Quantum Correlations and Non-Locality as a Consequence of the Connectivity of Random Graphs

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By appealing to theorems of "random graph theory" we demonstrate that quantum correlations and the inherent non-local structure of quantum theory as evidenced in violations of Bell's theorem are a manifestation of the connectivity of quantum matter through random graphs independent of the specific details of particle interactions.

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Introduction

After the initial success of quantum theory [1], it soon became apparent that general features of the theory were at variance with determinism, locality, separability and the general notions of realism and completeness. The famous paper of Einstein, Podolsky and Rosen [2] demonstrated that quantum theory is incomplete and in a sense has to be treated as a theory suggested to us by experiment. Bohr's response [3] to the paper by Einstein, Podolsky and Rosen [3] centered around the principle of complementarity which essentially describes the incomplete nature of quantum theory as the result of the simultaneous description of nature by particles and waves. To be specific, Einstein was unsatisfied with stochastic theories that violated certain "realistic principles," and viewed them as predictive instruments to ultimately be replaced by a complete theory. Bohr believed that complementarity was an inevitable consequence of the wave particle duality and did not believe an ultimate theory would replace the notion of complementarity. In the years following these papers a rather pragmatic attitude toward quantum theory pervaded the theoretical community that was primarily preoccupied with choosing between the Einstein DeBroglie "ensemble interpretation" of quantum theory and the Copenhagen interpretation suggested by Bohr. [4] Following this era came the monumental paper of Bell [5] demonstrating that quantum mechanics was clearly at variance with the foundations of a deterministic causal "hidden variable" theory. [6] [7] In short the years of research in this direction have demonstrated that quantum systems have an inherent initial correlation that defies any deterministic causal mechanism to explain it. [8] [9] This fact coupled with the understanding that Q.M. is a theory of "subjective probability" as opposed to classical physics that admits to "objective determinism" has left the theorist with no "basic principles" with which to construct a sub-microscopic understanding of quantum theory. On formal grounds the Von Neumann "no go" theorem concerning hidden variables has discouraged us to modify the formal structure of quantum theory in order to accommodate hidden variables. [10] If we however go back to the prophetic ideas suggested by Pauli, [11] Heisenberg [12] and Wheeler [13] [14] we find that it could very well be that it is the arena of "space time" that has provided us with an incomplete background to describe quantum theory. Driven by this recognition numerous authors [15], [16], [17], [18], [19] have sought to replace the continuum with a set of "points" (or vertices) and possible connections between them (links or edges) to represent the pregeometric origin of space-time. In fact Nagels [20] has shown how such a construction coupled with ideas from graph theory leads to 3 space with positive curvature. Somewhat related to these developments Penrose [21] with his celebrated "Penrose spin-network" and Wootters [22] have sought to construct a combinatoric discrete theory of space time using spin as the fundamental "building block" or "link" between points in space time.

The purpose of this note is to point out that based on general principles of "random graph theory" and the assumption of countability, all quantum properties of particles should be correlated with no deterministic mechanism necessary to explain this result. Such a result suggests that the question of "how and why" Q.M., might be replaced by the pregeometric hypothesis that the concept of space and time and quantum theory can be expressed as a random graph with the links representing correlations between the vertices of the graph (here the vertices could represent particles or particle properties).

Quantum Correlations and the Connectivity of Random Graphs

As mentioned in the introduction (Ref. 6) $\vec{a}$ Bell demonstrated that no deterministic hidden variable theory [23] could explain the results of simultaneously measuring the spins of the two fermions in an anti-symmetric state given by

$$\Psi = \frac{\alpha\,\beta - \beta\,\alpha}{\sqrt{2}}$$

(α = spin up function, β = spin down function)

This result for the expectation value of $\left(\vec{\sigma}_1 \bullet \vec{a}\right)\left(\vec{\sigma}_2 \bullet \vec{b}\right)$ where 1 spin detector is set in direction and the other in direction $\vec{b}$ is

$$E(a,b) = \frac{\psi^{+}(\vec{\sigma}_1 \bullet \vec{a})(\vec{\sigma}_2 \bullet \vec{b})\psi}{\psi^{+}\psi} = -\vec{a} \bullet \vec{b} = -\cos\theta .$$

demonstrating that the expectation value is 1 for detecting particle spins in opposite directions. Bell's theorem essentially involves showing that a deterministic evaluation of $E(a,b)$ leads to results at variance with Eq. (2.2). Eq. (2.2) can also be thought of as the inevitable result of the correlation of spin 1 with spin 2 through the anti-symmetric wave function Eq. (2.1) which emerges from the Pauli principle for fermions. It might better have been stated that the failure of trying to invent deterministic mechanisms to explain quantum measurements is a result of accepting the Pauli principle for correlated symmetrical and anti-symmetric wave functions (for Bosons and Fermions respectively). Various authors have pointed out that the Pauli principle is the most *ad-hoc* and "blind" tenet of quantum theory [24], [25] and could be related to misunderstood topological properties in spin space. [261] Another taken for granted feature of many particle systems is that the exclusion principle influences the binding energy of a system of particles not alone through the dynamical hamiltonian, but through the assumption of a correlated symmetric or anti-symmetric state. This suggests that the particles "feel" a connectivity even prior to any dynamical interactions, which could suggest their relationship to some pre-geometric ordering or notion of connectivity. In this spirit we borrow from ideas contained in earlier works [27], [28], [29] where prior to the geometric state of Minkowski space time each particle is a "microuniverse of its own" [30] and only after some as yet not understood evolutionary process space time evolved to the continuum we now perceive. In the evolutionary process, particles also became correlated and at this level correlations of symmetric and anti-symmetric states are a manifestation of the properties of each vertex (particles) and the connection it has with other vertices.

To prove the connectivity of all particle states (vertices) we consider n vertices with at least ($n-1$) edges. Here for n vertices we need at least ($n-1$) edges for a graph to be connected. We also allow only one edge between two vertices, for the probability of connection of n vertices with q edges we have (p = probability of 1 edge, $p < 1$).

$$P_{n,q} = p^q (1-p)^{\binom{n}{2}-q} C_{n,q}$$

(Here $C_{n,q}$ = number of connected graphs with n vertices and q edges, $\binom{n}{2} = \frac{n!}{(n-2)!\,2!}$ = number of possible links between all vertices). For the total probability of connection we have

$$P_n = \sum_{q=n-1}^{\frac{n(n-1)}{2}} C_{n,q}\, p^q (1-p)^{\binom{n}{2}-q}$$

According to calculations by Gilbert [31] we have

$$P_n \rightarrow as\ n \rightarrow \infty$$

Thus for a large number of vertices the probability of connectivity approaches 1. From a physical point of view it suggests that if even prior to the state of space and time as we now perceive them there existed "particles" or "particle properties", if there were a large enough number of them they would of necessity be connected or correlated according to Eq. (2.5) by just drawing links between pairs of vertices and studying the probability of connection. We note the maximum number of links for n particles is $\frac{n(n-1)}{2}$, which is a "complete graph" in the language of graph theory. It also suggests that "fermions" and "bosons" attain their anti-symmetric or symmetric properties through this mechanism suggested by "random graph theory".

There is also an alternate proof of "connectivity" that stems from constructing the quantity [32]

$$r_{n,q} = \frac{C_{n,q}}{G_{n,q}}$$

Eq. (2.6) is the ratio of the number of connected graphs of q edges and n vertices to that of the total number of graphs with q edges and n vertices, note that for $q < n-1$, $C_{n,q} = 0$; for $q = \rule{2em}{0.4pt}$, $C_{n,q} = \ = G_{n,q}$. Thus the ratio in Eq. (2.6) increases from 0 to 1. If we define the ratio $\frac{C_{n,q}}{G_{n,q}}$ as the probability of connection, for a specific choice of $q = q(n)$ we may show this ratio will approach 1 as $n \rightarrow \infty$. This is in a strict sense not a proof of connectivity but a strong suggestion that given enough links (edges) a graph will "always be connected" as n grows large. It is also noteworthy to point out here that as mentioned earlier the "Pauli Principle" determines the correlation between fermions and bosons which in turn determines the outcome of two or more particle correlation experiments. For N fermions the Slater determinant

$$\Psi = \frac{1}{\sqrt{N!}} \begin{vmatrix} \Psi_1(r_1) --- \Psi_1(r_n) \\ \Psi_2(r_1) --- \\ \Psi_N(r_1) --- \end{vmatrix} \tag{2.7}$$

involves exactly $N!$ separate products, it is an amazing coincidence that for N vertices of complete graph there are exactly $\frac{N!}{(N-2!)2!} = \frac{N(N-1)}{2}$ edges or "links" wherein each vertex is connected to every other vertex once. This might suggest that "particle statistics" are a result of maximizing the number of links (complete graph) between particles or vertices where there can be only 1 link between each pair of vertices.

Conclusion

In the above analysis we have shown how random graph theory can be used to demonstrate why QM is non-local not on the basis of a causual mechanism but rather on the basis of pre-geometric connectivity not involving continuous space time but rather links with no geometric properties. With regard to the emergence of the continuum "astrophysical studies" have indicated that space time might have a "self-similar" fractal structure [33] which might suggest evolution from a self similar graph as the foundation of Riemannian geometry. There has also recently been evidence suggested by Coleman and Glashow [34] that a violation of "special relativity" for neutrino propagation might suggest an absolute "preferred-frame" or "ether" framework of space time having a lattice structure. Lastly studies on the variation of the electron g factor [35] have indicated that there might be a "preferred frame" in the universe that admits a "Finsler geometry" which is actually a generalization of Riemannian geometry. Other investigators [36] have shown that Finsler geometry can lead to modifications of the geodesic motion that can be tested within the solar system. It is important here to note that lattice studies have indicated that Lorentz invariance can never be restored on a lattice and any violation of special relativity [37] might echo a faint reminder that space time emerged from a set of points with links between them. It is hoped that studies in graph theory and combinatorics, as well as studies in general relativity and quantum theory can meet on a common ground in an effort to understand how the paradoxes of non-locality, the exclusion principle, gauge theory and geometric structure of space-time emerged from a few basic principles that have been obscured by the apparent complexity of present physical theory.

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