

Signatures of Quantum-like Chaos in Dow Jones Index

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Dow Jones Index time series exhibit irregular or *fractal* fluctuations on all time scales from days, months to years. The apparently irregular (nonlinear) fluctuations are selfsimilar as exhibited in inverse power law form for power spectra of temporal fluctuations. Inverse power law form for power spectra of *fractal* fluctuations in space or time is generic to all dynamical systems in nature and is identified as *self-organized criticality*. Selfsimilarity implies long-range space-time correlations or non-local connections. It is important to quantify the total pattern of *fractal* fluctuations for predictability studies, e.g., weather and climate prediction, stock market trends, etc. The author has developed a general systems theory for universal quantification of the observed inverse power law spectra in dynamical systems. The model predictions are as follows. (1) The power spectra of *fractal* fluctuations follow the universal and unique inverse power law form of the statistical normal distribution. (2) The non-local connections or long-range correlations in space or time

exhibited by the *fractal* fluctuations are signatures of quantum-like chaos in dynamical systems. (3) The apparently irregular geometry of the *fractal* fluctuations forms the component parts of a unified whole precise geometrical pattern of the logarithmic spiral with quasiperiodic *Penrose* tiling pattern for the internal structure. Conventional power spectral analyses will resolve the logarithmic spiral pattern as an eddy continuum with progressive increase in eddy phase angle. (4) Continuous periodogram power spectral analyses of normalised daily, monthly and annual Dow Jones Index for the past 100-years show that the power spectra follow the universal inverse power law form of the statistical normal distribution in agreement with model prediction. The *fractal* fluctuations of the non-stationary Dow Jones Index time series therefore exhibit signature of quantum-like chaos on all time scales from days to years.

Keywords: inverse power law spectra, Dow Jones index, chaos, fractals, nonlinear dynamics, 1/f noise, self-organized criticality

1. Introduction

1.1 Fractal fluctuations

Irregular (nonlinear) fluctuations on all scales of space and time are generic to dynamical systems in nature such as fluid flows, atmospheric weather patterns, heart beat patterns, stock market fluctuations, etc. Mandelbrot (1977) coined the name *fractal* for the non-Euclidean geometry of such fluctuations which have *fractional* dimension, for example, the rise and subsequent fall with time of the Dow Jones Index traces a zig-zag line in a two-dimensional plane and therefore has a fractional Euclidean (*fractal*) dimension greater than one but less than two. However, the evaluation of a clear *fractal* dimension from a non-stationary finite time series is an unsolved

problem. Mathematical models of dynamical systems are nonlinear and finite precision computer solutions also exhibit irregular or unpredictable *fractal* fluctuations similar to real world dynamical systems. Further, the computed solutions are sensitively dependent on initial conditions resulting in chaotic solutions, identified as *deterministic chaos*. The physics underlying *nonlinear dynamics, fractals and chaos* is now (since 1980s) an area of intensive research in all branches of science (Gleick, 1987).

1.2 Fractals, self-organized criticality and Fibonacci series

The *fractal* fluctuations of dynamical systems exhibit scale invariance or selfsimilarity manifested as the widely documented (Bak, Tang, Wiesenfeld, 1988; Bak and Chen, 1989; 1991; Schroeder, 1991; Stanley, 1995; Buchanan, 1997; Goldberger *et al.*, 2002) inverse power law form $f^{-\alpha}$ where f is the frequency and α the exponent for power spectra of space-time fluctuations. The amplitudes of the large and small-scale fluctuations are related to each other by the scale factor α alone. A constant value for the scale factor α indicates monofractals which exhibit the same scaling properties for all the time scale ranges. Real world dynamical processes however exhibit multifractal characteristics, i.e., the scale factor α varies with time scale range. In general the scale factor α decreases with decrease in frequency f and approaches 1 for large time scales indicating a multifractal structure for the geometry of the fluctuations.

The power law is a distinctive experimental signature seen in a wide variety of complex systems. In *economy* it goes by the name fat tails, in *physics* it is referred to as critical fluctuations, in *computer science and biology* it is the edge of chaos, and in *demographics* it is called *Zipf's law* (Newman, 2000). Power-law scaling is not new to economics. The power law distribution of wealth discovered by

Vilfredo Pareto (1848-1923) in the 19th century (Eatwell, Milgate and Newman, 1991) predates any power laws in physics (Farmer, 1999).

Bak *et al.* (1987; 1988) postulated in 1987 that *fractal* geometry to spatial pattern and associated *fractal* fluctuations of dynamical processes in time are signatures of *self-organized criticality* in the spatiotemporal evolution of dynamical system. The relation between spatial and temporal power-law behaviour was recognized much earlier in condensed matter physics where long-range spatiotemporal correlations appear spontaneously at the critical point for continuous phase transitions. The amplitudes of large and small-scale fluctuation are obtained from the same mathematical function using appropriate scale factor, i.e. ratio of the scale lengths. This property of self-similarity is often called a renormalization group relation in physics (Wilson, 1979; West, 1990a,b; Peitgen *et al.*, 1992) in the area of continuous phase transitions at critical points (Weinberg, 1993; Back *et al.*, 1995). When a system is poised at a critical point between two macroscopic phases, e.g., vapour to liquid, it exhibits dynamical structures on all available spatial scales, even though the underlying microscopic interactions tend to have a characteristic length scale (Back *et al.*, 1995). But, in order to arrive at the critical point, one has to fine-tune an external control parameter, such as temperature, pressure or magnetic field, in contrast to the phenomena described above for dynamical systems which occur universally without any fine-tuning. The explanation is that open extended dissipative dynamical systems, i.e., systems not in thermodynamic equilibrium may go automatically to the critical state as long as they are driven slowly.

Time series analyses of global market economy also exhibits power law behaviour (Bak *et al.*, 1992; Mantegna and Stanley, 1995; Sornette *et al.*, 1995; Chen, 1996a, b; Stanley *et al.*, 1996;

Gopikrishnan *et al.*, 1999; Plerou *et al.*, 1999; Stanley *et al.*, 2000; Gabaix *et al.*, 2003) with possible *multifractal* structure (Farmer, 1999) and has suggested an analogy to fluid turbulence (Ghasgaie *et al.*, 1996; Arneodo *et al.*, 1998; Sornette, 2002). The stock market can be viewed as a self-organizing cooperative system presenting power law distributions, large events in possible co-existence with synchronized behaviour (Sornette *et al.*, 1995). Sornette *et al.* (1995) also conclude that the observed power law represents structures similar to 'Elliott waves' of technical analysis first introduced in the 1930s. It describes the time series of a stock price as made of different waves; these waves are in relation to each other through the *Fibonacci* series. Sornette *et al.* (1995) speculate that 'Elliott waves' could be a signature of an underlying critical structure of the stock market. Chen (1996b) has identified color chaos and persistent cycles with characteristic period of around three to four years in time series analyses of Standard and Poor stock price indices.

Incidentally the *Fibonacci* series represent a *fractal* tree-like branching network of selfsimilar structures (Stewart, 1992). The general systems theory presented in this paper shows (Section 2) that *Fibonacci* series underlies *fractal* fluctuations on all space-time scales.

3. $1/f$ noise and multifractal scaling

In the past few years, scientists have been making rapid progress in developing models and theories for understanding the observed scale-invariant behaviour in driven, nonlinear dynamical systems. It is important to make quantitative comparisons between theoretical models and experimental systems (Sethna *et al.*, 2001). Multifractal signals generic to dynamical systems are intrinsically more complex and inhomogeneous than monofractals. The quantification of the commonly observed multifractal signals in dynamical systems is of

interest for a number of reasons. In the case of physiological time series, a majority of previous investigations have focused only on the quantification of a single exponent (i.e., monofractal behaviour) (Goldberger *et al.*, 2002) to characterize the observed $1/f$ -like scaling. Sornette (2002) has investigated a wide range of dynamical systems and concludes that the dynamical processes can exhibit power law behaviour with superimposed log-periodic oscillations. Empirical analyses of aggregate stock market price fluctuations have identified simple power-law (monofractal) behaviour (Stanley *et al.*, 2002). The physics underlying the multifractal structure generic to space-time fluctuations of dynamical systems is yet to be identified and quantified.

Self-organized criticality implies long-range correlations or non-local connections in temporal (or spatial) fluctuations of the dynamical system. Prediction of the future evolution of the dynamical system requires precise quantification of the observed multifractal fluctuations. The author has developed a *general systems theory* (Capra, 1996), which predicts the observed *long-range correlations* as a signature of quantum-like chaos in the macro-scale dynamical system (Mary Selvam, 1990; Mary Selvam, Pethkar and Kulkarni, 1992; Selvam and Fadnavis, 1998). The model also provides universal and unique quantification for the observed *quantum-like chaos* characterizing dynamical systems in terms of the statistical normal distribution.

Continuous periodogram power spectral analyses of Dow Jones Index time series of widely different time scales (days, months, years) and data lengths (*100* to *10000* in the case of daily data sets) agree with model prediction, namely, the power spectra follow the universal inverse power law form of the statistical normal distribution. Dow Jones Index time series therefore exhibit long-range temporal correlations or persistence (memory), which is a signature of

quantum-like chaos. Earlier studies by the author have identified quantum-like chaos exhibited by dynamical systems underlying the observed fractal fluctuations of the following data sets: (1) time series of meteorological parameters (Mary Selvam, Pethkar and Kulkarni, 1992; Selvam and Joshi, 1995; Selvam *et al.*, 1996; Selvam and Fadnavis, 1998) (2) spacing intervals of adjacent prime numbers (Selvam and Suvarna Fadnavis, 1998; Selvam, 2001a) (3) spacing intervals of adjacent non-trivial zeros of the Riemann zeta function (Selvam, 2001b).

2. A General systems theory for universal quantification of fractal fluctuations of dynamical systems

As mentioned earlier (Section 1: Introduction) power spectral analyses of *fractal* space-time fluctuations exhibits inverse power law form, i.e., a selfsimilar eddy continuum. The cell dynamical system model (Mary Selvam, 1990; Selvam and Fadnavis, 1998, and all references contained therein; Selvam, 2001a, b) is a general systems theory (Capra, 1996) applicable to dynamical systems of all size scales. The model shows that such an eddy continuum can be visualised as a hierarchy of successively larger scale eddies enclosing smaller scale eddies. Eddy or wave is characterised by circulation speed and radius. Large eddies of root mean square (r.m.s) circulation speed W and radius R form as envelopes enclosing small eddies of r.m.s circulation speed w_* and radius r such that

$$W^2 = \frac{2}{p} \frac{r}{R} w_*^2 \quad (2.1)$$

Large eddies are visualised to grow at unit length step increments at unit intervals of time, the units for length and time scale increments

being respectively equal to the enclosed small eddy perturbation length scale r and the eddy circulation time scale t .

Since the large eddy is but the average of the enclosed smaller eddies, the eddy energy spectrum follows the statistical normal distribution according to the *Central Limit Theorem* (Ruhla, 1992). Therefore, the variance represents the probability densities. Such a result that the additive amplitudes of eddies when squared, represent the probabilities is an observed feature of the subatomic dynamics of quantum systems such as the electron or photon (Maddox 1988a, 1993; Rae, 1988). The *fractal* space-time fluctuations exhibited by dynamical systems are signatures of quantum-like mechanics. The cell dynamical system model provides a unique quantification for the apparently chaotic or unpredictable nature of such *fractal* fluctuations (Selvam and Fadnavis, 1998). The model predictions for quantum-like chaos of dynamical systems are as follows.

- (a) The observed *fractal* fluctuations of dynamical systems are generated by an overall logarithmic spiral trajectory with the quasiperiodic *Penrose* tiling pattern (Nelson, 1986; Selvam and Fadnavis, 1998) for the internal structure.
- (b) Conventional continuous periodogram power spectral analyses of such spiral trajectories will reveal a continuum of periodicities with progressive increase in phase.
- (c) The broadband power spectrum will have embedded dominant wavebands, the bandwidth increasing with period length. The peak periods (or length scales) E_n in the dominant wavebands will be given by the relation.

$$E_n = T_s (2 + \mathbf{t}) \mathbf{t}^n \quad (2.2)$$

where \mathbf{t} is the golden mean equal to $(1 + \sqrt{5})/2$ [≈ 1.618] and T_s the primary perturbation time (length) scale.

The model predicted periodicities (or length scales) in terms of the primary perturbation length scale units are 2.2, 3.6, 5.8, 9.5, 15.3, 24.8, 40.1, 64.9, 105.0, 170.0, 275.0, 445.0 and 720.0 respectively for values of n ranging from -1 to 11. Periodicities close to model predicted have been reported in weather and climate variability (Burroughs 1992; Kane 1996).

- (d) The ratio r/R (Equation 2.1) also represents the increment $d\theta$ in phase angle θ . Therefore the phase angle θ represents the variance. Hence, when the logarithmic spiral is resolved as an eddy continuum in conventional spectral analysis, the increment in wavelength is concomitant with increase in phase (Selvam and Fadnavis, 1998). Such a result that increments in wavelength and phase angle are related is observed in quantum systems and has been named 'Berry's phase' (Berry 1988; Maddox 1988b; Simon *et al.*, 1988; Anandan, 1992). The relationship of angular turning of the spiral to intensity of fluctuations is seen in the tight coiling of the hurricane spiral cloud systems. The overall logarithmic spiral flow structure is given by the relation

$$W = \frac{w_*}{k} \log z \quad (2.3)$$

where, the constant k is the steady state fractional volume dilution of large eddy by inherent turbulent eddy fluctuations. The constant k is equal to $1/t^2$ ($\cong 0.382$) and is identified as the universal constant for deterministic chaos in fluid flows (Selvam and Fadnavis, 1998). The steady state emergence of *fractal* structures is therefore equal to

$$\frac{1}{k} \cong 2.62 \quad (2.4)$$

In Equation 2.3, W represents the standard deviation of eddy fluctuations, since W is computed as the instantaneous r.m.s. (root mean square) eddy perturbation amplitude with reference to the earlier step of eddy growth. For two successive stages of eddy growth starting from primary perturbation w^* the ratio of the standard deviations W_{n+1} and W_n is given from Equation 2.3 as $(n+1)/n$. Denoting by \mathbf{s} the standard deviation of eddy fluctuations at the reference level ($n=1$), the standard deviations of eddy fluctuations for successive stages of eddy growth are given as integer multiple of \mathbf{s} , i.e., \mathbf{s} , $2\mathbf{s}$, $3\mathbf{s}$, etc., and correspond respectively to

$$\text{statistical normalized standard deviation} = 0, 1, 2, 3, \text{etc.} \quad (2.5)$$

The conventional power spectrum plotted as the variance versus the frequency in log-log scale will now represent the eddy probability density on logarithmic scale versus the standard deviation of the eddy fluctuations on linear scale since the logarithm of the eddy wavelength represents the standard deviation, i.e., the r.m.s. value of eddy fluctuations (Equation 2.3). The r.m.s. value of eddy fluctuations can be represented in terms of statistical normal distribution as follows. A normalized standard deviation $t=0$ corresponds to cumulative percentage probability density equal to 50 for the mean value of the distribution. Since the logarithm of the wavelength represents the r.m.s. value of eddy fluctuations the normalized standard deviation t is defined for the eddy energy as

$$t = \frac{\log L}{\log T_{50}} - 1 \quad (2.6)$$

where L is the period (in time units) and T_{50} is the period up to which the cumulative percentage contribution to total variance is equal to 50 and $t = 0$. The variable $\log T_{50}$ also represents the mean value for the r.m.s. eddy fluctuations and is consistent with the concept of the mean

level represented by r.m.s. eddy fluctuations. Spectra of time series of fluctuations of dynamical systems, for example, meteorological parameters, when plotted as cumulative percentage contribution to total variance versus t follow the model predicted universal spectrum (Selvam and Fadnavis, 1998, and all references therein).

The period (or length scale) T_{50} up to which the cumulative percentage contribution to total variances is equal to 50 is computed from model concepts as follows. The power spectrum, when plotted as normalised standard deviation t versus cumulative percentage contribution to total variance represents the statistical normal distribution (Equation 2.6), i.e., the variance represents the probability density. The normalized standard deviation values t corresponding to cumulative percentage probability density P equal to 50 is equal to θ from statistical normal distribution characteristics. Since t represents the eddy growth step n (Equation 2.5) the dominant period (or length scale) T_{50} up to which the cumulative percentage contribution to total variance is equal to 50 is obtained from Equation 2.2 for corresponding value of n equal to θ . In the present study of *fractal* fluctuations of Dow Jones Index, the primary perturbation length scale T_s is equal to *unit time interval* (days, months or years) and T_{50} is obtained as

$$T_{50} = (2 + \mathbf{t})\mathbf{t}^{\theta} \cong 3.6 \text{ unit time interval} \quad (2.7)$$

The above model predictions are applicable to all real world and computed model dynamical systems. Continuous periodogram power spectral analyses of Dow Jones Index of widely different time scales and data lengths give results in agreement with the above model predictions. Different data lengths of the non-stationary Dow Jones Index time series follow the model predicted universal inverse power law form of the statistical normal distribution.

3. Data and Analysis

Dow Jones Index values were obtained from Dow Jones Industrial Average History File, Dow Jones closing prices starting in 1900: 3 Jan 1900 to 5 June 2000 (27523 trading days). Data from: Department of Statistics at Carnegie Mellon Univ., (<http://www.stat.cmu.edu/cmu-stats>) Quote.com (<http://quote.com/>) Yahoo! (<http://quote.yahoo.com/>).

The normalised day-to-day changes in the Dow Jones Index values were computed as percentages of the earlier day value. Monthly and annual mean values were then computed from the normalised day-to-day changes in the Dow Jones Index.

A total of 27,500 daily values of normalised Dow Jones Index were used for the study. Starting from the Dow Jones Index values on day numbers 1, 10001, and 20001 respectively, the number of days used for the spectral analyses were in increments of 100 days up to 2500 days (25 data sets) and thereafter, in increments of 500 days till 10000 days (15 data sets) giving a total of 115 data sets.

A total of 1200 monthly mean values of Dow Jones Index were available for the study. A total of 11 data sets were subjected to spectral analyses. Starting from the first month, the number of months used for the spectral analyses for the first 10 data sets were in increments of 100 months till 1000 months and the 11th data set contains 1200 months.

A total of 100 annual mean Dow Jones Index values were available for the study. Starting from the first year, the number of years used for power spectral analysis for successive data sets were in increments of 20 years, thereby giving a total of 5 data sets.

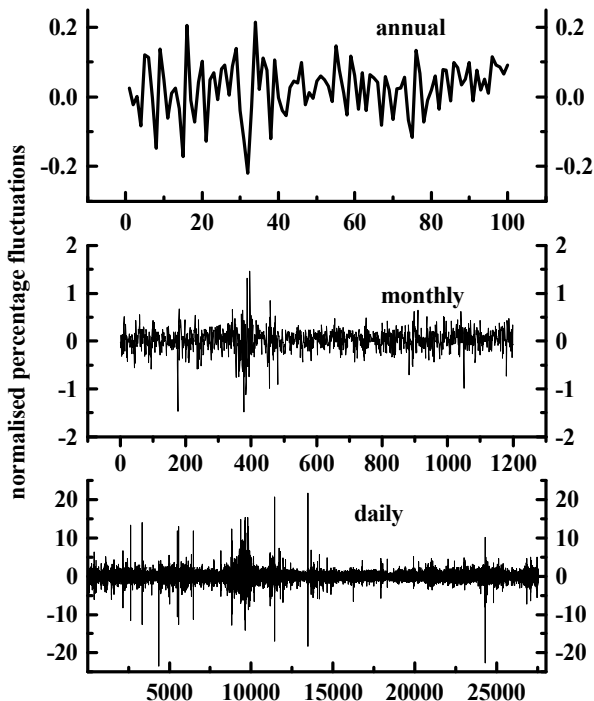
Details of data sets used for the study are shown in Figures 4a, 4b and 4c.

3.1 Fractal nature of Dow Jones Index fluctuations on time scales of days to years

The normalised daily, monthly and annual fluctuations of Dow Jones Index exhibit irregular *fractal* fluctuations as shown for representative data sets in Figure 1.

Figure 1: Fractal fluctuations of normalised daily, monthly and annual fluctuations in Dow Jones Index for the 100-year (1900 to 1999) data set

fractal fluctuations of Dow Jones Index



normalised day- to- day changes =
% of earlier day value

3.2 Continuous periodogram power spectral analyses

The broadband power spectrum of space-time fluctuations of dynamical systems can be computed accurately by an elementary, but very powerful method of analysis developed by Jenkinson (1977) which provides a quasi-continuous form of the classical periodogram allowing systematic allocation of the total variance and degrees of freedom of the data series to logarithmically spaced elements of the frequency range $(0.5, 0)$. The periodogram is constructed for a fixed set of $10000(m)$ periodicities L_m which increase geometrically as $L_m = 2 \exp(Cm)$ where $C = .001$ and $m = 0, 1, 2, \dots, m$. The data series Y_t for the N data points was used. The periodogram estimates the set of $A_m \cos(2\pi n_m S - f_m)$ where A_m , n_m and f_m denote respectively the amplitude, frequency and phase angle for the m^{th} periodicity and S is the time interval in days, months or years. The cumulative percentage contribution to total variance was computed starting from the high frequency side of the spectrum. The period T_{50} at which 50% contribution to total variance occurs is taken as reference and the normalized standard deviation t_m values are computed as (Equation 2.6).

$$t_m = \frac{\log L_m}{\log T_{50}} - 1$$

The cumulative percentage contribution to total variance, the cumulative percentage normalized phase (normalized with respect to the total phase rotation) and the corresponding t values were computed. The power spectra were plotted as cumulative percentage contribution to total variance versus the normalized standard deviation t as given above. The period L is in time interval units (days, months or years). Periodicities up to T_{50} contribute up to 50% of total variance. The phase spectra were plotted as cumulative percentage normalized (normalized to total rotation) phase.

3.3 Power spectral analyses: Representative variance and phase spectra

The variance and phase spectra along with statistical normal distributions are shown in Figure 2 for representative data sets of normalised daily, monthly and annual Dow Jones Index. The 'goodness of fit' (statistical chi-square test) between the variance spectra and statistical normal distribution is significant at less than or equal to 5% level for all the daily and monthly spectra. In the case of annual data sets, the variance spectra follow normal distribution for all data sets except for the first set consisting of the first 20-years (1900 to 1919). Phase spectra are close to the statistical normal distribution, with the 'goodness of fit' being statistically significant for all monthly and annual data sets and 66% of daily data sets. Further, in all the cases, the 'goodness of fit' between variance and phase spectra are statistically significant (chi-square test) for individual dominant wavebands, in particular, for longer periodicities. A representative example of daily data set is shown, where, though the phase spectrum does not follow normal distribution (Figure 2), the phase and variance spectra are the same in dominant wavebands (Figure 3).

Figure 2: Representative spectra of variance and phase along with statistical normal distribution for normalized daily, monthly and annual fluctuations of Dow Jones Index. The variance spectra for all data sets and phase spectra for monthly and annual data sets follow the model predicted statistical normal distribution. The phase spectrum does not follow the normal distribution for the sample daily data set, but the variance and phase spectra are the same in individual dominant eddies as shown in Figure 3 for the same data set

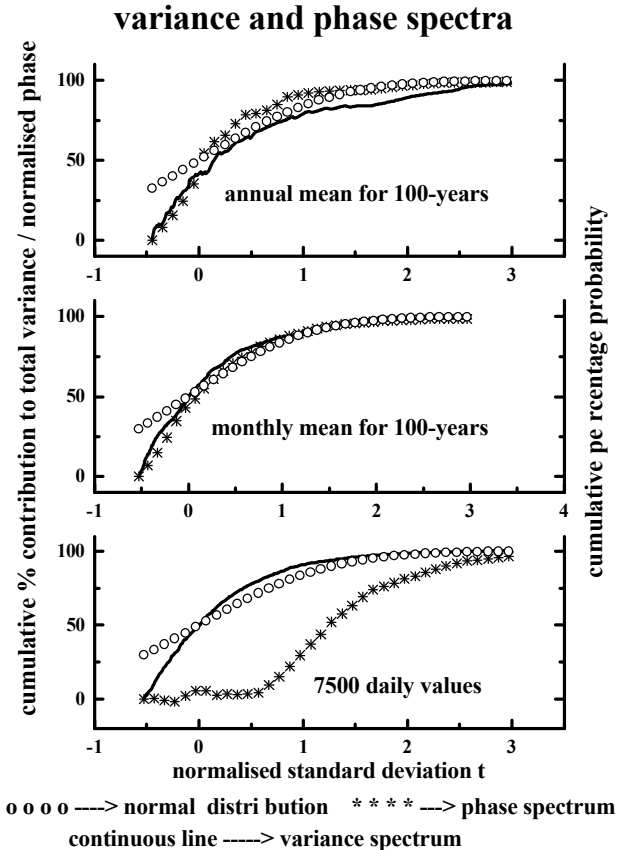
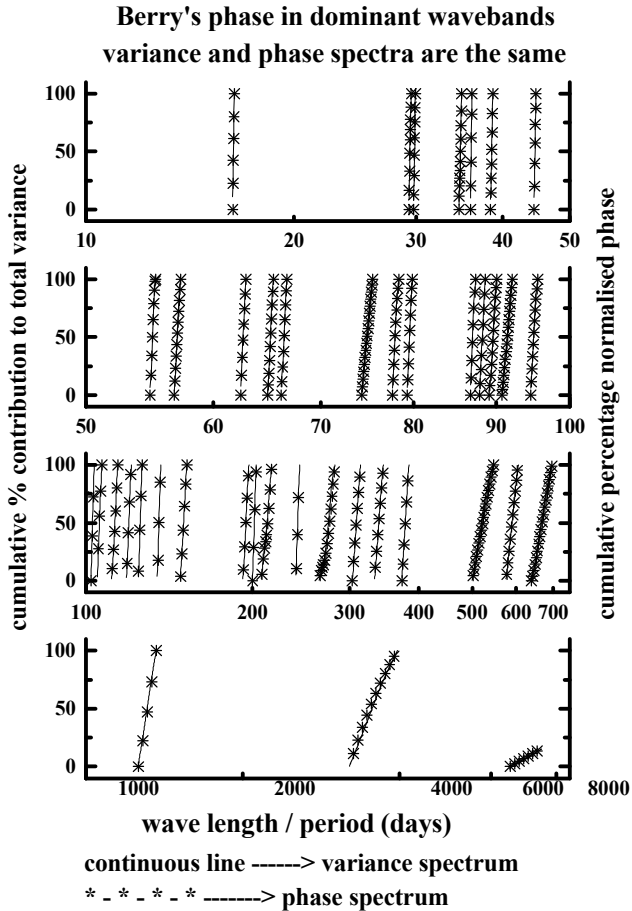


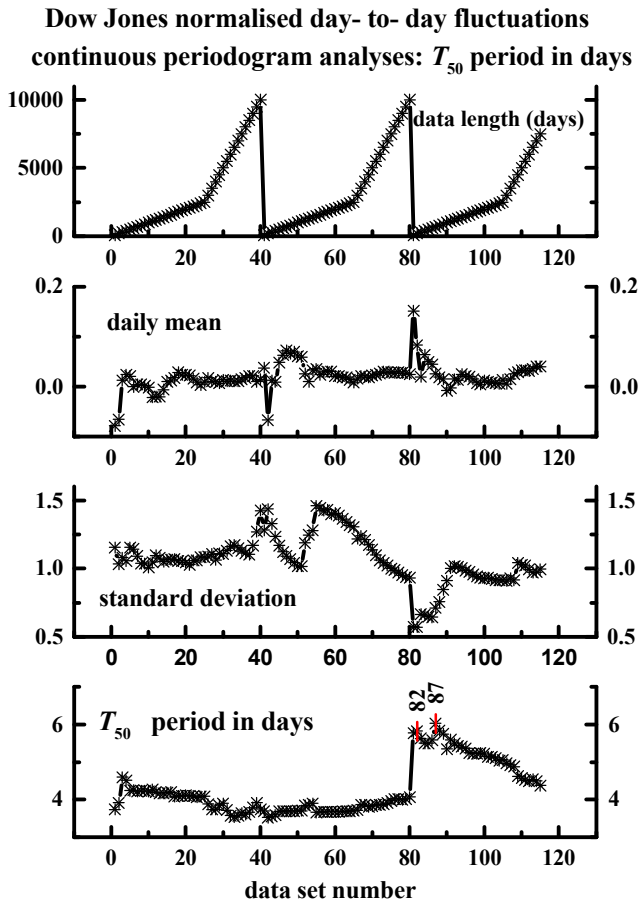
Figure 3: A representative example of daily data set is shown, where, though the phase spectrum does not follow normal distribution (Figure 2), the phase and variance spectra are the same in dominant wavebands as shown below. The variance and phase spectra being the same is a signature of Berry's phase in quantum systems



3.4 Power spectral analyses: Summary of results

The periodicities T_{50} up to which the cumulative percentage contribution to total variance is equal to 50 are shown for the three groups of Dow Jones Index data sets, namely, daily (115 data sets), monthly (11 data sets) and annual (5 data sets) in Figures 4 (a, b and c) respectively.

Figure 4a: The period T_{50} up to which the cumulative percentage contribution to total variance is equal to 50 for 115 data sets of normalised daily Dow Jones Index. Details are also given of data set length, data mean and data standard deviation



the sharp peak at data set number 87 corresponds to
 data length 700 days from 18 September 1970 to 27 June 1973

Figure 4b: The period T_{50} up to which the cumulative percentage contribution to total variance is equal to 50 for 11 data sets of normalized monthly Dow Jones Index. Details are also given of data set length, data mean and data standard deviation

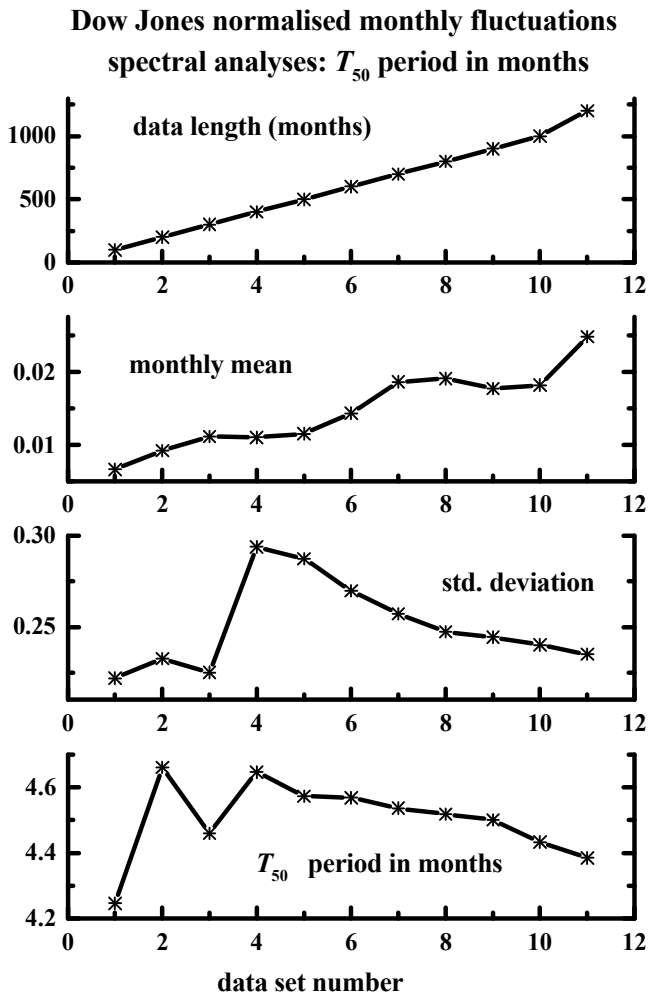
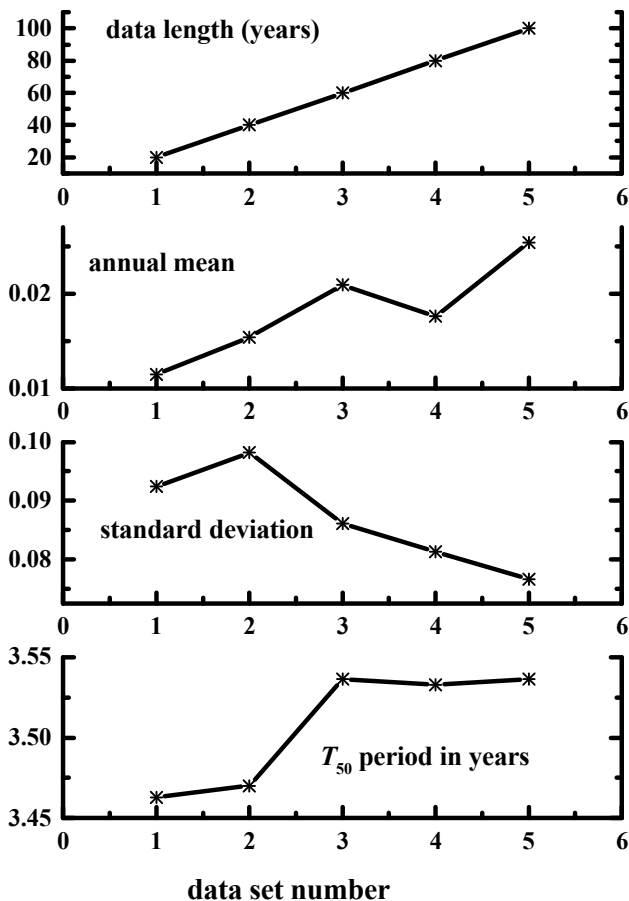


Figure 4c: The period T_{50} up to which the cumulative percentage contribution to total variance is equal to 50 for 5 data sets of normalized annual Dow Jones Index. Details are also given of data set length, data mean and data standard deviation

**Dow Jones normalised annual fluctuations
spectral analyses: T_{50} period in years**



4. Results and Discussion

The Dow Jones Index time series (daily, monthly and annual) exhibit *fractal* fluctuations (Figures 1) generic to dynamical systems in nature. The *fractal* fluctuations are basically a zig-zag pattern of successive upward and downward swings on all time scales in the Dow Jones Price Index. Such irregular fluctuations may be visualised to result from the superimposition of a continuum of eddies. Power spectral analysis is commonly applied to resolve the component periodicities and their phases. Continuous periodogram power spectral analyses of the *fractal* fluctuations in Dow Jones Index time series (daily, monthly and annual) follow closely the following model predictions given in Section 2.

- (1) The variance spectra follow statistical normal distribution for all the three (daily, monthly and annual) data groups except for the first data set of length 20 years (1900 to 1919) annual mean Dow Jones Index time series.
- (2) Phase spectra follow normal distribution for 66% of daily data sets and for 100% data sets of the monthly and annual data groups.
- (3) The period T_{50} up to which the cumulative percentage contribution to total variance is equal to 50% is very close to model predicted value of about 3.6 time units (daily, monthly or annual) for annual and monthly data sets. The daily data sets showed higher values for data set numbers 81 to 108. Incidentally these data sets 81 to 108 correspond to the period just prior to and following the *oil shock* of the year 1973 (Chen 1996a,b). Though the data length varied from 100 to 10000 for daily data sets, the value of T_{50} was relatively constant and close to the model predicted values (Figures 4a, b, c).

5. Conclusions

The observed inverse power law form for power spectra of *fractal* fluctuations is a signature of long-range temporal correlations and may signify *self-organized criticality* in aggregate market economy. The author had shown earlier (Selvam and Suvarna Fadnavis, 1998; Selvam 2001a,b) that (a) the observed long-range space-time correlations in dynamical systems can be quantified in terms of the universal inverse power law form of the statistical normal distribution and (b) selfsimilar *fractal* fluctuations imply long-range space-time correlations and is a signature of quantum-like chaos in macro-scale dynamical systems of all space-time scales.

Power spectra of normalised daily, monthly and annual fluctuations of Dow Jones Index time series follow the model predicted universal and unique inverse power law form of the statistical normal distribution. Inverse power law form for power spectra of temporal fluctuations imply long-range temporal correlations, or in other words, persistence or long-term memory of short-term fluctuations. The long-time period fluctuations carry the signatures of short-time period fluctuations. The cumulative integration of short-term fluctuations generates long-term fluctuations (eddy continuum) with two-way ordered energy feedback between the fluctuations of all time scales (Equation 2.1). The eddy continuum acts as a robust unified whole fuzzy logic network with global response to local perturbations. Increase in random noise or energy input into the short-time period fluctuations creates intensification of fluctuations of all other time scales in the eddy continuum and may be noticed immediately in shorter period fluctuations. Noise is therefore a precursor to signal.

Real world examples of noise enhancing signal has been reported in electronic circuits (Brown, 1996). Man-made, urbanization related,

greenhouse gas induced global warming (enhancement of small-scale fluctuations) is now held responsible for devastating anomalous changes in regional and global weather and climate in recent years (Selvam and Fadnavis, 1998).

The periodicities T_{50} up to which the cumulative percentage contribution to total variance is equal to 50 are close to model predicted value of 3.6 for the data groups of widely different time scale units (days, months and years) and different data lengths (100 to 10000 for daily data sets

The apparently irregular *fractal* fluctuations of the Dow Jones Index as a representative example in this study and dynamical systems in general, self-organize spontaneously to generate the robust geometry of logarithmic spiral with the quasiperiodic *Penrose* tiling pattern for the internal structure. Conventional power spectral analyses resolves such a logarithmic spiral geometry as an eddy continuum exhibiting inverse power law form of the statistical normal distribution. Power spectral analyses of different time period data sets of the non-stationary Dow Jones Index time series are in agreement with model predictions.

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