On the Lorentz-Covariant Theory of Gravity

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The main features of Lorentz-covariant theory of gravity (LCTG), the basis of which is the relativistic (4-vector) Newton potential, are presented. A further development of LCTG is becoming urgent since, as it has turned out, the main statement of the general theory of relativity contradicts the effect of gravitational shift. It is noted that the "electromagnetic-like" variant of the theory of gravity answers the Lorentz invariance demand. The strengths of the gravitational and cogravitational field are described by the antisymmetric tensor of rank 2. The field itself is described by two equations. The energy-momentum tensor of the gravitational field satisfies the conservation laws. The frequency decrease of light radiated in the gravitational field is treated as a consequence of the law of energy conservation.

Introduction

New and apparently insuperable difficulties of the general theory of relativity (GTR), which have recently been revealed [1,2], make vital a further development of the Lorentz-covariant theory of gravity (LCTG). The relativistic (4-vector) Newton potential serves as the basis for this theory. As to the strength of the gravitational field, one would, think, there are two possibilities here. In case of the antisymmetric field tensor, we have the "electromagnetic-like" theory of gravity, which has in fact been discussed in the literature (see, *e.g.*, [3]). It should be noted that the first attempts to build this type of theory were by Heaviside [4]. At one time, this problem was discussed by Lorentz [5], Poincaré [6], Minkowski [7] and others. An alternative version of LCTG when the gravitational field tensor is described by the symmetric 4-tensor of rank 2, as it proved, leads to a result similar to the relation of Abraham's theory [8]. In this theory as with GTR, the light velocity depends on the gravitational potential. Thus, the second possibility, as will be shown further, turns out to contradict the Lorentz invariance requirement.

Light velocity does not change in a gravitational field

The effect of the gravitational redshift of spectral lines was predicted by Einstein. As he marked [9]:

"The frequency of an atom situated on the surface of a heavenly body will be somewhat less than the frequency of an atom of the same element in which is situated in free space (or on the surface of a smaller celestial body. Now $\Phi = -kM/r$ where k is Newton's constant of gravitation, and M is the mass of the heavenly body. Thus a displacement towards the red ought to take place for spectral lines produced at the surface of stars as compared with the spectral lines of the same element produced at the surface of the earth, the amount of this displacement being

$$\frac{\nu_0 - \nu}{\nu} = \frac{kM}{c^2 r}$$

For the sun, the displacement towrds the red predicted by the theory amounts to two millionths of the wavelength."

These conclusions are the consequences of two equations:

$$\nu \cong \nu_0(1+\Phi/c^2), \ \ \lambda \cong \lambda_0(1-\Phi/c^2) \tag{1a,b}$$

presenting the frequency and wavelength of light emitted by a source in the gravitational field with the potential Φ; ν_0 and λ_0 are the corresponding quantities in the absence of the field.

The first statement, *i.e.*, equation (1a), was verified in the well-known Pound-Rebka-Snider experiments [10] based on Moessbauer's effect. Equation (1b) was directly confirmed, for example, in Brault's experiment [11] where the indicated shift of the wavelength of the D1 sodium line had been measured.

We now ask the following question: what is the velocity of light radiated by a source in the gravitational field? Based on eqs.(1a) and(1b), we easily obtain in a first approximation

$$c_g = \lambda\nu \cong \lambda_0\nu_0 = c\,, \tag{2L}$$

i.e., the light velocity remains an invariant quantity in the presence of the gravitational field [12].

The main statement of GTR, according to which the light velocity changes with a changing gravitational potential, in particular, by the formula (see, *e.g.*, [13])

$$c_g \cong (1+2\Phi/c^2)c \tag{2E}$$

obviously contradicts this result. What is more, based on eq.(2E) and the left half of eq.(2L), it follows that at least one of the factors, *i.e.* either λ or ν, must change when the gravitational potential is changed, but by a formula different from eqs.(1) [14]. However, this assumption directly contradicts the results of the above-mentioned experiments. According to these, the wavelength and the frequency of the emitted light depend on the gravitational potential (in the neighbourhood of the source), but they do not change when light propagates in a gravitational field.

Thus, in spite of the generally accepted opinion, *the gravitational field does not influence the light ray.* (Therefore, the observed deflection of the light ray by the Sun must be explained by other factors, for example, by refraction). The light velocity also remains invariant in the presence of the gravity field.

4-vector of the gravitational potential

The four-dimensional (covariant) formulation of relativity theory says that mass is a 4-scalar (Lorentzian invariant). Therefore, the covariant generalization of the known non-relativistic expression for the gravitational potential energy

$$E_g = m\Phi$$

takes the form

$$p_g^i = m\Phi^i \tag{3}$$

In the framework of LCTG, we have for the relativistic Poisson equation

$$\partial^2\Phi^i/\partial x^j\partial x_j = 4\pi G J^i\,. \tag{4}$$

Here the gravitational constant G and 4-current mass density figure on the right; whence it follows that the relativistic potential of gravity should be described by a 4-vector [15]. In particular, the exact expression for the gravitational (retarded) potential

$$\Phi^i = -G\frac{MU^i}{R^jU_j} \tag{5}$$

can be obtained by the Lorentz transformation of Newton's potential Φ (equal to Φ^0 in the rest frame). Here M is the mass of a moving particle, U^i its 4-velocity, R^i the retarded (light) distance.

The total energy and momentum of a particle with mass m and 4-velocity u^i in the gravitational field are described by the equation

$$P^i = p^i + p^i_g = m(u^i + \Phi^i / c). \tag{6}$$

For light, as the mass of photons is equal zero, $p^i_g = 0$, therefore the energy and momentum do not change when light propagates in a gravitational field.

The spin of the graviton. The vector nature of the gravity potential means that by analogy with photons, the quanta of the gravitational field (gravitons) must have spin 1. This fact, evidently, simplifies substantially the construction of the united theory of all interactions and, in particular, the quantum theory of gravity.

3. The gravitational field tensor

In LCTG the strength of the gravitational field is described by the antisymmetrical tensor of rank 2

$$G_{ik} = \partial\Phi_k / \partial x^i - \partial\Phi_i / \partial x^k \tag{7}$$

Based on eqs.(5) and (7) for the "retarded" tensor of the gravitational field (in the absence of acceleration), we obtain

$$G_{ik} = G\frac{Mc^2}{(R^j U_j)^3}(R_i U_k - R_k U_i). \tag{7'}$$

The relativistic force of gravity. Based on the covariant expression

$$F^i = -mG^{ik} u_k / c = -G^{ik} p_k / c, \tag{8}$$

where u_k is the 4-velocity of a "trial" particle of mass m, for the relativistic Newton force we have

$$\vec{F} = -G\frac{mM\Gamma^{-2}\gamma}{R^2(1-\vec{n}\vec{B})^3}[\vec{n}(1-\vec{\beta}\vec{B}) - \vec{B}(1-\vec{n}\vec{\beta})] \tag{9}$$

where $\vec{n} = \vec{R}/R$, $\vec{\beta} = u^\alpha / u^0$, $\gamma = (1-\beta^2)^{-1/2}$, $\vec{B} = U^\alpha / U^0$, $\Gamma = (1-B^2)^{-1/2}$, $\alpha = 1,2,3.$ It should be stressed that the space components of Minkowski's force figure here at left. Based on the known relativistic relation

$$u^i w_i = 0, \tag{10}$$

where w^i is 4-acceleration with use of eq.(8), we obtain

$$u_i G^{ik} u_k = 0 \tag{10'}$$

whence, in particular, it follows that

$$u_i G^{il} u_k + u_k G^{ki} u_i = u_i u_k (G^{ik} + G^{ki}) = 0\,;$$

i.e. the fulfilment of equality (10), and thereby the Lorentz invariance requirement, is possible if the gravitational field tensor is antisymmetric only. Here we have another example that the stated requirement strictly limits the character of physical laws.

Eq.(9) should be used in LCTG when calculating the anomalous secular displacement of Mercury and other planets. Here, however, we want to pay attention to the following. The important point is that all astronomical observations (and measurements) are conducted in the reference frame connected with the Earth (E-frame) whereas the calculations are carried out in another frame related to the Sun. But the first power of β figures in the Lorentz transformation that describe the transition between these frames. The value of the discussed effect is of the order of β^2. Therefore, the results of calculations and observations should belong to the same reference frame, for example, the E-frame. Strictly speaking, it is also necessary to take into account the Earth's rotation, in particular, performing the transition to a rotating E'-frame. And only the smallness of the linear velocity of rotation ($\beta_\omega c \approx 10^{-2} \beta c$) allows this effect to be neglected. The value of the "general precession" of a visible displacement of the perihelion, conditioned just by Earth rotation, is 5026", *i.e.*, on the contrary, it is two orders larger than the discussed effect. In general Clemence [17] said well about the difficulties associated with this problem: "The observations cannot be made in the Newtonian frame of reference. They are referred to the moving equinox, that is, they are affected by the precession of the equinoxes, and the determination of the precession motion is one of the most difficult problems of positional astronomy, if not most difficult. In the light of all these hazards, it is not surprising that a difference of opinions could exist regarding the closeness of agreement between the observed and theoretical motions."

Antigravitation. The substitution of $u_a^i = -u^i$ for antiparticles [16] in eq.(8) leads to the change of the force sign:

$$\vec{F}_a = -\vec{F}\,.$$

It means that antiparticles must be repelled by the gravitational field (the phenomenon of antigravitation). It should be noted that there is no place in LCTG for the known difficulty conditioned by the evident ascription of negative gravitational mass to antiparticles of positive inertial mass. It leads to a violation of the equivalence principle.

Observation of this phenomenon is of indubitable interest.

4. Equations of the gravitational field

Calculation of the 4-divergence of the gravitational field tensor (7) with the use of the Poisson relativistic equation (4) and the Lorentz gauge leads us to the first field equation

$$\frac{\partial G^{ik}}{\partial x^k} = \frac{4\pi}{c} G J^i\,. \qquad (11)$$

As we can see, this is like the corresponding Maxwell equation. Certainly, an essential difference is that the right-hand side (11) in case of electrodynamics can change the sign due to a change in the charge sign.

The second equation of the gravitational field is

$$\varepsilon^{ijkl} \frac{\partial G_{kl}}{\partial x^k} = 0\,, \qquad (12)$$

where ε^{ijkl} is Levi-Civita's pseudo-tensor, corresponds, evidently, to another pair of Maxwell's equations.

By analogy with electrodynamics, we suppose

$$G_{ik} = (\vec{E}_g, \vec{H}_g), \qquad G^{ik} = (-\vec{E}_g, \vec{H}_g)$$

and name E_g the strength of the gravitational field, and H_g the strength of cogravitational field, following [3].

5. The energy-momentum tensor

The energy- momentum tensor of the gravitational field takes the known form

$$T^{ik} = \frac{1}{4\pi G}(-G^{il}G_l^k + \frac{1}{4}\eta^{ik}G_{lm}G^{lm}), \tag{13}$$

where η^{ik} is Minkowski's tensor. Note that the space components $T^{\alpha\beta}$ of the energy-momentum tensor form the three-dimensional tensor of the momentum flow density or the "gravitational stress" tensor.

It is easy to show that the calculation of the 4-divergence of the energy-momentum tensor, taking field equations (11) and (12) into account, leads to the equality

$$\frac{\partial T^{ik}}{\partial x^k} = -\frac{1}{c}G^{il}J_l \tag{14}$$

the right-hand side of which becomes zero in the absence of matter, *i.e.*, for a free gravitational field.

Let us take now the kinetic tensor (the energy-momentum tensor of a system of non-interacting particles)

$$\theta^{ik} = (J^i u^k + J^k u^i)/2. \tag{15}$$

Form its 4-divergence. Then, taking into account the continuity equation for "mass current"

$$\partial J^k / \partial x^k = 0 \tag{16}$$

and the relation $J^i = \mu^* u^i$, where μ^* is the proper density of mass, we obtain

$$\frac{\partial \theta^{ik}}{\partial x^k} = \mu^* c \frac{du^i}{d\tau}. \tag{17}$$

Using eq.(8), which, taking the equality $F^i m du^i / d\tau$ into account, is written in the form

$$\frac{du^i}{d\tau} = -\frac{1}{c}G^{ik}u_k \tag{18}$$

we find that the total energy-momentum tensor of the gravitational and matter is conserved:

$$\frac{\partial}{\partial x^k}(T^{ik} + \theta^{ik}) = 0. \tag{19}$$

6. The proportionality principle: the redshift of light frequency

The total energy of a particle with mass M placed in the gravitational field (with the gravitational potential $\Phi = -|\Phi|$) decreases according to (6)

$$E = Mc^2(1 + \Phi / c^2). \tag{20}$$

Let this particle be an atom or another radiator of light. As noted earlier [18], the decrease of the total energy of the radiator must in accordance with the energy law conservation lead to the proportional decrease of the energy (frequency) of radiated light (Here we have a definite analogy with the displacement of spectral lines in the electric field - Stark's effect). Indeed, in the limiting case of the zero total energy, the radiator cannot radiate at all. In other words, for the frequency of light emitted in the gravitational field, we have

$$\nu = a\nu_0 = \nu_0(1 + \Phi / c^2) \tag{21}$$

Here a is the indicated coefficient of proportionality (the ratio of the corresponding total energies of the radiator). As is known, the Pound-Rebka-Snider experiments [10], based on Moessbauer's effect, confirm this formula.

7. The gravitational lengthening of scales

Taking into account that in LCTG the light velocity in the gravitational field does not change, from the formula

$$c_g = \lambda \nu \tag{22}$$

and eq.(21) we have for the wavelength

$$\lambda = \lambda_0 (1 + \Phi / c^2)^{-1} \tag{23}$$

whereas it follows from eq.(2E) that $\lambda_E \cong \lambda_0 (1 + \Phi / c^2)$ in contradiction with the result of Brault's experiment.

However, here we want to pay attention to the following. At the present time, the wavelength of the orange line krypton-86 is in fact assumed to be the standard of length ("microscale"). Therefore, the previous result, in particular, can be interpreted as an increase of microscale in the gravitational field [19]. It should be particularly emphasized that this conclusion is evidently in logical correspondence with the relativistic elongation of moving scales, being the consequence of the concept of covariant (radar) length (see, *e.g.*, [20]).

Conclusion

Since the main GTR statement of the dependence of the light velocity on the gravitational potential contradicts the effect of the gravitational shift, a further development of LCTG is becoming urgent. Its base is Newton's relativistic (4-vector) potential. Only the first possibility of two conceivable variants of the description of the gravitational field strength (by symmetric or antisymmetric tensor of rank 2) satisfies the Lorentz invariance requirement. Thus, we have an "electromagnetic-like" theory of gravity. For this, the vector character of the gravitational field quanta facilitates essentially the solution of the problem of constructing the united theory of all interactions. The frequency decrease of light, radiated in the gravitational field, is the consequence of the proportionality principle. The corresponding increase of the wavelength (microscale) is in logical correspondence with relativistic lengthening of moving scales (following from the concept of the covariant, or radar, length).

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