Anisotropic Velocity Distribution in the Solar Plasma: Solution to the Neutrino Problem?

Emilio S. Corchero* and Enrique Santos-Leal
Departamento de Física
Universidad de Cantabria
Santander

Arguments are given for the existence of a slightly anisotropic velocity distribution of electrons and nuclei in the central region of the Sun. A simple model of the Sun with such local anisotropy is exhibited, containing a single free parameter, which predicts neutrino fluxes in agreement with the observations within two standard deviations.
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1. The Hypothesis of Local Anisotropy in the Sun

A serious discrepancy exists between model predictions of solar neutrino fluxes and the rates observed (Bahcall *et al.* 1996 a). In particular the measurements show that the rate of ^{8}B neutrinos is less than half the prediction of the standard solar model (Bahcall and Pinsonneault 1995) and the ratio between the observed and the predicted rates of ^{7}Be neutrinos seems to be still smaller. A possible solution to the problem is an extension of standard electroweak theory in which neutrinos have small masses and lepton flavor is not conserved. But recent experiments, which seem to imply an oscillation between muon and tau neutrinos, do not give any evidence for oscillations involving solar (electron) neutrinos (Fukuda *et al.* 1998). Consequently the question whether the solar neutrino problem can be solved by changing the solar model is pertinent.

In this article we give arguments supporting the possibility of an astrophysical solution to the solar neutrino problem. Specifically we argue that: a) the plasma near the center of the Sun probably possess a slightly anisotropic velocity distribution, and b) this fact may lead to neutrino fluxes in agreement with observations. In this section we present a heuristic point of view about the whole problem. The argument is presented in more detail in section 2, where some properties of an anisotropic plasma are summarized. In section 3 we describe a solar model which predicts neutrino fluxes calculated in section 4. Finally in section 5 we discuss the reliability of our model, specially in relation with recent measurements of helioseismology.

Before giving the arguments for an anisotropic velocity distribution in the solar plama, we review the standard theory (that is, with local isotropy) of the Sun (Kippenhahn and Weigert 1990). A solar model involves the solution of a set of coupled nonlinear differential equations, namely the equation of gravitational hydrostatic equilibrium plus the equations for the production and transport of energy. Except in the convective (external) region of the Sun they may be written (with units such that $G = c = 1$)

* Address: E. Santos Corchero, Facultad de Ciencias, Avenida de los Castros *s/n*, 39005 Santander. Spain. FAX nº 34-42-201402; Email <santose@besaya.unican.es>

$$\frac{\mathrm{d}r(r)}{\mathrm{d}r} = -\frac{m\rho}{r^2}, \tag{1}$$

$$m(r) = 4\pi \int_0^r \rho r^2 \mathrm{d}r , \tag{2}$$

$$\frac{\mathrm{d}T(r)}{\mathrm{d}r} = -\frac{3\kappa\rho L}{16\pi a r^2 T^3} \tag{3}$$

$$L(r) = 4\pi \int_0^r \rho\varepsilon r^2 \mathrm{d}r , \tag{4}$$

where a is the Stefan-Boltzmann constant, $\rho(r)$ is the density at the distance r from the center and $T(r)$ the temperature. The emissivity ε (power per unit mass produced in the hydrogen burning) and the opacity κ are complicated functions of the chemical composition, the density and the temperature. The pressure, p, is essentially due to the plasma (*i.e.* the radiation pressure is negligible) and may be related to the temperature by the ideal gas law

$$p = \rho k_B T(\mu m_H)^{-1}, \tag{5}$$

where m_H is the mass of the hydrogen atom (more properly the mean mass of a nucleon plus an electron) and μ is the mean molecular weight defined by

$$\mu^{-1} = \sum \frac{\chi_j m_H}{m_j} \cong 2\chi + \tfrac{3}{4}(1-\chi) . \tag{6}$$

where m_j (χ_j) is the mass (mass fraction) of the species j of particles. In the last expression we have labelled χ the mass fraction of protons and neglected the fraction of metals (*i.e.* nuclei with $A > 4$). Actually the set of eqs. (1) to (4) is not complete and we should add the equations involving the gradients of χ_j, which determine the spatial distribution of nuclear species. But we shall ignore them for the sake of simplicity, that is we shall take the chemical composition as given.

Eqs. (1) to (6) have a unique solution for a given mass of the star provided that the chemical composition (*i.e.* the functions $\chi_j(r)$) and the expressions of ε and κ are known. For instance, if we exclude the external (convective) region of the Sun, where eq. (3) is not valid, and the most internal region, where the emissivity ε is relevant, we may write

$$\varepsilon \approx 0, \;\; \kappa \approx C\rho T^{-7/2}, \tag{7}$$

where C is a function of the chemical composition. If we simplify the model by assuming that the mass fractions, χ_j, do not depend on r, then inserting eqs. (5) and (7) in eqs. (1) to (4) it is easy to check that the solution is

$$\rho = \text{const.} \;\; T^n = \text{const.}\, r^s , \tag{8}$$

with $n(s)$ slightly above 3 (below 1). The first eq. (8), toghether with (6), shows that the Sun has a density distribution close to a polytrope with index $n = 3$.

Our criticism of the standard solar model (SSM, Bahcall and Pinsonneault 1995) derives from the fact that eqs. (1) to (4) are not the most general equation of stellar structure, because they involve the constraint of local isotropy. In particular, the most general equation of hydrostatic equilibrium for spherical symmetry is (see, *e.g.*, Bowers and Liang 1974, or Herrera and Santos 1997)

$$\frac{\mathrm{d}p_r}{\mathrm{d}r} + 2\frac{p_r - p_t}{r} = -\frac{m\rho}{r^2}, \tag{9}$$

which allows for an anisotropic stress tensor. That is, the radial pressure, p_r, may be different from the transverse pressure, p_t. The choice of one particular instance of eq. (9), namely eq. (1), in the SSM rest upon the *unproven hypothesis* that the solar plasma is isotropic everywhere. The standard argument for the full isotropy of the velocity distribution is that Coulomb collisions redistribute the directions of the plasma particles (nuclei and electrons) in characteristic times of order 10^{-12} sec., which is much less than the characteristic times for nuclear reactions, 10^{16} sec. During that small time interval the direction of the velocity of a particle may change by an angle of order $\pi/2$ and therefore the directions are rapidly randomized. What this argument proves is that randomization is a very rapid process, but it does not tell us what is the stationary velocity distribution of the particles. Intuitively randomness suggests isotropy, but the intuition alone may be misleading. The correct choice between local isotropy or some degree of anisotropy should be based on fundamental principles of physics rather than on intuition. Actually the rigorous measure of randomness is the *entropy,* so that randomization precisely means increase of the entropy. Consequently the correct model of the Sun will involve finding the configuration of maximum entropy compatible with the appropriate structure equations, that is equations which generalize eqs. (1) to (4) with allowance for local anisotropy. In particular, eq. (9) should be substituted for eq. (1) (a similar theory of star structure has been shown to predict local anisotropy in relativistic stars like white dwarfs (Corchero 1998a) and supermassive stars (Corchero 1998b)).

A well-known fact, which we shall illustrate with a simple example in the next section, is that the entropy of a mass of ideal gas is a maximum, for given average density and temperature, if the gas is homogeneous and isotropic. But in the solar plasma there is a trade-off between isotropy and homogeneity. In fact, a local anisotropy such that the transverse pressure is greater than the radial pressure, *i.e.* $p_t > p_r$, reduces the pressure gradient needed to counterbalance the gravitational force [see eq. (9)]. This will lead to a more flat density profile, with a smaller density at the center of the Sun but possibly a greater density outside that region. The homogenization of the density, with the same average, tends to rise the entropy and this may compensate for the entropy lowering caused by the local anisotropy.

This mechanism will modify the prediction of neutrino fluxes in the correct direction to improve the agreement with observations. In fact, as the temperature is an increasing function of the density, we arrive at a solar model with smaller temperature than the SSM at the center but greater at other places, say from $r \approx 0.1\ R_o$ outwards (this value follows from the estimate to be made in section 3). As more than 80% of the ^{8}B and ^{7}Be neutrinos are produced inside that region, their fluxes will be lowered. In contrast, the flux of pp neutrinos and the luminosity, which are produced within a much greater region may not change. Indeed, only 45% of the luminosity is produced inside a sphere of radius $0.1\ R_o$ (Bahcall and Pinsonneault 1995).

2. Properties of a Plasma with Anisotropy

If we do not assume local isotropy *a priori*, the determination of the structure of the Sun will require the knowledge of the phase-space distribution functions, $f_j(\mathbf{r},\mathbf{v})$, of the constituent particles, essentially electrons, protons and helium nuclei. In the SSM it is supposed that the velocity dependence is given by the Boltzmann distribution, but here we shall assume that the actual distribution corresponds to the maximum of the entropy

$$S = -k_B \sum_i \int f_j \log f_j \, d^3\mathbf{r}\, d^3\mathbf{v}\,, \tag{10}$$

where k_B is Boltzmann´*s* constant and we normalize f_j so that its integral over phase space gives the total number of particles of species j. We shall study the implications of the fact that the functions f_j depend on the velocity vector, $\mathbf{v}$, rather than only on its modulus, v. Actually, the spherical symmetry of our solar model implies that f_j depends only on r, v and the angle θ between the velocity and the radial direction.

We shall consider small local anisotropies so that we may use slightly distorted Boltzmann distributions of the form

$$f_j(\mathbf{r},\mathbf{v}) = C_j n_j \exp\left(-\frac{m_j \mathbf{v}^2}{2k_B \Theta y}\right),\ y = 1 + x\left(1-3u^2\right),\ u = \cos\theta\,, \tag{11}$$

where m_j is the mass of the particles j, $n_j(r)$ the number density of these particles and $C_j(r)$ a normalization factor. All the radial dependence goes on the parameters $\Theta(r)$ and $x(r)$, the former related to the mean temperature (see below eq. (14)) and the latter, which we assume small, measuring the fractional anisotropy. The SSM corresponds to the choice $x = 0$.

Before going to the problem of maximizing the entropy we shall derive some useful properties of an anisotropic plasma. The local moments of the kinetic energy distribution are the same for all particle species and we get

$$\left\langle E^n \right\rangle = \int \left(\tfrac{1}{2} m v^2\right)^n f(\mathbf{v}) \mathrm{d}^3\mathbf{v} = \frac{\left\langle E^n \right\rangle_{x=0} I_n(x)}{I_0(x)}\,, \tag{12}$$

$$\left\langle E^n \right\rangle_{x=0} = (2n+1)!!\left(\tfrac{1}{2}k_B\Theta\right)^n,\ I_n(x) = \int_0^1 y^{n+3/2}\mathrm{d}u\,, \tag{13}$$

where $f(\mathbf{v})$ is normalized to unity. For n=1 eq. (12) gives the mean kinetic energy, which allows defining the mean local temperature, T, as follows

$$T = \frac{2\langle E\rangle}{3k_B} = \frac{\Theta I_1(x)}{I_0(x)} \cong \left(1+\tfrac{6}{5}x^2\right)\Theta + O\left(x^3\right). \tag{14}$$

Hence we may calculate the local moments of the energy distribution in terms of the mean temperature and we obtain

$$\begin{aligned} &\left\langle E_n \right\rangle = (2n+1)!!\left(\tfrac{1}{2}k_B\right)^n T^n F_n(x), \\ &F_n(x) = \frac{I_n(x) I_0(x)^{n-1}}{I_1(x)^n} \sim \int_0^1 \exp\left[nx\left(1-3u^2\right)\right]\mathrm{d}u \approx 0.5117\,(nx)^{-\frac{1}{2}}\exp(nx), \end{aligned} \tag{15}$$

where the departure of the function $F_n(x)$ from unity measures the effect of the local anisotropy. The assymptotic expression of F_n(x) is valid in the limit $n \to \infty$ with nx finite; the last equality requires, in addition, $nx \gg 1$. Eq. (15) will be used in the study of neutrino fluxes of the Sun.

It is convenient to work with the mass density, ρ, and the mass fractions, χ_j, instead of the number densities, n_j, of the different species of particles, the relations being

$$\rho = \sum m_1 n_1,\ \ \chi_j = \frac{m_j n_j}{\rho}. \tag{16}$$

Now we may calculate the radial pressure, p_r (in the direction θ= 0), and the transverse pressure, p_t (in the directions $\theta = \pi/2$). We get

$$p_r = \rho\sum \chi_j \left\langle v_1^2 \cos^2\theta \right\rangle = \rho k_B T\left(\mu m_H\right)^{-1}\left(1-2x\right) + O\left(x^2\right), \tag{17}$$

$$p_t = \rho\sum \chi_j \left\langle \tfrac{1}{2} v_1^2 \sin^2\theta \right\rangle = \rho k_B T\left(\mu m_H\right)^{-1}\left(1+x\right) + O\left(x^2\right). \tag{18}$$

The mean pressure is given by $1/3p_r + 2/3p_t$, which agrees with eq. (5).

Now we may write the entropy in the form of a modified Sackur-Tetrode formula. We shall insert eq. (11) into (10) and perform the integral over velocities, taking eq. (14) into account in order to eliminate the function Θ. We get

$$S = S_0 + \tfrac{1}{2} k_B \sum_j \int n_1 \left[\log\left(\frac{T^3}{n_1^2} \right) - 3x^2 \right] \mathrm{d}^3\mathbf{r} + O\left(x^3\right). \tag{19}$$

Hence, using eqs. (6) and (17) we obtain

$$S = \frac{S_0 + \frac{1}{2} k_B}{\mu m_H} \int \left[\log\left(\frac{T^3}{\rho^2} \right) - 3x^2 \right] \mathrm{d}m - k_B \int \sum_j \left(\frac{\chi_j}{m_j} \right) \log\left(\frac{\chi_j}{m_j} \right) \mathrm{d}m + O\left(x^3\right), \tag{20}$$

where dm is the mass element as defined in eq. (2). The maximum of (20) for fixed $\rho(r)$, $T(r)$ and $\chi_j(r)$ gives obviously $x = 0$, that is a locally isotropic plasma. But we should not maximize eq. (20) with ρ, T and χ_j fixed, but with the weaker constraint posed by the structure equations of the Sun which, in particular, includes eq. (9) rather than eq. (1) . This will require a quite involved calculation which will not be made here. But it is not difficult to see the trade-off between anistropy and inhomogeneity mentioned above. If, for the sake of simplicity, we assume a fixed chemical compositon (that is we fix the mass fractions, χ_j, as functions of the variable m defined in eq. (2)) and we also assume the validity of the first eq. (8) even in the presence of anisotropy, then eq. (20) becomes

$$S - S_0 = -A \int \rho \log \rho r^2 \mathrm{d}r - B \int x^2 \rho r^2 \mathrm{d}r , \tag{21}$$

with both constants A and B positive. We see that the second term tends to isotropize whilst the first one tends to homogenize the density function $\rho(r)$. In fact, the maximum of the second term of (22) happens obviously for the isotropic case $x = 0$. On the other hand, the maximum of the first term corresponds to the minimum average value of log ρ, which is ρ = constant. We see that the trade-off between isotropy and homogeneity, derived from eq. (9), will very likely induce an anisotropy of the plasma velocity distribution in order to make the density more homogeneous.

3. Locally Anisotropic Model of the Sun

Our aim is now to estimate the change in model predictions, in particular for neutrino fluxes, when we pass from the SSM to the solar model with local anisotropy which would be obtained by solving the general structure equations (*e.g.* eq. (9) substituted for eq. (1)) and maximization of the entropy as said above. We shall get a rough estimate by constructing two simplified solar models, one with local isotropy (LISM) and another one locally anisotropic (LASM), and comparing the predictions of both. A simple, although not very accurate, LISM may be obtained taking the Sun as an n=3 polytrope and assuming homogeneous chemical composition. Then the radial coordinate and the density may be written in terms of the Lane-Emden variables, θ and ξ, as follows

$$r = \frac{R_o \xi}{\xi_1}, \; \xi_1 = 6.89, \; \rho = \rho_C \theta^3 , \tag{22}$$

where R_o is the radius of the Sun. With an appropriate value for the central density, ρ_C, the LISM may give the correct (observed) mass. The density (22) is the solution of eqs. (1) and (2) if the pressure is related to the density by the first eq. (8). In the following we shall use units $\rho_C = G = c = 1$, ρ_C being the central density in the LISM.

In order to define our LASM, we firstly assume that local anisotropy will not produce a dramatic change so that the first eq. (8) is still valid, but the density is different from the one given by eq. (22).

Then eq. (9) may be formally solved in order to get the anisotropy parameter, x, and we obtain (neglecting terms of order x^2)

$$x(r) = \left[2r^3 p(r)\right]^{-1} \int_0^r \left[r^2 \frac{\mathrm{d}\rho}{\mathrm{d}r} + m\rho \right] r\,\mathrm{d}r \,. \tag{23}$$

At first glance this equation has a solution for an arbitrary density $\rho(r)$, but there is a constraint because the pressure, p, should vanish at the surface of the Sun and the integral in eq. (23) should also vanish when $r \to R_0$. As our aim here is just to explore whether local anisotropic models have enough flexibility to possibly solve the solar neutrino problem, we shall ignore that requirement (in any case we are not concerned with the most external region of the Sun), and assume that the density (in dimensionless, Lane-Emden, variables) is

$$\rho = \rho_0 - \rho_1, \;\; \rho_0 = \theta^2, \;\; \rho_1 = \alpha\theta^3 \left(1 - \gamma\xi^2\right) \exp\left(-\beta\xi^2\right). \tag{24}$$

With this choice we get from eq. (23)

$$x(\xi) = \eta\xi^2 + O\left(\xi^4\right), \; \eta = \tfrac{1}{15}\left[2(1-\alpha)^{2/3} - 1 + 2\alpha \frac{(\beta+\gamma)}{1-\alpha} \right]. \tag{25}$$

The parameters α, β and γ should be so chosen that the solar mass, radius and luminosity of our LASM agree with those of our LISM. We shall see that this condition fixes 2 out of the 3 parameters and only 1 free parameter remains.

The condition on the mass and radius is

$$\int_0^{\xi_1} \rho_1 \xi^2 \mathrm{d}\xi = 0 \,. \tag{26}$$

In order to implement the condition on the luminosity, L_o, we assume that it may be given in terms of the density and temperature by eq. (4) with

$$\varepsilon = \text{const.}\;\; \rho(r) T(r)^3 = C\rho(r)^2 \,, \tag{27}$$

where C is a constant and the first eq. (8) has been taken into account. For this calculation we may take $F_n \cong 1$ (see eq. (15)). The linear dependence of ε on ρ in eq. (27) follows from the fact that the main source of energy in the Sun is the nuclear reaction chain ppI, which produces a power per unit mass proportional to the proton density. The cubic dependence on temperature follows from a fit to the predictions of the SSM for the nuclear power per unit mass *versus* temperature. Actually the dependence of the nuclear emissivity on the density, the mass fraction of protons, and the temperature in the SSM may be roughly approximated by

$$\varepsilon = C\rho(r) T(r)^{\lambda} \chi(r)^{\nu} \,, \tag{28}$$

where $\nu \approx 2$ and the parameter λ is close to 5.5 at the center of the Sun decreasing slowly outwards (see, *e.g.*, Bahcall & Pinsonneault 1995). Also, the product χT is roughly constant in the central region of the Sun (actually it first slightly increases and then decreases with the result that the value at $r \approx 0.27\, R_o$ is the same as in the center). This justifies our simple choice eq. (27), where we simulate the χ dependence by lowering the T dependence. The constant C should be chosen so that, when eq. (27) is used in combination with the SSM density and temperature, we get the observed luminosity of the Sun. However, the actual value of C is not needed in our calculation. All we need is that the luminosity is the same in LASM as in LISM, which gives

$$\int_0^{\xi_1} \left[\left(\rho_0 - \rho_1\right)^3 - \rho_0^3 \right] \xi^2 \mathrm{d}\xi = 0 \,. \tag{29}$$

Eqs. (26) and (29) yield the values of two of the model parameters in terms or the third one. For instance, a numerical calculation gives $\beta = 0.4$, $\gamma = 0.5$ for $\alpha = 0.2$ and $\beta = 0.5$, $\gamma = 0.6$ for $\alpha = 0.3$.

4. The Flux of Neutrinos

As is well known the flux of pp neutrinos is almost model independent once the luminosity is fixed. Also, the SSM prediction for that flux agrees well with the observations (Bahcall *et al.* 1996 a). However, as said in the introduction, the observed ^{8}B neutrino flux, $\phi(^8\mathrm{B})$, is less than half the SSM prediction, and the predicted ratio $\phi(^7\mathrm{Be})/\phi(^8\mathrm{B})$ is higher than the observed one. Here we shall calculate the ratios $\phi^{\mathrm{LASM}}/\phi^{\mathrm{LISM}}$ and show that the problem is alleviated if we allow for local anisotropy.

Neutrino fluxes should be calculated from the integral

$$\phi = \mathrm{const.} \int \rho \langle T^n \rangle \mathrm{d}^3\mathbf{r}\,, \tag{30}$$

if the flux depends of the nth power of the temperature. Here we propose a linear dependence on density, in contrast with the quadratic dependence of eq. (27), because it is more appropriate for the chain of nuclear reactions leading to ^{7}Be and ^{8}B. It is known that $n \approx 10$ for ^{7}Be neutrinos and $n \approx 24$ for ^{8}B neutrinos (Bahcall & Ulmer 1996). In a locally anisotropic medium, the dependence on the local nth moment of the temperature, $\langle T^n \rangle$, must be interpreted as a dependence on the $2n$th power of the relative velocities of the colliding nuclei, which means a dependence on $\langle E^n \rangle$ (see eq. (15)). We get

$$\begin{aligned} \phi^{LASM} &= \mathrm{const.}\int_0^{\xi_1} \langle E^n \rangle (\rho_0 - \rho_1) = \mathrm{const.}\int_0^{\xi_1} T(r)^n F_n(x)(\rho_0 - \rho_1)\xi^2 \mathrm{d}\xi \\ &= D\int_0^{\xi_1} (\rho_0 - \rho_1)^{1+n/3} F_n(x)\xi^2 \mathrm{d}\xi \end{aligned}\,, \tag{31}$$

where the first eq. (8) and eq. (15) have been used. The value of the constant D is not needed because we want only the ratio $\phi^{\mathrm{LASM}}/\phi^{\mathrm{LISM}}$, where

$$\phi^{LISM} = D\int_0^{\xi_1} \rho_0^{1+\frac{n}{3}} \xi^2 \mathrm{d}\xi\,. \tag{32}$$

The neutrino flux predictions of LASM may be easily derived from eq. (31). For small α a decrease, with respect to the LISM, of the ^{8}B neutrino flux is predicted much stronger than the decrease of the ^{7}Be neutrino flux. This is in disagreement with the observations, but the situation is not so bad for higher α (although for $\alpha > 0.31$ our model predicts a density increasing outwards at some points, which is unphysical). If $\alpha = 0.31$, which corresponds to about 12% decrease in the central temperature of the Sun, the density of the central region may be approximated by

$$\rho \approx (1-\alpha)\exp(-\sigma\xi^4),\ \sigma \cong 0.26. \tag{33}$$

Hence, using eqs. (31) and (32) we get

$$\frac{\phi^{LASM}}{\phi^{LISM}} = 0.71(1-\alpha)^{\frac{n}{3}+1}\left(\frac{n}{3}+1\right)^{3/4} G(z),\ z = \left[3\eta^2 \frac{n^2}{(n+3)}\sigma\right]^{1/2}, \tag{34}$$

where $\eta \cong 0.104$ in this case (see eq. (25) for the definition of η). The presence of the function $F_n(x)$ in eq. (31), which leads to $G(z)$ in eq. (34), produces a decrease of the ratio $\phi(^7\mathrm{Be})/\phi(^8\mathrm{B})$ less rapid than it should be expected from the decrease in the central temperature of the Sun. In fact, for ^{7}Be neutrinos $z = 0.98$ and $G(z) = 1.3$, whilst for ^{8}B neutrinos $z = 1.63$ and $G(z) = 2.1$. Nevertheless the effect is smaller than needed and we obtain

$$\phi^{LASM}\left({}^7\mathrm{Be}\right) \approx 0.51\phi^{LISM}\left({}^7\mathrm{Be}\right),\ \phi^{LASM}\left({}^8\mathrm{B}\right) \approx 0.25\phi^{LISM}\left({}^8\mathrm{B}\right).$$

There is an additional effect contributing to increase the ^{8}B neutrino flux, namely the rise in the mass fraction of hydrogen, χ, at the center of the Sun to be discussed in the next section. In fact, the rate of the reaction chain $^7\mathrm{Be} + \mathrm{p} \rightarrow {}^8\mathrm{Be} \rightarrow {}^8\mathrm{B} + \nu$ is proportional to χ whilst the rate of the competing reaction $^7\mathrm{Be} + \mathrm{e}^- \rightarrow {}^7\mathrm{Li} + \nu$ depends but slightly on χ (it is proportional to $1 + \chi$). This may rise the ^{8}B neutrino flux by about 20% relative to the ^{7}Be neutrino flux and we get finally

$$\phi^{LASM}\left({}^7\mathrm{Be}\right) \approx 0.51\phi^{LISM}\left({}^7\mathrm{Be}\right),\ \phi^{LASM}\left({}^8\mathrm{B}\right) \approx 0.30\phi^{LISM}\left({}^8\mathrm{B}\right).$$

This result is only in rough agreement with observations (within about 2 standard deviations, see Bahcall *et al.* 1996 a). But our model is too crude and we would not expect a better agreement.

5. The Problem of Helioseismology

An argument against any substantial change of the SSM has been given recently resting upon the results of helioseismology (Bahcall *et al.* 1997). Indeed an excellent agreement exists between the SSM predicted and the observed values of the velocity of sound in the central region of the Sun. We think that the argument does not rule out models with local anisotropy because the expected change in sound velocity will be probably very small. In fact, the sound speed is given by

$$c^2 \propto \frac{T}{\mu} \propto T(\chi + 0.6), \tag{35}$$

where T is the temperature and μ the mean molecular weight, which is related to the mass fraction of hydrogen, χ, by eq. (19). We may assume that the decrease in χ from the primordial value $\chi_0 \cong 0.73$, at every point inside the Sun is roughly proportional to the nuclear emissivity. The local emissivity is related to the temperature by eq. (27) with $\lambda \approx 5$ and $\rho \propto T^3$, that is

$$\chi_0 - \chi \propto \chi^2 T^8. \tag{36}$$

Now assuming that the relation $\rho \propto T^3$ holds for the modified model, from eqs. (35) and (36) we get

$$\begin{aligned} \frac{\delta c}{c} &= \frac{1}{2}\left[\frac{\delta T}{T} + \frac{\delta\chi}{(\chi + 0.6)}\right] = \frac{1}{2}\frac{\delta T}{T}(1-\kappa), \\ \kappa &= 8\chi\frac{\chi_0 - \chi}{\left[(\chi + 0.6)(2\chi_0 - \chi)\right]}. \end{aligned} \tag{37}$$

The interesting feature is that the parameter κ is remarkably close to 1 throughout the central region of the Sun. For instance, if $\chi \cong 0.33$ (at the center) we get $\kappa = 1.015$, and if $\chi \cong 0.42$ (at 0.05 R_o) $\kappa = 0.982$. This means that the change in mean molecular weight almost counterbalances the effect of the change in temperature with the result that the speed of sound changes very little with the proposed modification of the SSM. For instance a 7% decrease in T_C gives only 0.05% increase (at the center) or 0.06% decrease (at 0.05 R_o) in the sound speed, although the hydrogen mass fraction increases by almost 20% in both places.

6. Conclusions

We conclude that: 1) There is probably a local anisotropy in the central region of the Sun making the transverse pressure greater than the radial pressure, 2) If this is true, the predictions for the neutrino fluxes may be compatible with the observations.

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