. In the energetic form this corresponds to the following linear equation:

$$\hat{a}_o e_p \pm c\hat{\vec{a}}\cdot\vec{p}_p = \mp\hat{b}\, m_e c^2, \tag{3.6}$$

Using (3.6), from (3.4) and (3.5) we will obtain the usual kind of Dirac equation with mass:

$$y^{+}\left[\left(\hat{a}_o\hat{e} - c\hat{\vec{a}}\cdot\hat{\vec{p}}\right) - \hat{b}\, m_e c^2\right] = 0, \tag{3.7}$$

$$\left[\left(\hat{a}_o\hat{e} + c\hat{\vec{a}}\cdot\hat{\vec{p}}\right) - \hat{b}\, m_e c^2\right] y = 0, \tag{3.8}$$

We now find which value corresponds to the mass term in the electromagnetic form of the Dirac equation.

3.2. Electrodynamics form of the Dirac equation with mass

Consider two Hermitian-conjugate equations, corresponding to the minus sign of the expression (2.3):

$$\left[\left(\hat{a}_o\hat{e} + c\hat{\vec{a}}\cdot\hat{\vec{p}}\right) + \hat{b}\ m_e c^2\right] y\ = 0\,, \tag{3.9}$$

$$y^{+}\left[\left(\hat{a}_o\hat{e} + c\hat{\vec{a}}\cdot\hat{\vec{p}}\right) + \hat{b}\ m_e c^2\right] = 0\,, \tag{3.10}$$

Using (2.5), from (3.9) and (3.10) we obtain:

$$\begin{cases} \frac{1}{c}\frac{\P\ E_x}{\P\ t} - \frac{\P\ H_z}{\P\ y} + i\frac{w}{c}E_x = 0, \\ \frac{1}{c}\frac{\P\ E_z}{\P\ t} + \frac{\P\ H_x}{\P\ y} + i\frac{w}{c}E_z = 0, \\ \frac{1}{c}\frac{\P\ H_x}{\P\ t} + \frac{\P\ E_z}{\P\ y} - i\frac{w}{c}H_x = 0, \\ \frac{1}{c}\frac{\P\ H_z}{\P\ t} - \frac{\P\ E_x}{\P\ y} - i\frac{w}{c}H_z = 0, \end{cases} \tag{3.11}$$

$$\begin{cases} \frac{1}{c}\frac{\P\ E_x}{\P\ t} - \frac{\P\ H_z}{\P\ y} - i\frac{w}{c}E_x = 0, \\ \frac{1}{c}\frac{\P\ E_z}{\P\ t} + \frac{\P\ H_x}{\P\ y} - i\frac{w}{c}E_z = 0, \\ \frac{1}{c}\frac{\P\ H_x}{\P\ t} + \frac{\P\ E_z}{\P\ y} + i\frac{w}{c}H_x = 0, \\ \frac{1}{c}\frac{\P\ H_z}{\P\ t} - \frac{\P\ E_x}{\P\ y} + i\frac{w}{c}H_z = 0, \end{cases} \tag{3.12}$$

where $w = \frac{m_e c^2}{\hbar}$. The equations (3.11) and (3.12) are Maxwell equations with imaginary currents, which differ in direction. These currents have the following form:

$$\vec{j}_e = i\frac{w}{4p}\vec{E}\,,\ \vec{j}_m = i\frac{w}{4p}\vec{H}\,, \tag{3.13}$$

and it is interesting that together with the electric current j_e the magnetic current j_m also exists here. The latter must be equal to zero according to the Maxwell theory [24], but its existence according to Dirac does not contradict quantum theory. (See the Dirac theory of the magnetic monopole [27].)

Thus the term that in the Dirac equation contains the electron mass, corresponds to the term that in the Maxwell equation contains the imaginary electric and the imaginary “magnetic” current.

Now we will consider the origin of the appearance of the current-mass term in the electromagnetic form by using differential geometry.

3.3. The displacement ring current

We will consider the Maxwell equations without current (2.10) or (2.11), as the equations of the initial photon field. The reaction (3.1) can then be formally understood in such a way that while moving through the nucleus field, the electromagnetic wave fields may undergo a transformation and convert into the electron-positron pair.

We will show that the reason why the current-mass term appears in the equations (2.10) and (2.11) is the transition of the initial electromagnetic wave field from the linear to the curvilinear trajectory, and also that this term is the supplementary Maxwell displacement current.

Let the plane-polarized wave, which has the field vectors (E_x, H_z), be spun with some radius r_p in the plane (X', O', Y') of a fixed co-ordinate system (X', Y', Z', O') so that E_x is parallel to the plane (X', O', Y') and H_x is perpendicular to it.

According to Maxwell [24] the displacement current is defined by the equation:

$$\vec{j}_{dis} = \frac{1}{4p} \frac{\P \vec{E}}{\P t}, \tag{3.14}$$

The above electrical field vector $\vec{E}$, which moves along the curvilinear trajectory (let it have a direction from the centre), can be written in the form:

$$\vec{E} = -E\,\vec{n}, \tag{3.15}$$

where $E = \left|\vec{E}\right|$ and $\vec{n}$ is the normal unit-vector of the curve (having direction to the center). The derivative of $\vec{E}$ with respect to t can be represented as:

$$\frac{\P \vec{E}}{\P t} = -\frac{\P E}{\P t}\vec{n} - E\frac{\P \vec{n}}{\P t}, \tag{3.16}$$

Here the first term has the same direction as $\vec{E}$. The existence of the second term shows that when the wave spins the additional displacement current appears. It is not difficult to show that it has a direction tangential to the ring:

$$\frac{\P \vec{n}}{\P t} = -\frac{u_p}{r_p}\vec{t}, \tag{3.17}$$

where $\vec{t}$ is the tangential unit-vector, $u_p \equiv c$ is the electromagnetic wave velocity. Thus, the displacement current of the ring wave can be written in the form:

$$\vec{j}_{dis} = -\frac{1}{4p}\frac{\P E}{\P t}\vec{n} + \frac{1}{4p}w_p E\,\vec{t}\,, \tag{3.18}$$

where $w_p = \frac{u_p}{r_p}$ is the angular velocity, $\vec{j}_n = \frac{1}{4p}\frac{\P E}{\P t}\vec{n}$ and $\vec{j}_t = \frac{w_p}{4p}E\,\vec{t}$ are the normal and tangent components of the displacement current of the spinning electromagnetic wave, correspondingly. Thus:

$$\vec{j}_{dis} = \vec{j}_n + \vec{j}_t\,, \tag{3.19}$$

The currents $\vec{j}_n$ and $\vec{j}_t$ are always mutually perpendicular, so that we can write them in the complex form: $j_{dis} = j_n + ij_t$, where $j_t = \frac{w_p}{4p}E$. It corresponds exactly to the vector equation (3.18).

Thus, as we see, the transition of the initial electromagnetic wave from the linear to the curvilinear trajectory corresponds to the production of the Dirac bispinor theory. We can prove this by analysis of the free electron equation solution.

4.0. Electromagnetic form of the free electron equation solution

In accordance with the above results the electromagnetic form of the solution of the Dirac free electron equation must be a spinning electromagnetic wave. Let us show that this supposition is actually correct.

From the above point of view for the y-direction photon two solutions must exist:

1) for the wave, spinning around the OZ-axis

$$^{oz}y = \begin{pmatrix} E_x \\ 0 \\ 0 \\ iH_z \end{pmatrix} = \begin{pmatrix} y_1 \\ 0 \\ 0 \\ y_4 \end{pmatrix}, \tag{4.1}$$

2) for the wave, spinning around the OX-axis

$$^{ox}y = \begin{pmatrix} 0 \\ E_z \\ iH_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ y_2 \\ y_3 \\ 0 \end{pmatrix}, \tag{4.2}$$

Let us compare (4.1) and (4.2) with the Dirac theory solutions.

It is known [15, 21-23] that the solution of the Dirac free electron equation (2.1) has the form of the plane wave:

$$y_j = B_j \exp\left(-\frac{i}{\hbar}(e\ t - \vec{p}\vec{r}) \right), \tag{4.3}$$

where $j = 1,\ 2,\ 3,\ 4$; $B_j = b_j e^{if}$; the amplitudes b_j are the numbers and f is the initial wave phase. The functions (4.3) are the eigenfunctions of the energy-momentum operators, where e and $\vec{p}$ are the energy-momentum eigenvalues. Here, according to equation (2.3), for each $\vec{p}$, the energy e has either positive $e_+ = +\sqrt{c^2\vec{p}^2 + m_e^2c^4}$ or negative values $e_- = -\sqrt{c^2\vec{p}^2 + m_e^2c^4}$.

Note, that generally for the particles, the equation (2.3) stands. But here we are concerned with the fields of the motionless particle in the electromagnetic representation. For the electron's nonlinear wave fields the equation $e = m_e c^2$ stands, as it is half of the initial photon energy.

For e_+ we have two linearly independent sets of four orthogonal normalizing amplitudes:

$$1)\ B_1 = -\frac{cp_z}{e_+ + m_e c^2},\ B_2 = -\frac{c(p_x + ip_y)}{e_+ + m_e c^2},\ B_3 = 1,\ B_4 = 0, \tag{4.4}$$

$$2)\ B_1 = -\frac{c(p_x - ip_y)}{e_+ + m_e c^2},\ B_2 = \frac{cp_z}{e_+ + m_e c^2},\ B_3 = 0,\ B_4 = 1, \tag{4.5}$$

and for e_-:

$$3)\ B_1 = 1,\ B_2 = 0,\ B_3 = \frac{cp_z}{-e_- + m_e c^2},\ B_4 = \frac{c(p_x + ip_y)}{-e_- + m_e c^2}, \tag{4.6}$$

$$4)\ B_1 = 0,\ B_2 = 1,\ B_3 = \frac{c(p_x - ip_y)}{-e_- + m_e c^2},\ B_4 = -\frac{cp_z}{-e_- + m_e c^2}, \tag{4.7}$$

Let us analyze these solutions.

1) The existence of two linear independent solutions corresponds with two independent orientations of the electromagnetic wave vectors, and gives a unique logical explanation for this fact.

2) Since here $y = y(y)$ and the electron-positron fields, like the spinning waves, have momentum equal to half of the photon momentum, i.e. $m_e c$, we have $p_x = p_z = 0,\quad p_y = m_e c$. Then for the field vectors we obtain: from (4.4) and (4.5) for "positive" energy

$$B_+^{(1)} = \begin{pmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \cdot e^{if}, \quad B_+^{(2)} = \begin{pmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{pmatrix} \cdot e^{if}, \tag{4.8}$$

and from (4.6) and (4.7) for "negative" energy:

$$B_-^{(1)} = \begin{pmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{pmatrix} \cdot e^{if}, \quad B_-^{(2)} = \begin{pmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \cdot e^{if}, \tag{4.9}$$

which exactly correspond to (4.1) and (4.2).

3) Calculate the correlations between the components of the field vectors. Putting $f = \frac{p}{2}$ for $e_+ = m_e c^2$ and $e_- = -m_e c^2$ we obtain correspondingly:

$$B_+^{(1)} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ i \cdot 1 \\ 0 \end{pmatrix}, \quad B_+^{(2)} = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ 0 \\ i \cdot 1 \end{pmatrix}, \tag{4.10}$$

$$B_-^{(1)} = \begin{pmatrix} i \cdot 1 \\ 0 \\ 0 \\ -\frac{1}{2} \end{pmatrix}, \quad B_-^{(2)} = \begin{pmatrix} 0 \\ i \cdot 1 \\ \frac{1}{2} \\ 0 \end{pmatrix}, \tag{4.11}$$

The imaginary unit in these solutions indicates that the field vectors $\vec{E}$ and $\vec{H}$ are mutually orthogonal. Also we see that the electric field amplitude is two times less than the magnetic field amplitude. This fact corresponds to the electromagnetic spinor contrary to the linear wave of the Maxwell theory, where the field vectors $\vec{E}$ and $\vec{H}$ are equal. It can be shown that this relation is needed for electron stability. (Note also, that the appearance of a minus sign before the electric field vector in $B_{+}^{(1)}$ and $B_{-}^{(1)}$ automatically corrects the field directions in (4.1) – (4.2)).

4) It is easy to show that the electromagnetic form of the solution of the Dirac equation (4.3) is the standing wave. Really, since in the case of the spinning circle wave the vector $\vec{r}$ is the radius-vector of the circle trajectory and the vector $\vec{p}$ has the tangential direction to the circle, we have $\vec{p} \perp \vec{r}$. Hence $\vec{p} \cdot \vec{r} = 0$ and instead (4.3) we obtain:

$$y_{j} = b_{j} \exp\left(-\frac{i}{\hbar} e \; t \right), \tag{4.12}$$

Thus, the equation (4.12) is the oscillation equation and in this sense it is the standing wave equation.

5) According to the Euler formula $e^{ij} = \cos j + i \sin j$ the solution of the Dirac equation (4.12) describes a circle.

It is appropriate to note here that in case when the function (2.5) is the solution of the Dirac equation in the electromagnetic form we can call it an "electromagnetic bispinor". In other words the electromagnetic bispinor is the electromagnetic wave, moving along the curvilinear trajectory.

From this follows that the Dirac electron equation in the electromagnetic form must be the nonlinear electromagnetic field equation. Below, we will clarify the explicit form of this equation.

5.0. Nonlinear electrodynamic representation of the Dirac electron theory

Stability of a spinning photon is possible only by the self-action of the photon's parts. We could introduce self-action of fields in the Dirac equation, just as external interaction is introduced in the quantum field theory equations, putting the photon mass equal to zero [15, 21-23]. But this equation may be obtained more easily using relation (3.6), for example, the expression

$$\hat{b}\; m_e c^2 =- \left(\boldsymbol{e}_p - c\hat{\vec{a}} \cdot \vec{p}_p\right), \tag{5.1}$$

By substituting (5.1) for the Dirac electron equation we obtain the nonlinear integral equation:

$$\left[\hat{a}_0\left(\hat{e} - \boldsymbol{e}_p\right) + c\hat{\vec{a}} \cdot \left(\hat{\vec{p}} - \vec{p}_p\right)\right] y = 0 , \tag{5.2}$$

where:

$$\boldsymbol{e}_p = \int_0^t U \; dt , \tag{5.3}$$

$$\vec{p}_p = \int_0^t \vec{g} \; dt = \frac{1}{c^2}\int_0^t \vec{S} \; dt , \tag{5.4}$$

and we take that the upper limit t is the variable. (Here U, $\vec{g}$, $\vec{S}$ are the energy density, momentum density and Poynting vector, correspondingly.)

We suppose that (5.2) is the common form of the nonlinear electron structure equation, which describes the electron in both quantum and concurrent electromagnetic forms.

In the electromagnetic form we have [24]:

$$\begin{cases} U = \dfrac{1}{8p}\left(\vec{E}^2 + \vec{H}^2\right) \\ \vec{g} = \dfrac{1}{4p}\left[\vec{E} \times \vec{H}\right] \end{cases}, \tag{5.5}$$

and in the quantum form we have [1]:

$$\begin{cases} U = \dfrac{1}{8p} y^{+} \hat{a}_0 y \\ \vec{g} = -\dfrac{1}{8p\, c} y^{+} \hat{\vec{a}}\, y \end{cases}, \tag{5.6}$$

Let us show that in the approximate form the equation (5.2) gives the forms that are known in modern theory [11,12], particularly the form of the known nonlinear spinor equation:

$$g_m \frac{\partial y}{\partial x_m} + \frac{1}{2} l^2 \left[g_m y \left(\bar{y} g_m y \right) + g_m g_5 y \left(\bar{y} g_m g_5 y \right) \right] = 0 , \tag{5.7}$$

which was investigated by Heisenberg *et al*. [11,12] and which for a while played the part of the unitary field theory equation

Since the main part of the electron energy consists of some finite volume, we can write approximately:

$$t \approx \Delta t = V\, r_p^{\,3} , \tag{5.8}$$

where V is a constant. Using the quantum form of U and $\vec{g}$ (see (5.6)) and taking into account that the free electron Dirac equation solution is the plane wave (4.3), we can write (5.6) in the next approximate form:

$$e_p \approx U\, \Delta t = \frac{\Delta t}{8p} y^{+} \hat{a}_0 y , \tag{5.9}$$

$$\vec{p}_p \approx \vec{g}\ \Delta t = -\frac{\Delta t}{8p\ c}\ y^{+}\hat{\vec{a}}\ y\ , \tag{5.10}$$

By substituting (5.9) and (5.10) into (5.2), we obtain the following approximate equation:

$$\frac{\P\ y}{\P\ t} - c\hat{\vec{a}}\vec{\nabla} y + i\frac{V}{8p\ \hbar c}\cdot r_p^{\ 3}\left(y^{+}\hat{a}_0 y - \hat{\vec{a}} y^{+}\hat{\vec{a}} y\right) y = 0\ , \tag{5.11}$$

It is not difficult to see that equation (5.11) is the nonlinear equation of the same type as (5.7), if instead of g -set matrices we use a -set matrices. Contrary to the latter, the equation (5.11) is obtained in a logical and correct way and the self-action constant r_p appeared in (5.11) automatically.

5.2. Lagrangian density of the nonlinear Dirac equation

The Lagrangian density of the linear Dirac equation in quantum form is [23]:

$$L_D = y^{+}\left(\hat{e} + c\hat{\vec{a}}\ \ \hat{\vec{p}} + \hat{b} m_e c^2\right) y\ , \tag{5.12}$$

or in the electromagnetic form [1]:

$$L_D = \frac{\P\ U}{\P\ t} + div\ \vec{S} - i\frac{w}{8p}\left(\vec{E}^2 - \vec{H}^2\right), \tag{5.13}$$

The Lagrangian density of a nonlinear equation is not difficult to obtain from the Lagrangian density of the linear Dirac equation [23] using the method by which we found the nonlinear equation. By substituting (5.1) we obtain:

$$L_N = y^{+}\left(\hat{e}\ - c\hat{\vec{a}}\cdot\hat{\vec{p}}\right) y + y^{+}\left(e_p - c\hat{\vec{a}}\cdot\vec{p}_p\right) y\ , \tag{5.14}$$

We suppose that the expression (5.14) represents the common form of the Lagrangian density of the nonlinear spinning electromagnetic wave equation.

Using (5.9) and (5.10) we can represent (5.12) in the approximate quantum form:

$$L_N = i\hbar\left[\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\psi^{+}\psi\right)\right] - c\,div\left(\psi^{+}\hat{\vec{\alpha}}\psi\right)\right] + \frac{\Delta t}{8\pi}\left[\left(\psi^{+}\psi\right)^2 - \left(\psi^{+}\hat{\vec{\alpha}}\psi\right)^2\right], \quad (5.15)$$

By normalizing the ψ-function with the expression $L'_N = \frac{1}{8\pi\, mc^2} L_N$ and transforming (5.14) into electrodynamic form using equations (5.5) and (5.7), we will obtain from (5.15) the following approximate electromagnetic form:

$$L'_N = i\frac{\hbar}{2m_e}\left(\frac{1}{c^2}\frac{\partial U}{\partial t} + div\,\vec{g}\right) + \frac{\Delta t}{m_e c^2}\left(U^2 - c^2\vec{g}^2\right), \qquad (5.16)$$

It is not difficult to transform the second term, using the known electrodynamic transformation:

$$(8\pi)^2\left(U^2 - c^2\vec{g}^2\right) = \left(\vec{E}^2 + \vec{H}^2\right)^2 - 4\left(\vec{E}\times\vec{H}\right)^2 = \left(\vec{E}^2 - \vec{H}^2\right)^2 + 4\left(\vec{E}\cdot\vec{H}\right)^2, \quad (5.17)$$

Thus, taking into account that $L_D = 0$ and using (5.13) and (5.17), we obtain from (5.16) the following expression:

$$L'_N = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right) + \frac{\Delta t}{(8\pi)^2 mc^2}\left[\left(\vec{E}^2 - \vec{H}^2\right)^2 + 4\left(\vec{E}\cdot\vec{H}\right)^2\right], \qquad (5.18)$$

As we see, the approximate form of the Lagrangian density of the nonlinear equation of the spinning electromagnetic wave contains only the invariants of the Maxwell theory and is similar to the known Lagrangian density of the photon-photon interaction [15].

Let us now analyze the quantum form of the Lagrangian density (5.18). The equation (5.12) can be written in the form:

$$L_Q = y^{+}\hat{a}_m \P_m y + \frac{\Delta t}{8p}\left[\left(y^{+}\hat{a}_0 y\right)^2 - \left(y^{+}\hat{\vec{a}}\, y\right)^2\right], \qquad (5.19)$$

It is not difficult to see that the electrodynamic correlation (5.17) in quantum form has the known form of the Fierz identity [28]:

$$\left(y^{+}\hat{a}_0 y\right)^2 - \left(y^{+}\hat{\vec{a}}\, y\right)^2 = \left(y^{+}\hat{a}_4 y\right)^2 + \left(y^{+}\hat{a}_5 y\right)^2, \qquad (5.20)$$

Using (5.20) from (5.19) we obtain:

$$L_Q = y^{+}\hat{a}_m \P_m y + \frac{\Delta t}{8p}\left[\left(y^{+}\hat{a}_4 y\right)^2 - \left(y^{+}\hat{a}_5 y\right)^2\right], \qquad (5.21)$$

The Lagrangian density (5.21) coincides with the Nambu and Jona-Lasinio Lagrangian density [29], which is the Lagrangian density of the relativistic superconductivity theory. It is well known that this Lagrangian density is used for the solution of the problem of appearance of elementary particles mass by the mechanism of the spontaneous breakdown of vacuum symmetry. (It also corresponds to Cooper pair production in superconductivity theory.)

Conclusion

We have showed that the description of electron-positron pair production corresponds to a reformation of the Maxwell linear theory into a nonlinear theory of the electromagnetic field, the approximation of which is equivalent to the Dirac theory. We can say that this transformation corresponds to the spontaneous breakdown of the photon, which corresponds to the creation of two massive particles, the electron and the positron.

The representation of QED in the form of a nonlinear theory of electromagnetic waves leads to an explanation of many features of modern quantum theory [30].

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