Analysis of the Lorenz Gauge

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The multivaluedness of solutions of the wave equation, which depend on a gauge choice, is established. It is shown that the Lorenz gauge of the Maxwell equations predicts the existence of longitudinal electromagnetic waves. These are waves of scalar potential and vector potential. The cancellation of longitudinal waves is analysed. It is shown that the Lorenz gauge is inconsistent with the phenomena of electrodynamics. No hypotheses are made.

Keywords: refutation of Lorenz gauge; longitudinal scalar and vector potentials; refutation of gauge invariance.

Introduction

Modern physical theories have many difficulties. Analysing these difficulties we have made the conclusion that Lorenz's theory is a main source of difficulties. The analysis of Lorenz's theory was made by us. In this paper we offer attention of readers a small part of our research which is connected to the Lorenz gauge. We shall consider as follows:

1) Mathematical analysis of uniqueness of a solution;

2) Law of energy conservation for the Lorenz gauge.

3) Analysis of singularities of the Lorenz gauge.

We propose to publish other fragments of our research in later papers.

1. Uniqueness of solution

In the mathematical textbooks (Tikhonov, Samarsky, 1953) they conclude that a solution of a wave equation exist under the known initial and boundary conditions, and also the solution is unique. We shall check up this assertion. It is known that any negative example can limit area of use of the assertion or reject it. Such example we offer to attention of readers.

We shall consider a simple example for the illustration. Let function U satisfies a homogenous wave equation without boundary conditions.

$$\frac{\partial^2 U}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = 0 \tag{1.1}$$

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Initial conditions are zero.

$$U\Big|_{t=0} = 0; \qquad \frac{\partial U}{\partial t}\Big|_{t=0} = 0.$$

First solution. Function U is equal to zero. U_1=0. (1.2)

Second solution. We may write very much other solution of the task. For example, the second solution is

$$U_2(x,t) = -\frac{A}{2}[e^{-a(x+ct)^2} + e^{-a(x-ct)^2}] - \frac{Ab}{2c}\int_{x-ct}^{x+ct} e^{-ax^2}dx + \frac{1}{2c}\int_0^t [\int_{x-c(t-t)}^{x+c(t-t)} f(x,t)dx]\, dt + Ae^{-ax^2}e^{-bt} \tag{1.3}$$

where $f(x,t) = -Ae^{-bt}e^{-ax^2}[2a(1-2ax^2)+\frac{b^2}{c^2}]$. α, β, A are constant (α >0, β>0).

The second solution U_2 (x,t) is not equal to zero, and it has no singularities.

Thus, we have two different solutions of wave equation under the given initial conditions. The obtained outcome is not exclusive or unique. Always it is possible to find similar examples, in which there is a second solution, for the inhomogeneous equations and tasks with boundary conditions. In other words, we can generalise this example. The further research of the problem is a task for mathematicians.

Choice of Second Solution

Now we shall show one of many ways for searching a second solution. Earlier we investigated the problem in (Kuligin *et al.,* 1989), (Kuligin *et al.,* 1990). To simplify the analysis we write a wave equation without boundary conditions for free space. This condition not is limitation. We can use this way for the tasks with boundary conditions.

$$\Delta U - \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2} = f(\mathbf{r};t) \tag{1.6}$$

where: U is some function; v is propagation speed of it; f is source of function U;

$$f(\mathbf{r};t) = \begin{cases} F(\mathbf{r};t) & \text{when} \quad r \le a \;; \\ 0 & \text{when} \quad r > a \end{cases} \tag{1.7}$$

A solution of equation (1.6) should satisfy to the following initial condition.

$$U(\mathbf{r};0) = \vartheta(\mathbf{r}); \quad \frac{\partial U}{\partial t}\Big|_{t=0} = \psi(\mathbf{r}) \tag{1.8}$$

It is a standard initial value problem for a hyperbolic equation without boundary conditions.

When the solution exists (in general case it exist (Tikhonov, Samarsky, 1953)) then we write it as $U(\mathbf{r};t)$. The instantaneous functions in the solution are absent.

Now we shall begin to seek a solution of equation (1.6) as a sum of two functions.

$$U = u(\mathbf{r};t) + V(\mathbf{r};t) \tag{1.9}$$

After substitution of expression (1.9) in equation (1.6) we find

$$\Delta u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} + \Delta V - \frac{1}{v^2}\frac{\partial^2 V}{\partial t^2} = f(\mathbf{r};t) \qquad (1.10)$$

As we have entered two new unknown functions we should add an appropriate condition. There are many variants to give a side condition. For instance, we may use following equations.

$$\Delta V_1 = f(\mathbf{r};t); \quad \Delta V_2 + \alpha V_2 = f(\mathbf{r};t); \quad \Delta V_3 + \beta\frac{\partial V_3}{\partial t} = f(\mathbf{r};t) \qquad (1.11)$$

where α and β are some factors.

We assume that function u satisfies to the Poisson equation. We should specially notice that the instantaneous functions *do not break the principle of causality* in physics (Kuligin, 1987).

$$\Delta V_1 = f(\mathbf{r};t); \qquad \lim_{r\to\infty} V_1 = 0 \qquad 1.12)$$

The solution of Poisson equation (1.12) exists also (Tikhonov, Samarsky, 1953). Naturally, function u_1 should satisfy to the new equation:

$$\Delta u_1 - \frac{1}{v^2}\frac{\partial^2 u_1}{\partial t^2} = -\Delta V_1 + \frac{1}{v^2}\frac{\partial^2 V_1}{\partial t^2} + f(\mathbf{r};t) = \frac{1}{v^2}\frac{\partial^2 V_1}{\partial t^2} \qquad (1.13)$$

Function u_1 has the new initial conditions.

$$u_1(\mathbf{r};0) = \vartheta(\mathbf{r}) - V_1(\mathbf{r};0); \qquad \left.\frac{\partial u_1}{\partial t}\right|_{t=0} = \psi(\mathbf{r}) - \left.\frac{\partial V_1}{\partial t}\right|_{t=0}. \qquad (1.14)$$

Other conditions are conserved.

We shall consider that in general case a solution of equation (1.12) exists, and it is known to us. In this case the right part of equation (1.13) and initial conditions (1.14) are determined and are known. In general case a solution of equation (1.14) exists (Tikhonov, Samarsky, 1953). We write it as $V_1(\mathbf{r};t)$.

Thus, we receive the new solution of equation (1.6), which differs from solution U($\mathbf{r}$;t). It has the following form.

$$U_1 = u_1(\mathbf{r};t) + V_1(\mathbf{r};t) \qquad (1.15)$$

Expression (1.15) is solution of equation (1.6) under the initial conditions. It can be confirmed by direct check.

In the same way we can find other solutions of equation (1.11): $U_2 = u_2 + V_2; \quad U_3 = u_3 + V_3$.

Moreover, any solution, which is composed with U, U_1, U_2, U_3 , is also one of same task.

$$U_{new} = a_0 U + a_1 U_1 + a_2 U_2 + a_3 U_3; \quad a_0 + a_1 + a_2 + a_3 = 1 \qquad (1.16)$$

where a_i are free constant (i=0,1,2,3).

The method of choice of new solutions can also be used, for example, for Maxwell equations and Shroedinger's equation.

Mathematical Gauges

The procedure of a choice of the second solution has immediate connection with a choice of kinds of functions having different space and time properties. It contains also the choice of some equations to have these properties. We shall name this procedure as mathematical gauge of primary equations. We mark the following singularities of mathematical gauge procedure.

1. The solution of any fixed gauge is unique.
2. In general case the solutions of different gauges differ from each other. The gauge invariance of mathematical gauges has no a place.

2. Mathematical Gauges in Electrodynamics

Maxwell equations

The problem of mathematical gauge is immediately connected to a problem of a physical covariance of equations. The violation of uniqueness of a solution reduces to a series of problems. We shall not consider the problem of physical covariance in this paper. We shall consider, that the inertial frame of observer, in which the equations are fair, exists, and we shall compare solutions of different gauges only in this frame of observer.

We write Maxwell equations.

$$\begin{aligned} &\mathrm{rot}\mathbf{H} - \varepsilon\frac{\partial \mathbf{E}}{\partial t} = \mathbf{j}; \qquad \mathrm{div}\,\mathbf{H} = 0 \\ &\mathrm{rot}\mathbf{E} + \mu\frac{\partial \mathbf{H}}{\partial t} = 0; \qquad \mathrm{div}\,\mathbf{E} = \frac{\rho}{\varepsilon} \end{aligned} \tag{2.1}$$

In set of equations (2.1) two vectors $\mathbf{E}$ and $\mathbf{H}$ are connected among themselves. We can note independent equations for $\mathbf{E}$ and $\mathbf{H}$.

$$\begin{aligned} &\mathrm{rotrot}\mathbf{H} + \frac{1}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} = \mathrm{rot}\mathbf{j}; \qquad \mathrm{div}\mathbf{H} = 0 \\ &\mathrm{rotrot}\mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{\partial \mathbf{j}}{\partial t}; \qquad \mathrm{div}\mathbf{E} = \frac{\rho}{\varepsilon} \end{aligned} \tag{2.2}$$

The equations (2.2) have wave solutions $\mathbf{E}_{w1}$ and $\mathbf{H}_{w1}$. The index “w” means that they are direct solutions of equations (2.2) without introduction of additional mathematical gauges.

Now we shall search for a solution of equation (2.1) as the following.

$$\mathbf{E} = \mathbf{E}_w + \mathbf{E}_{ins} \tag{2.3}$$

where: $\mathbf{E}_w$ is solution of a wave equation; $\mathbf{E}_{ins}$ is solution of Poisson equation.

The index “ins” means that we deal with a solution of Poisson equation.

We add the following equations to destroy indeterminacy.

$$\mathrm{div}\mathbf{E}_w = 0; \quad \mathrm{rot}\mathbf{E}_{ins} = 0 \tag{2.4}$$

Now the complete set of the equations is defined and one is

$$\mathrm{rotrot}\mathbf{H}_w + \frac{1}{c^2}\frac{\partial^2 \mathbf{H}_w}{\partial t^2} = \mathrm{rot}\mathbf{j}; \qquad \mathrm{div}\mathbf{H}_w = 0$$

$$\mathrm{rotrot}\mathbf{E}_w + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}_w}{\partial t^2} = -\frac{\partial \mathbf{j}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}_{ins}}{\partial t^2}; \qquad \mathrm{div}\mathbf{E}_w = 0 \tag{2.5}$$

$$\mathrm{rot}\mathbf{E}_{ins} = 0; \qquad \mathrm{div}\mathbf{E}_{ins} = \frac{\rho}{\varepsilon}$$

The equations (2.5) have electric field strength $\mathbf{E}_2 = \mathbf{E}_{ins} + \mathbf{E}_{w2}$ and magnetic intensity $\mathbf{H}_{w2}$. A solution of equation (2.2) differs from a solution of equation (2.5) even by that the first solution does not contain instantaneous electric field strength $\mathbf{E}_{ins}$.

Here there is a parallel between equations (2.2), (2.5) and gauges of Maxwell equations. If we replace in equation (2.5) electric field strength $\mathbf{E}_{ins}$ by -gradϕ, $\mathbf{E}_w$ by $-\frac{\partial \mathbf{A}}{\partial t}$ and magnetic intensity $\mathbf{H}_w$ by rot$\mathbf{A}$ then we obtain Coulomb's gauge of Maxwell equations.

$$\Delta \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu\, \mathbf{j} + \frac{1}{c^2}\frac{\partial}{\partial t}\mathrm{grad}\phi;$$
$$\Delta \phi = -\frac{\rho}{\varepsilon}; \qquad \mathrm{div}\mathbf{A} = 0 \tag{2.6}$$

If we replace $\mathbf{E}_w$ by $-\frac{\partial \mathbf{A}}{\partial t}$ and $\mathbf{H}_w$ by rot$\mathbf{A}$ in equation (2.2) then we receive the Lorenz gauge of Maxwell equations.

$$\Delta \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu\, \mathbf{j} \;;\; \Delta \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\varepsilon} \;;\; \mathrm{div}\, \mathbf{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \tag{2.7}$$

Thus, physical gauge of Maxwell equations, which is determined by a condition for div$\mathbf{A}$, as a matter of fact coincides with mathematical gauge. It is a particular case of mathematical gauge. Therefore there is no sense to distinguish these gauges further.

Now we consider properties of solutions of equations (2.2) and (2.5). The second solution (2.5) contains instantaneous field strength $\mathbf{E}_{ins}$, which is absent in solution (2.2). In the books (for example (Ginzburg, 1987)) they state that $\mathbf{E}_{ins}$ is compensated by other field strength $\mathbf{E}_w$. Due to this fact Coulomb's gauge is equivalent to the Lorenz gauge. We consider this conclusion as fallacy. It is shown in Appendix 1 that two gauges are not equivalent.

We can tell that the casual concurrence cannot be generalised without the proofs, and in the general case the uniqueness of a solution of Maxwell equations has no a place.

Solutions of the Lorenz gauge

Now we consider two solutions of the Lorenz gauge (2.7). The first solution is wave solution of equations (2.7) :

$$\mathbf{E}_{1w} = -\mathrm{grad}\phi - \frac{\partial \mathbf{A}}{\partial t}; \qquad \mathbf{H}_{1w} = \mathrm{rot}\mathbf{A}. \tag{2.8}$$

The second solution may be obtained with the standard way. Let potentials $\mathbf{A}$ and ϕ be a sum of other potentials.

$$\mathbf{A} = \mathbf{A}_{ins} + \mathbf{A}_w; \quad \phi = \phi_{ins} + \phi_w \tag{2.9}$$

We give special condition for potentials $\mathbf{A}_{ins}$ and ϕ_{ins}. The potentials should be solutions of Poisson equation.

$$\Delta\,\mathbf{A}_{ins} = -\mu\,\mathbf{j};\quad \Delta\,\phi_{ins} = -\frac{\rho}{\varepsilon};\quad \operatorname{div}\mathbf{A}_{ins} + \frac{1}{c^2}\frac{\partial}{\partial t}\operatorname{grad}\phi_{ins} = 0 \tag{2.10}$$

Hence, potentials $\mathbf{A}_w$ and ϕ_w are solutions of wave equations.

$$\Delta\,\mathbf{A}_w - \frac{1}{c^2}\frac{\partial^2\mathbf{A}_w}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2\mathbf{A}_{ins}}{\partial t^2}\;;\;\Delta\,\phi_w - \frac{1}{c^2}\frac{\partial^2\phi_w}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2\phi_{ins}}{\partial t^2}\;;$$

$$\operatorname{div}\ \mathbf{A}_w + \frac{1}{c^2}\frac{\partial\,\phi_w}{\partial\,t} = 0 \tag{2.11}$$

The second solution is the following.

$$\mathbf{E}_2 = \mathbf{E}_{2w} + \mathbf{E}_{2ins} = -\frac{\partial\mathbf{A}_w}{\partial t} - \operatorname{grad}\phi_{ins} - \frac{\partial\mathbf{A}_{ins}}{\partial t};$$
$$\mathbf{H}_2 = \mathbf{H}_{2w} + \mathbf{H}_{2ins} = \operatorname{rot}\mathbf{A}_w + \operatorname{rot}\mathbf{A}_{ins}. \tag{2.12}$$

We see that the first solution (2.8) differs from the second solution (2.12). Thus, the Lorenz gauge has many solutions too.

Limiting transfer

We should notice that the transfer to the limit $c\to\infty$ takes place

$$\mathbf{E}_{2ins} = \lim_{c\to\infty}\mathbf{E}_{1w};\quad \mathbf{H}_{2ins} = \lim_{c\to\infty}\mathbf{H}_{1w}. \tag{2.13}$$

Expression (2.13) is a casual concurrence, as the solution of equation (2.8) is not a solution of equation (2.12). We should use the transfer to limit $c\to\infty$ very cautiously. Otherwise, we shall make illegal transfer from one gauge in another. The solutions of wave equation (2.8) do not contain instantaneous fields. We can not force an instantaneous field to propagate with light velocity.

Now we shall draw some brief conclusions.

1. The different gauges of Maxwell equations have different solutions. In general case the gauge invariance is absent.
2. The electromagnetic potentials play the important role in classical electrodynamics. Due to this circumstance we should have special gauge, which else is necessary to find.
3. The transfer to limit $c\to\infty$ is not always lawful in an electrodynamics.

3. Law of Energy Conservation for Lorenz's Gauge

Before to investigate singularity law of energy conservation of the Lorenz gauge, we do some explanations.

At first, the interaction in the classical theories is a basis. The interaction has two aspects. They are the force and power aspects. The electric field strength is the reflecting of force aspect, and the electric potential is the reflecting of energy aspect. They speak that the potential has no physical sense due to it is defined to a free constant. It is incorrect. The potential energy also is defined with the same exactitude.

In second, in general case the solution of a wave equation can contain fields of different kinds. Each field has the own equation. Due to this circumstance each kind of a field has its law of energy conservation, energy density and energy flux density. Moreover, each law has its special interpretation (Kuligin, Kuligin*a*, 1986), (Kuligin *et al.,* 1996). Poynting's theorem is not universal, and one may not be used for all task of electrodynamics.

The Proof

Let us consider equations (2.7). If we multiple the first equation by $-(1/m)(\partial\mathbf{A}/\partial t)$ and the second equation by $-\varepsilon\frac{\partial\phi}{\partial t}$ then after simple transformations we receive

$$\frac{1}{\mu}\operatorname{div}[-\frac{\partial\mathbf{A}}{\partial t}\times\operatorname{rot}\mathbf{A}-\frac{\partial\mathbf{A}}{\partial t}\operatorname{div}\mathbf{A}]+\frac{1}{2\mu}\frac{\partial}{\partial t}[(\operatorname{rot}\mathbf{A})^2+(\operatorname{div}\mathbf{A})^2+(\frac{\partial\mathbf{A}}{\partial c t})^2]-\frac{\partial\mathbf{A}}{\partial t}\mathbf{j}=0 \quad (3.1)$$

$$\varepsilon\operatorname{div}[-\frac{\partial\phi}{\partial t}\operatorname{grad}\phi]+\frac{\varepsilon}{2}\frac{\partial}{\partial t}[(-\operatorname{grad}\phi)^2+(\frac{\partial\phi}{\partial c t})^2]-\rho\frac{\partial\phi}{\partial t}=0 \quad (3.2)$$

Equations (3.1) and (3.2) have the known form of a law of energy conservation:

$$\operatorname{div}\mathbf{S}+\frac{\partial}{\partial t}w+p=0 \quad (3.3)$$

Expressions (3.1) and (3.2) are laws of energy conservation within the framework of the Lorenz gauge.

Three energy flux densities

Now we must analyse the expressions and must show that three different fluxes exist in the framework of the Lorenz gauge. We give an explanation mathematically.

Vector potential **A** and current density **j** can be submitted as a sum of two independent components. They are solenoidal and irrotatational components of potential and current density.

$$\mathbf{A}=\mathbf{A}_1+\mathbf{A}_2;\ \mathbf{j}=\mathbf{j}_1+\mathbf{j}_2 \quad (3.4)$$

where: $\operatorname{div}\mathbf{A}_1=0$; $\operatorname{div}\mathbf{j}_1=0$; $\operatorname{rot}\mathbf{A}_2=0$; $\operatorname{rot}\mathbf{j}_2=0$.

This is not a new gauge.

The convenience of similar division of potentials and current densities is dictated also by following circumstance. Now we consider a *point* source that creates fields of vector potentials $\mathbf{A}_1$ and $\mathbf{A}_2$. The components of electric and magnetic fields created by potential $\mathbf{A}_1$ are always orthogonal to an electric field created by potential $\mathbf{A}_2$. By other words, the following relationships take place:

$$(\frac{\partial\mathbf{A}_1}{\partial t}\frac{\partial\mathbf{A}_2}{\partial t})=0;\ \ (\operatorname{rot}\mathbf{A}_1\frac{\partial\mathbf{A}_2}{\partial t})=0 \quad (3.5)$$

Expressions (3.5) remain fair for any current densities and charge densities provided that the fields and potentials are considered for very large distances from the source. We will use these relationships further.

With expression (3.4) we can write three wave equations in the Lorenz gauge (2.7).

$$\Delta \mathbf{A}_1 - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}_1}{\partial t^2} = -\mu\, \mathbf{j}_1 \tag{3.6}$$

$$\Delta \mathbf{A}_2 - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}_2}{\partial t^2} = -\mu\, \mathbf{j}_2 \tag{3.7}$$

$$\Delta \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\varepsilon} \tag{3.8}$$

$$\mathrm{div}\, \mathbf{A}_2 + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \tag{3.9}$$

Each wave equation describes own energy density, each wave equation has own vector of energy flux density and law of energy conservation (Kuligin *et al.,* 1990), (Kuligin *et al.,* 1997). These laws have an identical general form:

$$\mathrm{div}\, \mathbf{S}_k + \frac{\partial w_k}{\partial t} + p_k = 0 \quad (k = 1; 2; 3) \tag{3.10}$$

where: $\mathbf{S}_k$ is vector of flux density; w_k is energy density; p_k is power density of applied electromotive forces; $k = 1$ corresponds to the law of conservation of vector potential $\mathbf{A}_1$; $k = 2$ corresponds to potential $\mathbf{A}_2$; $k = 3$ corresponds to scalar potential ϕ.

Expressions of $\mathbf{S}_k$, p_k and w_k are given in Table 1. We see that within the framework of the Lorenz gauge there are three various energy fluxes, which are described by three various vectors of flux density. The first vector $\mathbf{S}_1$ is classical vector that describes a transversal electromagnetic wave. The second vector is vector $\mathbf{S}_2$ that describes a longitudinal wave of vector potential. The third vector $\mathbf{S}_3$ is vector of flux density of longitudinal wave of scalar potential ϕ.

Table 1.

Transversal Waves of Vector Potential		
$\mathbf{S}_1 = -\frac{1}{\mu}\frac{\partial \mathbf{A}_1}{\partial t} \times \mathrm{r(}$	$w_1 = \frac{1}{2\mu}[(\mathrm{rot}\mathbf{A}_1)^2 + (\frac{\partial \mathbf{A}_1}{\partial ct})^2]$	$p_1 = -\mathbf{j}_1 \frac{\partial \mathbf{A}_1}{\partial t}$
Longitudinal Waves of Vector Potential		
$\mathbf{S}_1 = -\frac{1}{\mu}\frac{\partial \mathbf{A}_2}{\partial t}\mathrm{div}\ldots$	$w_2 = \frac{1}{2\mu}[(\mathrm{div}\mathbf{A}_2)^2 + (\frac{\partial \mathbf{A}_2}{\partial ct})^2]$	$p_2 = -\mathbf{j}_2 \frac{\partial \mathbf{A}_2}{\partial t}$
Longitudinal Waves of Scalar Potential		
$\mathbf{S}_3 = -\varepsilon \frac{\partial \phi}{\partial t}\mathrm{grad}\,\phi$	$w_3 = \frac{\varepsilon}{2}[(\mathrm{grad}\phi)^2 + (\frac{\partial \phi}{\partial ct})^2]$	$p_3 = -\rho \frac{\partial \phi}{\partial t}$

Due to absence of uniqueness of the solution we cannot use expressions of energy density and energy flux density, which were given by Poynting, within the framework of the Lorenz gauge.

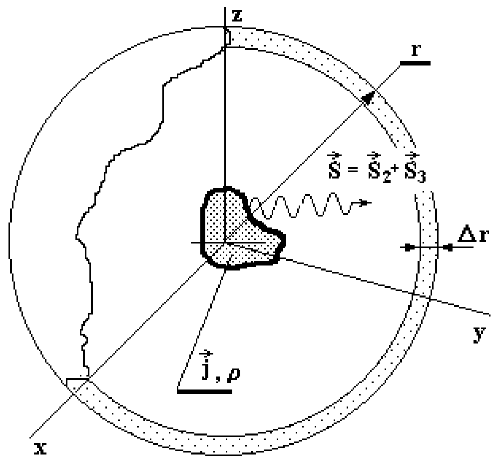

Figure 1.

4. Energy Cancellation of Longitudinal Waves

From Maxwell equations follow that a longitudinal electric field must decreases not slower than r^2 when $r \to \infty$ (Appendix 2). It is known, that any wave, which propagates from a local source to infinity, in general case decreases as r^{-1} for $r \to \infty$.

Thus, we deal with the contradiction. On the one hand, according to expression (B.4) the longitudinal electric field decreases not slower than r^{-2}. On the other hand, the wave equation predicts existence of longitudinal electrical fields of scalar and vector potentials, which decrease proportionally r^{-1}. This contradiction should be investigated.

For the erasure of the contradiction we may assume that vectors $\mathbf{S}_2$ and $\mathbf{S}_3$ should destroy each other even with $r \to \infty$. We write two integrals for search of a necessary and sufficient condition of longitudinal wave cancellation.

$$\Pi = \oiint \mathbf{S}\,\mathbf{r} r \sin\theta \, d\theta \, d\varphi \tag{4.1}$$

$$\Delta W = \Delta r \iint w \, r^2 \sin\theta \, d\theta \, d\varphi \tag{4.2}$$

where: Π is common flow of electromagnetic waves through the spherical surface; ΔW is energy inside thin layer Δr as shown in Figure 1;

$$\mathbf{S} = -\frac{\zeta}{\mu}\left[\frac{\partial \mathbf{A}}{\partial t} \times \mathrm{rot}\mathbf{A} - \frac{\partial \mathbf{A}}{\partial t}\mathrm{div}\mathbf{A}\right] - \eta\varepsilon \frac{\partial \phi}{\partial t}\mathrm{grad}\phi \tag{4.3}$$

$$w = \frac{\zeta}{2\mu}\left[(\mathrm{rot}\mathbf{A})^2 + (\mathrm{div}\mathbf{A})^2 + \left(\frac{\partial \mathbf{A}}{\partial ct}\right)^2\right] + \eta\frac{\varepsilon}{2}\left[(\mathrm{drad}\phi)^2 + \left(\frac{\partial \phi}{\partial ct}\right)^2\right] \tag{4.4}$$

η and ζ are factors, them modules are equal to 1, and signs will be established later.

Now we present vector potential $\mathbf{A}$ as sum (3.4), and we suppose that all sources of potentials locate inside some sphere with radius a ($r \gg a$). Let's consider potentials on the spherical surface r .We may use orthogonality of fields (3.5) when $r \to \infty$.

Integrals (4.1) and (4.2) can be easily divided into two independent groups.

$$\Pi_T = \lim_{r\to\infty} - \oiint [\frac{\zeta}{\mu}\frac{\partial \mathbf{A}_1}{\partial t} \times \mathrm{rot}\mathbf{A}_1]\mathbf{r}\, r \sin\theta\, d\theta\, d\varphi$$
$$\Delta W_T = \lim_{r\to\infty} \iint \Delta r \frac{\zeta}{2\mu}[(\mathrm{rot}\mathbf{A}_1)^2 + (\frac{\partial \mathbf{A}_1}{\partial ct})^2] r^2 \sin\theta\, d\theta\, d\varphi \qquad (4.5)$$

$$\Pi_L = \lim_{r\to\infty} - \oiint [\frac{\zeta}{\mu}\frac{\partial \mathbf{A}_2}{\partial t}\,\mathrm{div}\,\mathbf{A}_2 + \eta\varepsilon\frac{\partial \phi}{\partial t}\mathrm{grad}\phi]\mathbf{r}\, r\sin\theta\, d\theta\, d\varphi = 0$$
$$\Delta W_L = \lim_{r\to\infty} \iint \Delta r\{\frac{\zeta}{2\mu}[(\mathrm{div}\mathbf{A}_2)^2 + (\frac{\partial \mathbf{A}_2}{\partial ct})^2] + \eta\frac{\varepsilon}{2}[(\mathrm{grad}\phi)^2 + (\frac{\partial \phi}{\partial ct})^2]\} r^2 \sin\theta\, d\theta\, d\varphi = 0 \qquad (4.6)$$

where: Π_T and Π_L are flows of transversal and longitudinal waves; ΔW_T and ΔW_L are energy values inside thin layer Δr for transversal and longitudinal waves accordingly.

The transversal waves carry away energy at infinity. Therefore integrals (4.5) are not equal to zero.

As the longitudinal waves cannot transfer energy at infinity as two integrals (4.6) must be equal to zero. The first equation (4.6) establishes that the common flow of longitudinal waves does not carry away energy at infinity. The second equation (4.6) shows that field energy inside the thin spherical layer quickly decreases because of the previous condition

Two equations (4.6) are satisfied simultaneously if expressions inside integrals are equal to zero.

$$\lim_{r\to\infty} r\,\mathbf{r}\,[-\frac{\zeta}{\mu}\frac{\partial \mathbf{A}_2}{\partial t}\,\mathrm{div}\,\mathbf{A}_2 - \eta\,\varepsilon\frac{\partial \phi}{\partial t}\mathrm{grad}\,\phi] = 0 \qquad (4.7)$$

$$\lim_{r\to\infty} r^2\{\frac{\zeta}{2\mu}[(\mathrm{div}\,\mathbf{A}_2)^2 + (\frac{\partial \mathbf{A}_2}{\partial\, ct})^2] + \eta\frac{\varepsilon}{2}[(\mathrm{grad}\,\phi)^2 + (\frac{\partial \phi}{\partial\, ct})^2]\} = 0 \qquad (4.8)$$

In general case $\mathrm{div}\mathbf{A}_2$, $-\mathrm{grad}\phi$, $-\frac{\partial \mathbf{A}_2}{\partial t}$ and $-\frac{\partial \phi}{\partial c t}$ may decrease as r^{-1}. The sums of square terms inside braces of equation (4.8) are always positive. Hence, the expression (4.8) may be equal to zero only provided that $\eta = -\zeta = -1$ or $\eta = -\zeta = +1$.

Let $\eta = -1$ and $\zeta = +1$. Taking into account this condition and equation (3.8) we transform expressions (4.7) and (4.8).

$$\lim_{r\to\infty} r\,\mathbf{r}\,\varepsilon\frac{\partial \phi}{\partial t}[-\frac{\partial \mathbf{A}_2}{\partial t} - \mathrm{grad}\phi] = 0; \qquad \lim_{r\to\infty} r^2\frac{\varepsilon}{2}[-\frac{\partial \mathbf{A}_2}{\partial t} - \mathrm{grad}\phi][-\frac{\partial \mathbf{A}_2}{\partial t} + \mathrm{grad}\phi] \qquad (4.9)$$

From expression (4.9) follows that both equations are equal to zero simultaneously if and only if

$$\lim_{r\to\infty} \mathbf{r}\,\mathbf{E}_L = \lim_{r\to\infty} \mathbf{r}\,[-\frac{\partial \mathbf{A}_2}{\partial t} - \mathrm{grad}\,\phi] = 0 \qquad (4.10)$$

where $\mathbf{E}_L$ is longitudinal component of an electric field strength.

Expression (4.10) is always right due to expression (B.4). In other words, the set of simultaneous equations (4.6) will be identically equal to zero if only the summarised longitudinal electric field strength decreases faster than r^{-1}. It is possible only under the condition $\eta = -\zeta$.

So, we can formulate the final conclusion. The longitudinal waves of scalar and vector potentials can compensate each other under $c \to \infty$ only provided that energy flux density and energy density of scalar potential field should have an opposite sign with respect to the appropriate densities of vector potential.

The return thesis also is fair. *If the longitudinal waves of scalar and vector potentials, which are described by the wave equations, compensate each other with r ® ¥, then potential energy and energy flow of scalar potential and, accordingly, potential energy and energy flow of vector potential should have opposite signs***.**

5. Crash of Lorenz's Gauge

As is evident from the equations (3.7)–(3.8) there are always longitudinal waves of scalar and vector potentials within the framework of the Lorenz gauge. However, these waves are compensated in the solutions of Maxwell equations, *i.e.* mutually cancellated each other with $r \to \infty$. Hence, the Lorenz gauge either deals with negative energy flux density and negative energy density of charge fields of scalar potential or deals with negative flux density of transversal electromagnetic waves because of the necessary and sufficient condition of cancellation of longitudinal waves.

Firstly, $\eta = -1$ and $\zeta = +1$. We deal with violation of Coulomb's law. If energy of scalar potential is negative then the same charges should be attracted, and the unlike charges should be repelled! But it contradicts reality!

Secondly, $\eta = +1$ and $\zeta = -1$. Here a radiation of negative energy by aerials also is absurd.

So, we see that the Lorenz gauge is in conflict with the physical phenomena. The gauge cannot and should not be used for the description of electrodynamical phenomena. This is a crash of the Lorenz gauge. Now it is essential to find a new transformation of the equations. It is possible that light velocity is not the limit of all velocities. We consider that not only the wave fields exist. Instantaneous fields also can exist in a nature.

We could add following argument. We have shown that the limiting transfer $c \to \infty$ is illegal within the framework of the Lorenz gauge. Therefore, no the the Lorenz gauge may explain quasistatical phenomena of electrodynamics.

Special Relativity theory, which use mathematical construction of Lorenz's theory and the Lorenz gauge, now may be considered as *questionable theory***.** Our conclusions are not original. Similar conclusions are found in other works (Wallace, 1969), (Hayden, 1993), where the authors compare experimental results to results following from the Special Relativity theory, in our research (Kuligin *et al.,* 1990), (Kuligin *et al.,* 1989), (Kuligin *et al.,* 1996), (Kuligin *et al.,* 1994) and many papers of other authors.

Conclusions

We have proved that the wave equation has no a unique solution. Is shown that the gauge procedure is a choice of different kinds of fields, which will give the contribution to a solution of a wave equation. Generally, gauge invariance is not existing.

We have proved laws of energy conservation for wave fields in framework of the Lorenz gauge, which are expressed with wave potentials.

The problem of cancellation of longitudinal waves is investigated. Is shown with the laws that the Lorenz gauge can not correctly to describe electrodynamics effects.

We consider that the experimental inspection of Maxwell equations, which will define the exact gauge and equations for electromagnetic potentials, is necessary.

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Appendix1. Compensation of Instantaneous Potential by Retarded Potential

Till present day it was considered that the initial problem of wave equation has the unique solution. By virtue of it the natural conclusion about independence of the solution from gauge selection. Differently, the solutions within the framework of the Lorenz gauge and within frameworks of Coulomb's gauge were considered equivalent.

Now we should show that such conclusion is not right. Let's consider Coulomb's gauge of Maxwell equations (2.6).

$$\Delta \mathbf{A}_w - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}_w}{\partial t^2} = -\mu\,\mathbf{j} + \frac{1}{c^2}\frac{\partial}{\partial t}\mathrm{grad}\,\phi_{ins}; \qquad (?.1)$$

$$\Delta\,\phi_{ins} = -\frac{\rho}{\varepsilon}; \quad \mathrm{div}\,\mathbf{A}_w = 0$$

The solution of these equations should contain a field of instantaneous potential ϕ_{ins} , *i.e.* -gradϕ_{ins} .

They assert that the field of a vector potential $-\frac{\partial \mathbf{A}_w}{\partial t}$ compensates this instantaneous component of field strength $-\mathrm{grad}\phi_{ins}$ in all free space. To show an inaccuracy of this conclusion we shall transform the right member of vector equation using an equation of continuity of scalar potential ϕ_{ins}

$$\mathrm{div}\mathbf{v}\phi_{ins} + \frac{\partial \phi_{ins}}{\partial t} = 0 \qquad (A.2)$$

For simplicity we shall consider the charge velocity **v** is constant.

In outcome the right part of the vector equation of expression (A.1) is:

$$-\mu\,\mathbf{j} + \frac{1}{c^2}\frac{\partial}{\partial t}\mathrm{grad}\phi_{ins} = -\mu\,\rho\mathbf{v} - \frac{1}{c^2}\mathrm{grad}\frac{\partial\phi_{ins}}{\partial t} =$$
$$-\mu\varepsilon\,\mathbf{v}\Delta\,\phi_{ins} - \mu\varepsilon\,\mathrm{grad}\,\mathrm{div}\,\mathbf{v}\phi_{ins} = \frac{1}{c^2}\mathrm{rot}\,\mathrm{rot}\,\mathbf{v}\phi_{ins} \qquad (A.3)$$

A compensation of field - gradϕ_{ins} by field $-\frac{\partial \mathbf{A}_w}{\partial t}$ is impossible for following reasons. At first, the right part of the equation of the vector potential is proportional v/c. Therefore, when v/c equal or close to zero we can not have full compensation in free space. In second, the source of the vector potential field (the right part of equation (A.3)) is solenoidal. It can create only solenoidal vector potential **A**. The attempt to construct in free space a field of polar vector, using only sources of solenoidal fields, is senseless.

Sometimes they speak, that compensation of instantaneous field by field of vector potential follows directly from gauge invariance linking Coulomb's gauge to the Lorenz gauge. Let's consider also this approach.

It is known that electric field **E** and the magnetic field **H** are saved with a following transformation.

$$\mathbf{A}' = \mathbf{A} + \mathrm{grad}\ f; \qquad \phi' = \phi - \frac{\partial f}{\partial t} \qquad (A.4)$$

where: **A** and ϕ old electromagnetic potentials; **A**' and ϕ' new electromagnetic potentials; $f = -\frac{\partial \phi}{\partial t}$ is some function (gauge potential) satisfying to a homogenous wave equation:

$$\Delta f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} = 0 \tag{A.5}$$

Obviously from equation (A.5), the gauge potential f is not instantaneous.

Let's now record the Lorenz gauge of Maxwell equations.

$$\Delta \mathbf{A}_w - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}_w}{\partial t^2} = -\mu \mathbf{j}\ ; \ \Delta \phi_w - \frac{1}{c^2}\frac{\partial^2 \phi_w}{\partial t^2} = -\frac{\rho}{\varepsilon}\ ; \ \operatorname{div}\mathbf{A}_w + \frac{1}{c^2}\frac{\partial \phi_w}{\partial t} = 0 \tag{A.6}$$

To receive from these equations of Coulomb's gauge of Maxwell equation they enter the condition $f = -\frac{\partial \phi_w}{\partial t}$. Using this condition and expression (A.4) we shall substitute **A** and ϕ, which are expressed with **A**' and ϕ', into expression (A.6).

In outcome we find:

$$\Delta \mathbf{A'}_w - \frac{1}{c^2}\frac{\partial^2 \mathbf{A'}_w}{\partial t^2} = -\mu \mathbf{j} + \frac{1}{c^2}\frac{\partial}{\partial t}\operatorname{grad}\phi'_{ins}\ ; \tag{?.7}$$

$$\Delta \phi'_w = -\frac{\rho}{\varepsilon}; \quad \operatorname{div}\mathbf{A'}_w = 0$$

It would seem that now we really deal with Coulomb's gauge. Actually we should remember that potential f and, therefore, potentials ϕ'_w and ϕ_w should be the solutions of homogeneous wave equation (see (A.5)).

$$\Delta f_w - \frac{1}{c^2}\frac{\partial^2 f_w}{\partial t^2} = \frac{\partial}{\partial t}(\Delta\phi_w - \frac{1}{c^2}\frac{\partial^2 \phi_w}{\partial t^2}) = \frac{\partial}{\partial t}(\Delta\phi'_w - \frac{1}{c^2}\frac{\partial^2 \phi'_w}{\partial t^2}) = 0 \tag{A.8}$$

It contradicts with Poisson equation of scalar potential (A.7)

$$\Delta \phi'_w = -\frac{\rho}{\varepsilon} \tag{A.9}$$

The equations (A.8) and (A.9) are incompatible. Therefore, Coulomb's gauge is not a corollary of the Lorenz gauge because of gauge invariance. This conclusion has basic value for the quantum electrodynamics.

Appendix 2. Longitudinal Electric Field at Infinity

Let's consider pattern of electric charges, which are taking place inside sphere limited in some radius a. To estimate a rate of electric field change for r > > a we write integral

$$\oiint \varepsilon\, \mathbf{E}\, \mathbf{n}^o r^2 \sin\theta\, d\theta\, d\varphi = \sum q_i \tag{B.1}$$

where: **E** is electric intensity; $\mathbf{n}^o$ is unit normal to the surface of sphere; q_i is i-charge inside sphere of radius a.

It is obvious that the transversal electrical field of vector potential $\mathbf{A}_1$ does not give the contribution to integral. The electric field strength is orthogonal to vector $\mathbf{n}^o$. Therefore when $r \to \infty$ we can estimate a rate of change of a longitudinal electrical field formed by potentials $\mathbf{A}_2$ and ϕ.

$$\mathbf{E}_L = -\frac{\partial \mathbf{A}_2}{\partial t} - \operatorname{grad}\phi \tag{B.2}$$

Now with expressions (B.1) and (B.2), we estimate a rate of change of longitudinal electrical field $\mathbf{E}_L$. For the aim we write expression (B.1) as an approximation.

$$\varepsilon E_L 4\pi r^2 \approx \sum q_i \tag{B.3}$$

With expression (B.3) it is easily to write:

$$E_L \approx \frac{\sum q_i}{4\pi\varepsilon r^2} \quad (B.4)$$

Thus, when the charge sum inside sphere is different from zero then the longitudinal electrical field decreases as r^{-2}. If the sum is equal to zero and the pattern of electric charges has a form of a multipole source then the longitudinal field decreases even faster.

References

Ginzburg V.L., 1987. *Teoreticheskaia Fizika i Astrofizika*, Moscow, Nauka, GIFML (in Russian).

Hayden H.C., 1993. Stellar aberration. *Galilean Electrodynamics*, vol. 4, # 5.

Kuligin V.A., 1987. Causality and physical interactions. In "*Determinism in a modern science*" Voronezh University Press. (In Russian).

Kuligin V.A., Kuligina G.A., 1986. The mechanics of quasineutral systems of charged particles and laws of conservation of nonrelativistic electrodynamics, deposited with *VINITI*, Sep. 4, # 6451-V86. (in Russian).

Kuligin V.A., Kuligina G.A., Korneva M.V., 1989. Lorentz's transformation and theory of knowledge, deposited with *VINITI*, Jan. 24, # 546-V89. (in Russian).

Kuligin V.A., Kuligina G.A., Korneva M.V., 1990. The paradoxes of relativistic mechanics and electrodynamics, deposited with *VINITI*, July 24, # 4180-V90. (in Russian).

Kuligin V.A., Kuligina G.A., Korneva M.V., 1994. Epistemology and Special Relativity. *Apeiron*, (20:21).

Kuligin V.A., Kuligina G.A., Korneva M.V., 1996. The electromagnetic mass of a charged particle. *Apeiron,* vol.3, # 1

Kuligin V.A., Kuligina G.A., Korneva M.V., 1997. Energy of radiators and transfer lines with dispersion, deposited with *VINITI*, Feb. 20, # 538-V97. (in Russian).

Tikhonov A.N., Samarsky A.A., 1953. *The equations of mathematical physics*. GIFML. (in Russian).

Wallace B.G., 1969. Radar testing of the relative velocity of light in space. *Spectroscope Letters,* vol.2, # 12. Pp. 361-367.

Addendum

We write the action integral of an electromagnetic field

$$S = \frac{1}{ic}\int [\frac{\varepsilon}{4}F_{ik}^2 - j_k A_k] d\Omega \quad (C.1)$$

where: $F_{ik} = \frac{\partial A_k}{\partial x_i} - \frac{\partial A_i}{\partial x_k}$ is the tensor of the electromagnetic field; $d\Omega$ is elementary 4-volume (dx; dy; dz; icdt).

We shall vary only potentials A_k to find equations of the electromagnetic field (*Landau,Lifshitz,* 1975) . Also we shall consider that j_k does not depend on potentials A_k.

$$\delta S = \frac{1}{ic}\int \{\frac{\varepsilon}{2}[\frac{\partial A_k}{\partial x_i}\frac{\partial}{\partial x_i}\delta A_k - \frac{\partial A_k}{\partial x_i}\frac{\partial}{\partial x_k}\delta A_k - \frac{\partial Ak}{\partial x_i}\frac{\partial}{\partial x_k}\delta A_i] - j_k \delta A_k\} d\Omega = 0 \quad (?.2)$$

After integration by parts we obtain

$$\delta S = \frac{1}{ic}\int [-\varepsilon \frac{\partial^2 A_k}{\partial x_i^2}\delta A_k - j_k]\delta A_k d\Omega - \frac{1}{ic}\int_{S_i} \varepsilon \frac{\partial Ai}{\partial x_k}\delta A_k dS_i = 0 \quad (?.3)$$

where dS_i is an element of a 4-surface.

In the second integral we should evaluate values for limits of integration. The integration limits of spatial co-ordinates is infinity. Here field is equal to zero. Under the initial condition the variation of potentials is equal to zero an initial point t_a and final point t_b when we integrate over time. Thus, the second integral is equal to zero (Landau,Lifshitz, 1975). The variation dA_k is arbitrary. Therefore expression in brackets of the first integral is equal to zero.

$$\varepsilon\frac{\partial^2 A_k}{\partial x_i^2} - j_k = 0 \tag{C.4}$$

We write down the expression in the classical form

$$\frac{1}{\mu}[\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}] = -\mathbf{j}; \qquad \varepsilon[\Delta\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}] = -\rho \tag{C.5}$$

Lorenz's condition ∂? $_i$ / ∂ x_i=0 follows from equation ∂j_i / ∂ x_i=0 and expressions (C.5).

As the second integral of expression (C.3) do not give any components in equations (C.4) and (C.5) we can give other expression of the action integral.

$$S = \frac{1}{ic}\int[\Lambda - j_k\delta A_k]d\Omega = 0 \tag{C.6}$$

where

$$\Lambda = \frac{\varepsilon}{2}(\frac{\partial A_k}{\partial x_i})^2 \tag{C.7}$$

is LaGrange function density of the electromagnetic field;

$$S = \int_{t_a}^{t_b}\iiint_\infty \{\frac{1}{2\mu}[(\mathrm{rot}\mathbf{A})^2 + (\mathrm{div}\mathbf{A})^2 - (\frac{\partial\mathbf{A}}{\partial ct})^2] - \varepsilon[(\mathrm{grad}\phi)^2 - (\frac{\partial\phi}{\partial ct})^2] - \mathbf{jA} + \rho\phi\}dxdydzdt \tag{C.8}$$

With expression (C.7) and using (Landau,Lifshitz, 1975) we can write an energy-momentum tensor of the electromagnetic field.

$$T_{il} = \varepsilon[\delta_{il}\frac{1}{2}(\frac{\partial A_l}{\partial x_i})^2 - \frac{\partial A_k}{\partial x_i}\frac{\partial A_k}{\partial x_l}] \tag{C.9}$$

From expression (C. 9) follows that energy density of the electromagnetic field (T_{44}) is

$$w = \frac{1}{2\mu}[(\mathrm{rot}\mathbf{A})^2 + (\mathrm{div}\mathbf{A})^2 + (\frac{\partial\mathbf{A}}{\partial ct})^2] - \varepsilon[(\mathrm{grad}\phi)^2 + (\frac{\partial\phi}{\partial ct})^2] \tag{C.10}$$

Now we write energy of the electromagnetic field in a volume V

$$W = \int_V w dv = \int_V\{\frac{1}{2\mu}[(\mathrm{rot}\mathbf{A})^2 + (\mathrm{div}\mathbf{A})^2 + (\frac{\partial\mathbf{A}}{\partial ct})^2] - \varepsilon[(\mathrm{grad}\phi)^2 + (\frac{\partial\phi}{\partial ct})^2]\}dv \tag{C.11}$$

We consider energy change in the volume.

$$\frac{dW}{dt} = \int_V\{\frac{1}{\mu}[\mathrm{rot}\mathbf{A}\frac{\partial}{\partial t}\mathrm{rot}\mathbf{A} + \mathrm{div}\mathbf{A}\frac{\partial}{\partial t}\mathrm{div}\mathbf{A} + \frac{1}{c^2}\frac{\partial\mathbf{A}}{\partial t}\frac{\partial^2\mathbf{A}}{\partial t^2}] - [\mathrm{grad}\,\phi\frac{\partial}{\partial t}\mathrm{grad}\phi + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\frac{\partial^2\phi}{\partial t^2}]\}dv \tag{C.12}$$

After integration of expression (C.12) by parts and using of equations (C. 5) we obtain:

$$\int_V\{\frac{1}{2\mu}[(\mathrm{rot}\mathbf{A})^2 + (\mathrm{div}\mathbf{A})^2 + (\frac{\partial\mathbf{A}}{\partial ct})^2] - \varepsilon[(\mathrm{grad}\phi)^2 + (\frac{\partial\phi}{\partial ct})^2]\}dv = \oint_S[\frac{1}{\mu}(\frac{\partial\mathbf{A}}{\partial t}\times\mathrm{rot}\mathbf{A} + \frac{\partial\mathbf{A}}{\partial t}\mathrm{div}\mathbf{A}) - \varepsilon\frac{\partial\phi}{\partial t}\mathrm{grad}\phi]\mathbf{n}^o ds + \int_V(\mathbf{j}\frac{\partial\mathbf{A}}{\partial t} - \rho\frac{\partial\phi}{\partial t})dv \tag{?.13}$$

where $\mathbf{n}^o$ is a unit normal vector on surface S.

The potentials $\mathbf{A}$ and ϕ are independent and we may write the integral form of the law of energy conservation

$$\int_V \frac{1}{2\mu}[(\mathrm{rot}\mathbf{A})^2 + (\mathrm{div}\mathbf{A})^2 + (\frac{\partial \mathbf{A}}{\partial ct})^2]dv - \oint_S \frac{1}{\mu}(\frac{\partial \mathbf{A}}{\partial t} \times \mathrm{rot}\mathbf{A} + \frac{\partial \mathbf{A}}{\partial t}\mathrm{div}\mathbf{A})\mathbf{n}^o dS + \int_V (-\mathbf{j}\frac{\partial \mathbf{A}}{\partial t})dv = 0$$

$$-\int_V \frac{\varepsilon}{2}[(\mathrm{grad}\phi)^2 + (\frac{\partial \phi}{\partial ct})^2]dv + \oint_S (\frac{\partial \phi}{\partial t}\mathrm{grad}\,\phi)\mathbf{n}^o dS + \int_V (\rho\frac{\partial \phi}{\partial t})dv = 0 \qquad \text{(C.14)}$$

From expression (C.14) are visible that Pointing's law of energy conservation not is unique. The form of the law and its contents depend on gauge selection. It not is randomness, as the solution of Maxwell equations depends on gauge.

We also want to pay attention to expression (C.11). Within the framework of the Lorenz gauge the field energy of the scalar potential is negative. This fact is known for the experts of quantum electrodynamics. Often they admire with a mathematical formalism, but sometimes overlook about an essence of physical phenomena. The purpose of our article is to fill in this vacuum.

Reference

Landau L.D., Lifshitz E.M., 1975. *The Classical Theory of Fields*. Pergamon Press. New York.