The $\mathrm{B}^{(3)}$ Field Controversy

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The controversy generated by the theory of the $\mathbf{B}^{(3)}$ magnetic field is considered in the light of a paradox raised by E. Comay and its repudiation by M.W. Evans and S. Jeffers. Their arguments are examined and assessed.

Keywords: $\mathbf{B}^{(3)}$; Stokes' Theorem

1. Introduction

The theory of the $\mathbf{B}^{(3)}$ magnetic field originated in 1992 [Evans1992], and there is now a voluminous literature: E. Comay [Comay1996a] cites the articles up to 1996, and four volumes in the series entitled: *The Enigmatic Photon* have been published [Evans1994a, Evans1995a, Evans1996a, Evans1998]. A recent double-issue of this journal [Evans1997] was devoted to articles about the $\mathbf{B}^{(3)}$ theory.

Several scientists have written criticisms of the $\mathbf{B}^{(3)}$ theory [Comay1996a, Comay1996b, Barron1993, Lakhtakia1993, Grimes1993, Buckingham1994a, Buckingham1994b, Rikken1995, Akhtar1997], and rebuttals of most of these criticisms have been published [Evans1997, Evans1993, Evans1995b, Evans1996b]. A continuing controversy is manifest; the Editorial in [Evans1997, pp.35-36] presents a synopsis of the controversy.

Controversies in science are best resolved by reasoning supported by experimental evidence. The experiments of Rikken [Rikken1995] and Akhtar Raja [Akhtar1997] were designed to detect and measure the longitudinal magnetic field of circularly polarized radiation predicted by the $\mathbf{B}^{(3)}$ theory; *i.e.* predicted by formula (5) of [Evans1992, p.238] (quoted as formula (4) of [Akhtar1997]). The experiments have not confirmed the existence of the $I^{1/2}$ and $I^{3/2}$ terms in this formula. Nevertheless a rebuttal of Rikken's inference that $\mathbf{B}^{(3)}$ is not physical [Evans1997, p.35] has been published, as has a similar rebuttal of the experiments of Akhtar Raja, *et al*, [Evans1997, p.94].

Casual observers of the continuing controversy may be confounded, for the essence of the scientific method is to put all ideas to the tests of logical consistency and confirmation by experimental measurements. This article is intended to calibrate the credibility of the opponents in the controversy, specifically by a detailed analysis of the validity of the paradox raised by E. Comay [Comay1996a] and of its subsequent repudiation [Evans1996b].

2. Comay's Analysis

Comay's article [Comay1996a] concludes as follows:

> "It is proved here that Evans's modified electrodynamics yields physically unacceptable results, thereby establishing its inconsistency with Maxwell's equation in the vacuum $\nabla \times \mathbf{B} = \partial \mathbf{E}/\partial t$."

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Comay's conclusion (quoted above) is somewhat ambiguous, because his inference of "physically unacceptable results" *employed* the Maxwell equation $\nabla \times \mathbf{B} = \partial \mathbf{E}/\partial t$ in the latter steps of his analysis (as explained below), and one aspect of the $\mathbf{B}^{(3)}$ theory is that Maxwell's equations should be modified by the addition (to two of them) of a mass term $-\xi^2 \mathbf{A}$ to produce [Evans1994a, eqn.(243),p.124]:

$$\nabla \times \mathbf{B} = \partial \mathbf{E}/\partial t - \xi^2 \mathbf{A}$$

where the mass is believed to be "of the order of 10^{-51} kgm" as stated below eqn.(243) on page 124 of [Evans1994a]. This consideration leads to the inference that all that Comay proved was that the $\mathbf{B}^{(3)}$ concept is inconsistent with Maxwell's electrodynamics (in particular with the equation $\nabla \times \mathbf{B} = \partial \mathbf{E}/\partial t$), and since this inconsistency was already an established part of the $\mathbf{B}^{(3)}$ theory, Comay's inference of "physically unacceptable results" was not proven.

However, whether this tiny mass term is an essential part of the $\mathbf{B}^{(3)}$ theory is itself ambiguous, for this same source [Evans1994a] contains statements such as:

> "... the three field components $\mathbf{B}^{(1)}$, $\mathbf{B}^{(2)}$ and $\mathbf{B}^{(3)}$ obey the Maxwell equations in free space"[Evans1994a, p.71]

This statement (and similar statements elsewhere) justified Comay in employing the unmodified Maxwell equation to deduce "physically unacceptable results". Furthermore, the argument that it was Comay's use of the unmodified Maxwell equation that invalidated his analysis, was not employed in the subsequent rebuttal [Evans1996b], and since this article is primarily concerned with the credibility of Comay's analysis *vis a vis* its subsequent repudiation [Evans1996b] (*i.e.* the $\mathbf{B}^{(3)}$ field controversy), this ambiguity in the import of Comay's result is peripheral to what follows.

2.1 The Scenario

Comay reached his conclusion (quoted above) by considering the radiation from a rotating electric dipole. He begins his analysis by quoting Evans's definition of the $\mathbf{B}^{(3)}$ magnetic field (Comay's equation (1)). He also quotes 3 assertions that recur in the literature of the $\mathbf{B}^{(3)}$ field:

(A) A circularly polarized electromagnetic wave has an additional magnetic field $\mathbf{B}^{(3)}$ which is parallel (or antiparallel) to the wave's propagation. The amplitude of this field is proportional to that of the transverse magnetic field, but unlike the latter, it is independent of the angular frequency ω (see Ref.[12], p. 69 $\equiv$ [Evans1994b, p.69]).

(B) The longitudinal magnetic field $\mathbf{B}^{(3)}$ vanishes if the wave is linearly polarized (see Ref.[28], item (g) on p. 568 $\equiv$ [Evans1995b, p.568]).

(C) The magnetic field $\mathbf{B}^{(3)}$ is not associated with any real electric field (see Refs.[12], pp. 69 and 70; [24], etc. $\equiv$ [Evans1994b, Evans1994c]).

Comay uses assertions (A) and (B) to evaluate the line integral of the magnetic field around a closed path ($PQRSP$) in the radiation field which he defines as follows (see Figs.1 and 2 in [Comay1996a]):

- PQ is on the axis of rotation of the rotating dipole (chosen to be the z-axis). It is a straight line directed radially outwards whose length is chosen to be equal to the wavelength of the radiation.

- SR is a straight line segment of the x-axis whose length is also one wavelength. Thus it lies in the plane of rotation of the dipole. It is (like PQ) radial, but directed inwards.

- QR and SP are quarter-circle arcs of concentric circles whose center is at the center of the rotating dipole; *i.e.* the origin of coordinates: x=y=z=0. The radial distance between the two circular arcs is one wavelength everywhere.

The radius of the circular arcs is chosen to be large compared with the length of the dipole (*i.e.* the distance separating the two charges of the rotating electric dipole). This ensures that the entire path $PQRSP$ lies within the far field of the radiation emanating from the rotating dipole. This far field region of space is known as the *radiation zone*, its salient property being that the direction of propagation of the radiation is radially outwards everywhere. Comay cites Landau and Lifshitz [Landau1975] in support of this radially directed propagation in the far field (Comay's Ref.1); another analysis reaching the same conclusion is given in the book by Wangsness [Wangsness1979, pp.517-521].
Analysis of the radiation from a rotating dipole also leads to the inferences:

1. the radiation is circularly polarized along the axis of rotation (the z axis),

2. it is linearly (plane) polarized in the plane of rotation (the x-y plane),

3. at intermediate directions it is elliptically polarized.

These inferences derive from the time-varying electric field of the rotating dipole:

1. along its axis,

2. within its plane, and

3. at intermediate angles.

Comay proceeds to qualitatively compute the line integral of the magnetic field around the closed path $PQRSP$:[1]

- Using the $B^{(3)}$ theory assertion (A) (quoted above) he notes that $B^{(3)}$ makes a non-zero contribution to the integral along PQ because PQ is parallel to the direction of propagation (the scalar product is the product of the magnitudes since $\cos(0) = 1$), and because the radiation is circularly polarized along PQ (the z axis).

- Along the arcs QR and SP $B^{(3)}$ makes no contribution to the integral because these path-segments are perpendicular to the direction of propagation, and hence the scalar product is zero ($\cos(\pi/2) = 0$).

- Using the $B^{(3)}$ theory assertion (B) (quoted above) he infers that the radial segment RS (along the x axis) makes no contribution to the integral because although the scalar product is not zero ($\cos(\pi) = -1$), the radiation is linearly polarized along RS and hence $B^{(3)} = 0$.

[1] The integrand of a *line* integral is the scalar product of the vector field being integrated, with the element of the directed *line* segment.

- The transverse components of the magnetic field do not contribute to the line integral along the radial segments because these segments are perpendicular to the transverse direction (and hence the scalar product is zero).

- The transverse contributions to the integral are not zero along the circular arcs QR and SP, but Comay has cleverly chosen them to be a wavelength apart (the length of PQ and RS), which allows him to argue that these non-zero transverse contributions to the integral are equal in magnitude and opposite in sign; the opposite sign arises from the scalar product: $\cos(0) = 1$ along QR and $\cos(\pi) = -1$ along SP.

  The equality of the magnitudes arises because being a wavelength apart they are in phase, and although the arc SP is shorter than QR (because the latter is on a larger circle), the amplitude of the radiation field is reduced by the same ratio; hence the cancellation is exact.

Thus the line integral of the entire magnetic field around the closed path $L \equiv PQRSP$ is shown to be non-zero because of the contribution of $B^{(3)}$ along the radial segment PQ.

Having established that the line integral is not zero, Comay uses Stokes's theorem to re-express it as a surface integral over the area A bounded by by the closed path L:

$$\oint_L \mathbf{B}.\mathbf{dl} = \int_A (\nabla \times \mathbf{B})\,.ds = \int_A \frac{\partial \mathbf{E}}{\partial t}.ds = \frac{\partial}{\partial t}\int_A \mathbf{E}.ds \tag{1}$$

where the R.H.S. results from Comay's use of the Maxwell equation:

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} \tag{2}$$

Since the line integral is only non-zero on account of the contribution from $\mathbf{B}^{(3)}$ along PQ, and since its magnitude, $B^{(3)}$, is independent of time, it follows that the time derivative of the electric field $\mathbf{E}$ averaged over the surface A (the electric *flux* through the surface A) is a non-zero constant. Hence the time-integral of the electric flux through A must be a linear function of the time t:

$$\int_A \mathbf{E}.ds = a\,t + b \tag{3}$$

which implies that $\mathbf{E}$ itself has a linear dependence upon time rather than the oscillatory dependence of an electromagnetic wave. This non-wave electric field $\mathbf{E}$ must increase indefinitely with the passage of time, which is, of course, contrary to the physics of a radiation field whose intensity is stationary.

Thus Comay's analysis concludes that the assumption that $\mathbf{B}^{(3)}$ is the longitudinal component of the physical magnetic field in circularly polarized radiation, yields the non-physical result of an electric field through $PQRS$ whose magnitude increases linearly with time. In addition, Comay notes that this result disproves assertion (C) of the $\mathbf{B^{(3)}}$ theory (quoted above), thus making the theory self-contradictory.

The paradox was established via an apparently sound argument (summarized above), and thus it is of prime importance because, as he says:

> "one counter-example justifies a refutation of a theory."

It is a disproof by counter-example.

While there may be other possible explanations for the paradox, the most obvious explanation is that it is incorrect to regard $B^{(3)}$ as the longitudinal component of the physical magnetic field; *i.e.* assertion (A) of the $\mathbf{B}^{(3)}$ theory is incorrect.

3. Evans-Jeffers Reply to Comay

The following analysis was conducted in December 1996 in response to a request for a review of the proofs of [Evans1996b] by Dr. M.W. Evans; the review was communicated to Dr. Evans on December 23, 1996.

3.1 A Matter of Attribution

The first sentence of the Reply reads as follows:

> "Comay's definition [1] of B(3) ... in his Eq.(1) uses the fact that ... B(1)xB(2) is empirically irrotational."

However, the definition of $\mathbf{B}^{(3)}$ given in Comay's equation (1) is Evans's definition, rather than Comay's. Comay is simply quoting it so that his readers will know precisely what he is writing about. Furthermore, Comay's Comment doesn't mention anything about $\mathbf{B}^{(3)}$ being irrotational. These observations create the perception that the statement quoted above falsely attributes to Comay both a definition and the use of the irrotational property.

The same false attribution of the definition occurs in the first sentence of the paragraph preceding equation (4), where the Reply article refers to "Comay's *own* [sic] definition"; the definition (of $\mathbf{B}^{(3)}$) that occurs in Comay's Comment is simply quoted from Evans's publications. It is not Comay's *own* definition, but rather Evans's *own* definition.

The Reply article [Evans1996b] says that Comay "asserted the curl of B(3) to be non-zero", whereas what Comay actually did was to:

- evaluate the line integral of the complete magnetic field (with the result that the integral was non-zero because of the contribution of $\mathbf{B}^{(3)}$),
- convert this non-zero integral into a surface integral via Stokes's theorem, and (since its integrand is the curl of B)
- infer that the integrand (*i.e.* the curl of B) cannot be zero.

This is how he *deduced* that the curl of $\mathbf{B}$ (not specifically its $\mathbf{B}^{(3)}$ component) is non-zero. The Reply article's description of this deductive mode of reasoning as an "assertion" is invalid.

3.2 Line and Surface Integrals and Stokes's Theorem

The Abstract of the Reply article [Evans1996b] reads as follows:

> "The argument presented by E. Comay in Ref.1 is in error precisely at the point where he uses the Cartesian form of Stokes's theorem. His Comment is therefore erroneous and inconsequential."

Thus, notwithstanding that the Reply article contains several other, independent arguments in defense of the $B^{(3)}$ theory, it singles out Comay's use of the Cartesian form of Stokes's theorem as the point in his argument where he made a mistake.

However, a careful reading of Comay's Comment reveals that he does *not* use the *Cartesian form* of Stokes's theorem *anywhere* in his article; it simply isn't mentioned anywhere in his Comment. Hence the first sentence of the Abstract of the Reply article is another instance of false attribution; it attributes to Comay's Comment something that is not actually present in it.

3.2.1 Stokes's Theorem and its Cartesian Form

The Reply article addresses the question of the correct application of Stokes's theorem (and specifically its Cartesian form) in equations (4) and (5) and related text. The Reply article cites *The Vector Analysis Problem Solver* [Milewski1987, p.965 eq.7] for the explicit form of the Cartesian form of Stokes's Theorem:

$$\oint_C (F_1\,dx + F_2\,dy + F_3\,dz) = \int\int_S \left\{ \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) dy\,dz + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right) dx\,dz + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy \right\} \tag{4}$$

where **F** is a vector field which in terms of Cartesian components is:

$$\mathbf{F} = \mathbf{i}\,F_1 + \mathbf{j}\,F_2 + \mathbf{k}\,F_3 \tag{5}$$

that is F_1 is the x-component of **F** henceforth herein re-written as F_x

F_2 is the y-component of **F** henceforth herein re-written as F_y

F_3 is the z-component of **F** henceforth herein re-written as F_z

These components are functions of $x\ y\ z$:[2]

$$F_x \equiv F_x(x,y,z),\ F_y \equiv F_y(x,y,z),\ F_z \equiv F_z(x,y,z).$$

Note also that:

1. The L.H.S. is a *line integral* around the closed path C.
2. The R.H.S. is a surface (*i.e.* 2-dimensional) integral over the surface S which is bounded by the closed path C.

The more general (not necessarily Cartesian) form of Stokes' theorem is:

$$\oint_C \mathbf{F}.\mathbf{dl} = \int\int_S curl\,\mathbf{F}.\mathbf{n}\,ds \tag{6}$$

where $\mathbf{dl}$ is an increment of the line around the closed path C, and ds is a (2-dimensional) increment of the surface S; $\mathbf{n}$ is the unit vector perpendicular (normal) to the surface. Both integrals are thus *definite* integrals; they are related because the *closed* path C delineates the edge of the surface S. The dot (on both sides of the equation) signifies the *scalar product*.

This general form is readily related to the Cartesian form via the expression for $curl\,\mathbf{F}$ in Cartesian coordinates:

$$\begin{aligned} curl\,\mathbf{F} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \\ &= \mathbf{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \mathbf{j}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \mathbf{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) \end{aligned} \tag{7}$$

[2] and also of the time t, but the time-dependence is not relevant to the discussion here.

where the order of the two terms multiplying $\mathbf{j}$ was reversed on expanding the determinant in order to make the sign preceding $\mathbf{j}$ positive.

Since:

$$\mathbf{dl} = \mathbf{i}\,dx + \mathbf{j}\,dy + \mathbf{k}\,dz \tag{8}$$

and the Cartesian form of $\mathbf{F}$ is given by equation (5), taking the scalar product on the L.H.S. of (6) yields the integrand on the L.H.S. of equation (4)

The volume element of the surface integral in (6), ds, being defined in terms of the normal to the surface, $\mathbf{n}$, is given by:

$$\mathbf{n}\,ds = \mathbf{i}\,dy\,dz + \mathbf{j}\,dx\,dz + \mathbf{k}\,dx\,dy \tag{9}$$

and hence taking the scalar product on the R.H.S. of (6) yields the integrand on the R.H.S. of equation (4).

In this way we see how the Cartesian form of Stokes's theorem (4) is derived from the general form (6).

The Cartesian form of Stokes's theorem (4) applies to any *closed* path C which will in general be a curved line in 3-dimensional space; in particular the path C does not necessarily lie in a plane. Similarly, the surface, S is not necessarily planar. The three Cartesian terms in equation (4) take account of the generally curved nature of the path C and the surface S.

For a plane surface the integrals will be easier to compute if the coordinate system is set up so that the plane of the surface is coincident with one of the coordinate planes. Additionally, if parts (at least) of the path C are (perpendicular) straight lines then the line integral will be computed more easily if these portions of C are coincident with one (or two) of the coordinate axes.

3.2.2 The Comay and Evans-Jeffers Scenarios

Comay's scenario is an application of Stokes's theorem based upon an integration path consisting of two concentric quarter circles joined by radial segments. The integration path in the similar Evans-Jeffers scenario is rectangular. In both scenarios the surface S is planar, and wisely both articles place this surface in the x-z coordinate plane.

Thus in the line integral $\mathbf{dl}$ is perpendicular to $\mathbf{j}$ *everywhere*, and hence the second term on the L.H.S. of (4) is zero: $\mathbf{j}.\mathbf{dl} = 0$, or equivalently $dy = 0$. The reason is that a factor of the scalar product is the cosine of the angle between the two vectors, and $\cos(90^\circ) = 0$ when the vectors are perpendicular.[3]

For the surface integral, the normal unit vector $\mathbf{n}$ is parallel to $\mathbf{j}$, and perpendicular to *both* $\mathbf{i}$ and $\mathbf{k}$. Hence the only non-zero term on the R.H.S. of (4) is the second (middle) term; this (as for the line integral) is equivalent to $dy = 0$.

Thus for both the Comay and Evans-Jeffers paths and surfaces, the general Cartesian form of Stokes's theorem (4) simplifies to:

$$\oint_C (F_x\,dx + F_z\,dz) = \int\int_S \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) dx\,dz \tag{10}$$

[3] The detail in this discussion may appear to be needlessly elementary to some readers. Nevertheless it is given here for the sake of explicit clarity.

3.2.3 Evans-Jeffers Equation (4)

In the text preceding equation (4) Evans-Jeffers identify the contribution to the line integral from "the radial segment $\overline{PQ}$ (on the z-axis)" as the "precise point at which Comay's argument fails". This is correct in so far as it is the non-zero contribution from $\mathbf{B}^{(3)}$ along this radial segment that makes the line integral non-zero.

Equation(4) of the Reply article reads as follows:[4]

$$\oint B_z^{(3)}\,dz = \int\int\left(\frac{\partial B^{(3)}}{\partial y}d\,y\,d\,z - \frac{\partial B^{(3)}}{\partial x}d\,z\,d\,x\right) = 0 \qquad (\mathrm{EJ}-4) \qquad (11)$$

The absence of the $B_x^{(3)}\,dx$ term on the L.H.S. of this equation is explained in the preamble to (EJ-4) in the Reply article; this integral is only for *part* of the line integral; the part along the segment $\overline{PQ}$ which is along the z-axis. Hence: $dx = 0$.
However, comparison of (10) and (11) reveals that the presence of the term:

$$\frac{\partial B^{(3)}}{\partial y}d\,y\,d\,z \qquad (12)$$

in (EJ-4), and the absence of the term:

$$\frac{\partial B^{(3)}}{\partial z}d\,x\,d\,z \qquad (13)$$

appear to be algebraic errors.[5]

The two terms in the surface integral on the R.H.S. of (EJ-4) appear to have been selected because the authors argued (in the text preceding their equation (4)) that the two partial derivatives in the integrand are zero; in this author's opinion it is not clear that they are necessarily zero, for although $B^{(3)}$ is directed along $\overline{PQ}$ (the z-axis), one would expect its magnitude to vary with x and y.

A more serious concern is that there is a conceptual difficulty with (EJ-4) in so far as it apparently attempts to use Stokes's theorem for a *non-closed* path, this being specifically the straight-line segment $\overline{PQ}$. It is of course valid to compute the *contribution* to the *line* integral from the *non-closed* line-segment $\overline{PQ}$, but one can hardly equate this with a surface integral, simply because the area of the surface is zero ! Along the line segment $\overline{PQ}$, $d\,x = d\,y = 0$, and hence the "surface" integral is zero regardless of which derivatives of $B_z^{(3)}$ are being integrated.

The conceptual oversight on the part of the authors of the Reply article, is that one can only equate the line integral of Stokes's theorem to the corresponding surface integral, when the path of the line integral encloses a non-zero area. To do this for any non-closed path (and fortissimo for a straight line segment—as Evans-Jeffers do in writing their equation (4)) is mathematically invalid.

3.2.4 Evans-Jeffers Equation (5)

Equation (5) of the Reply article is introduced as follows:

[4]Evans-Jeffers use upper-case $X\ Y\ Z$ for the Cartesian coordinates; in quoting their equations here the corresponding lower-case letters are substituted.

[5]These surface integral terms $B^{(3)}$ should have a z subscript: $B_z^{(3)}$.

Consider now Comay's comment, " ... while the ordinary transverse (rotating) magnetic field makes no contribution because it is perpendicular to the line segment $\overline{PQ}$." This is also incorrect because the Stokes theorem for the transverse components reads

$$\oint B_x\,dx + \oint B_y\,dy = \int\int \frac{\partial B_x}{\partial z} d\,z\,d\,x - \int\int \frac{\partial B_y}{\partial z} d\,y\,d\,z = 0 \quad (\text{EJ}-5) \quad (14)$$

Comparison of this equation (14) with the Cartesian form of Stokes' theorem pertaining to the x-z plane (*i.e.* the plane of the Comay and Evans-Jeffers scenarios: eqn.10) reveals the following algebraic errors:

- The line integral term $B_y\,dy$ should be absent
- The line integral term $B_z\,dz$ should be present
- The surface integral term $\frac{\partial B_y}{\partial z} d\,y\,d\,z$ should be absent
- The surface integral term $\frac{\partial B_z}{\partial x} d\,x\,d\,z$ should be present

Furthermore, since the specific line segment under evaluation is the radial line $\overline{PQ}$ (along the z-axis) the other term of the line integral present in (EJ-5) ($B_x\,dx$) should be absent; *i.e.* both terms of the line integral in (EJ-5) are incorrect, and the correct term is missing. The same conceptual error noted above for the Evans-Jeffers equation (4) pertains to their equation (5) also: it is meaningless to apply Stokes' theorem when the path of integration C is not closed.

More serious than incorrect and/or missing terms, is the inference that the authors of the Reply article apparently do not understand:

1. that in a *line integral* the integrand is the *scalar product* of the vector function (field) being integrated with the direction of the line segment $\mathbf{dl}$.
2. that Stokes's theorem only relates the line integral around a *closed path* with the surface integral over a surface bounded by that path.
3. that in a surface integral the 2-dimensional integration is *within* the surface.

A misconception of the Reply article is to confuse the evaluation of a line integral (and its *subsequent* conversion into a surface integral via Stokes's theorem) with Stokes's theorem itself.

The above algebraic analysis demonstrates that the Reply article's use of the Cartesian form of Stokes's theorem is erroneous, and since the Reply article (in its Abstract) identifies the Cartesian form of Stokes's theorem as the kingpin of its rebuttal of Comay's Comment, we must conclude that the Reply article has no credibility as an effective rebuttal of Comay's proof (by counter-example) that the theory of the $B^{(3)}$ field is both internally inconsistent and incompatible with the physics of electromagnetic radiation.

3.2.5 Circular and Rectangular Paths

A problem around equations (9) and (10) of the Reply article is that the integration path being considered, $ABCDA$ is rectangular (rather than the perimeter of the segment of a circle $PQRSP$ considered by Comay), and yet the text reads:

"... because of Comay's assertion that there is no contribution in DA and BC."

How can the Reply article make such a statement, when in fact Comay never even considered the path $ABCD$? (it was introduced in the Evans-Jeffers Reply).

It appears that Evans-Jeffers adopted the rectangular path $ABCDA$ in order to facilitate their use of the Cartesian form of Stokes's theorem - rectangular (*i.e.* Cartesian) coordinates being appropriate for such a path. Regardless of their reason, choosing an entirely different path from Comay is hardly the way to refute his proof, especially since the model field introduced by them in equation (7) does not represent a radially propagating field.

Another flaw in their argument is that the rectangular path $ABCDA$ is manifestly not in the radiation zone because the point A is at the origin of their Cartesian coordinate system; *i.e.* at the centre of the rotating dipole.

4. Conclusions

The above detailed discussion establishes several factual, technical and mathematical errors in the Evans-Jeffers Reply [Evans1996b] to Comay's Comment [Comay1996a]. These errors have been demonstrated in specific, detailed terms with quotations from both Comay's Comment [Comay1996a] and the Evans-Jeffers Reply [Evans1996b]. This detail will enable any reader of this article to verify the correctness of the inferences of these errors. These errors are *not* in the trivial category of "typographical"; they are substantive errors that undermine the integrity, coherence and credibility of the Reply article.

The high level of conceptual and mathematical sophistication displayed in the extensive literature of the $\mathbf{B}^{(3)}$ field (a prodigious scholarly achievement) [Evans1994a, Evans1995a, Evans1996a, Evans1997, Evans1998, especially] is incongruous with the elementary nature of the errors (e.g. the incorrect evaluation of line integrals).

The Reply article is a protestation that Comay's result must be wrong because his conclusion is inconsistent with the belief that $\mathbf{B}^{(3)}$ is the third, longitudinal component of the physical magnetic field in radiation. This belief is a case of mistaken identify, for while the theory correctly recognizes that $\mathbf{B}^{(3)}$ is directly proportional to one of the four Stokes parameters of an electromagnetic field [Evans1994a, p.145], it fails to recognize that this parameter characterizes the polarization state of the field, so that it cannot be an independent (third, longitudinal) component of the physical magnetic field; this is expounded upon elsewhere [Hunter1999].

The Reply to Comay evaluated herein fails to directly address the question of what (if anything) is wrong with Comay's analysis.

While M.W. Evans was at the University of North Carolina some of his colleagues engaged in a series of experiments [Akhtar1997] to detect and measure the longitudinal magnetic field predicted by the $B^{(3)}$ theory; these experiments produced only negative results [Akhtar1997].

It is of course possible that electromagnetic radiation has longitudinal field components in accord with the claims of the protagonists of the theory of the $\mathbf{B}^{(3)}$ field, but it is clear from mathematical analysis [Hunter1999] that any such field is not the $\mathbf{B}^{(3)}$ field defined throughout the literature as the conjugate cross-product of the transverse magnetic field [Evans1994a, eqn.(4a),p.3]. Comay [Comay1999] has recently pointed out that this definition of $\mathbf{B}^{(3)}$ makes it proportional to charge *squared* (in the radiation from a rotating dipole), whereas the physical field must be proportional to charge (not squared), and for the same reason, $\mathbf{B}^{(3)}$ does not satisfy the principle of superposition unlike the physical electromagnetic field. $\mathbf{B}^{(3)}$ is simply a defined function of the transverse components of the field, which is properly interpreted as the Stokes parameter of the field that measures its polarization state [Hunter1999]. The possible existence of a longitudinal field in radiation must therefore be based upon a mathematical re-definition of this field. This re-definition must be different from the alternative definition of $\mathbf{B}^{(3)}$ in terms of the vector potential [Evans1994a, eqn.(12),p.6] (see also [Ogievetskii,Polubarinov,1967]).

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