Validity of Quantum Wiggler Electrodynamics Based on Analysis of the First Free-Electron Laser and Radiation in the First Smith-Purcell Configuration

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It is shown that the measured power of the induced radiation by the externally-launched laser field in first free-electron laser (FEL) experiment is much larger than the work against the laser field per unit time which would be done by all the electrons in the cavity even under the assumption that they are bunched into a single position of the phase at which a point charge can work at maximum. It is shown that the power of net stimulated emission due to the magnetic Wiggler is too small to account for the measured power, but the power of net stimulated free-electron two-quantum Stark (FETQS) emission driven by the transversely wiggling motion in the electrostatic wave generated by the same transversely wiggling motion and the radial density gradient can be large as the measured power. The wavelength of the FETQS emission is the same as the measured one. In order to prove that FETQS emission takes place and is extraordinarily strong, we make a scrutiny into the so-called Smith-Purcell (SP) radiation, which is another example of FETQS emission. It is shown that the wavelength of any emission conceived in the classical concept can not match that of the experimentally observed SP radiation, and the power of the measured SP radiation is 10^{14} times larger than that of any emission directly from the electrons in any equivalent condition that can be conceived based on the Liénard-Wichert potentials or quantum electrodynamics before the advent of QWD. We identify the SP radiation with FETQS emission driven by the axial motion in the axial component of the electric field from the surface charge induced by the electron itself. The power of the FETQS emission calculated by taking into account a fact that the radius of the electron beam in the SP experiment is about 50 times the grating period well agrees with the experimentally estimated power. Both the observed radiation and the FETQS emission have the same wavelength. Since the wavelengths and powers of both the first FEL and the SP radiation agree with those of the FETQS emissions, we conclude that the recently founded Quantum Wiggler Electrodynamics which gives reasons for the occurrence of FETQS emission is valid.

1. Introduction

In this paper, a Wiggler means a classical oscillating field whose wave vector is parallel to a particle beam, and whose potential amplitude in terms of the particle energy is far less than the axial kinetic energy. Quantum Electrodynamics (QED) means the ordinary quantum electrodynamics before the advent of Quantum Wiggler Electrodynamics (QWD) (Kim 1994c). Both the ordinary QED and QWD are quantum electrodynamics in the broad meaning.

We cannot fully describe vast phenomena in various areas such as emissions in various Wigglers with only the standard universal quantum-mechanical principles. Therefore, we have been using various quantum-mechanical approaches such as atomic spectroscopy, QED, and QWD, which not only include the standard universal quantum mechanical principles, but also adopt even some laws pertinent only to the phenomena in a particular region. The recent identification of the so-called Smith-Purcell radiation, which is defined as the observed emission in the Smith-Purcell type configurations (Smith and Purcell 1953; Doucas *et al.* 1992), as free-electron two-quantum Stark (FETQS) emission driven by macroscopic axial motion, which is many orders of magnitude (*e.g.*, 10^{19} times) stronger than any emission conceivable with classical or quantum electrodynamics in an equivalent condition (Kim 1993b, c, 1994a), and the discoveries of an emission in an electron-cyclotron maser (ECM) configurations, which is 10^9 times stronger than Larmor radiation from gyration (Kim 1994c), and of a radiative reaction force acting on an electron which is stronger than the Lorentz force driving the electron emit the Smith-Purcell radiation (Kim 1994a), have led to the formation of QWD which is a branch of QED in the broad meaning to describe free-electron emission in Wigglers (Kim 1994c). FETQS emission should take place if a law of QWD (Kim 1994c) which can be stated as that the wave function phase of the final state of a transition route involving an interaction with a Wiggler is random so that the transition-probability amplitudes of the two-transition routes of a two-quantum process do not cancel each other, is valid (Kim 1993d, 1994a). Raman scattering of a laser light by a plasma electron in a plasma electric field is a two-quantum process in which only the scattered radiation and the plasma field act as a quantum and a Wiggler, respectively, in contrast to that the incident laser wave does not act as a first-order perturbing field, but as a prime (or zero-order) field; Raman scattering of a laser light by a free electron can be viewed as FETQS emission in the presence of the laser field. Thus, Forward Raman scattering (FRS) can be observable if the third law of QWD is valid. On the contrary, if the ordinary QED concept on the wave function phase is valid, *i.e.*, if the above two transition-probability amplitudes subtract each other, FRS cannot be exhibited by plasma electrons. Since experiments (Kitagawa *et al.* 1992) show that FRS can be exhibited by plasma electrons, we confirm that the third law of QWD is valid.

As mentioned above, FETQS emission cannot be explained even with the pre-QWD quantum mechanical concept, still less with the concepts would-be or truly based on classical electrodynamics. Against this, one may raise an argument that free-electron emission in any Wiggler has been well explained by the classical FEL theory (Marshall 1985); such concept has been prevailing for last 10 years or so. Here, we first dismiss this concept as nothing but a groundless non-scientific faith by revealing that there has not been even a single published claim in reality that a quantity calculated with the classical FEL theory (Marshall 1985) agrees with the measured quantity in the first FEL experiment (Elias 1976). In the author's knowledge, so far there has been only the functional shape of the gain *vs.* the electron energy without showing the units of the gain and the electron energy, which resembles the shape of the measured gain function. The proponents of the classical FEL theory who wish that the results obtained with the classical FEL theory would agree with the experimental ones have simply believed that the results obtained with the classical FEL theory really agree with the experimental ones on the account that the above gain functions resemble each other in shape. Even though both resemble in shape, the gain function calculated with the classical FEL theory is far different from the measured gain function in magnitude. We will indeed show that the measured power, accordingly the magnitude of the gain, of the first FEL (Elias 1976) can not be accounted for with the classical FEL theory. Further, we will show that any classical concept cannot explain the first FEL unless the Lorentz force equation is revised. Then, we will show that in the present knowledge there is no other way to explain the first FEL than to identify lasing in the first FEL with an FETQS emission in the electrostatic wave generated within the electron beam by the external magnetic Wiggler. This is because the measured power is far larger than both the power obtained with the classical FEL theory and the power of net stimulated magnetic *bremsstrahlung* calculated with quantum mechanics, but can be in the range of the estimated power of the above FETQS emission.

It is easier to study FETQS emission with the SP radiation than with the first FEL because the electric Wiggler is generated by a simpler process in the SP configuration than in the first FEL. For this reason we will study the SP radiation with the intention to find whether the SP radiation can be accounted for with classical electrodynamics, and whether the power of FETQS emission agrees with the measured power of the SP radiation.

2. Proof of the Invalidity of the Classical Free-Electron Laser (FEL) Theory with the Measured Radiation Power of the First FEL

In this section, we investigate the classical FEL concept with the experimental data of the first FEL (Elias *et al.* 1976). We start it with brief review of the classical FEL theory (Marshall 1985).

In classical electrodynamics, radiation is uniquely determined by velocity and acceleration. The power of radiation is expressed by Liénard formula which is the formula classically derived from Maxwell's equations (Jackson 1975). It is illogical if there is another emission whose power can not be expressed by Liénard formula in the framework of classical electrodynamics. In the FEL, velocity and acceleration does not change significantly even if a laser wave whose wavelength is the same as that of the emission in the absence of the laser wave, *i.e.*, spontaneous emission, is externally launched. Thus, in classical electrodynamics the radiation in the presence of such a laser field, *i.e.*, induced emission by such a laser field, cannot be many-orders times stronger than spontaneous emission. However, in the first FEL (Elias *et al.* 1976) induced emission by an externally launched laser field is 10^9 times stronger than spontaneous emission. Thus, the classical FEL theory (Marshall 1985), which claims that it describes classically such induced emission, cannot be classical electrodynamics and must be to misunderstand some classical laws. We will pinpoint where the classical FEL theory is wrong.

The classical FEL theory (Marshall 1985) is stated as follows:

(a) The energy amount of the work done against the laser field by a lasing electron is entirely transformed to the laser field energy by some mechanism which is not identified by the concept, and the energy amount of the work received from the laser field is entirely transformed to the electron kinetic energy.

(b) No matter how strong is laser radiation from an electron, the reaction force due to the laser radiation is ignored, and phase bunching occurs only by the Lorentz forces of the laser field and the magnetic Wiggler.

(c) No matter how compactly are the electrons bunched, the EM waves emitted from the bunched electrons add incoherently.

(d) The laser wave conserves its coherence. There should be a laser wave at the inlet, that is, a laser wave must be injected into the cavity, or the classical FEL theory applies after the laser wave is sufficiently built.

The claim (c) is consistent with the claim (a). This is because the claim (a) states that no matter how compactly are the electrons bunched, the increase in the laser field energy is equal to the sum of the works done by the individual electrons, and the works do not add coherently.

The radiative-reaction force should not be ignored in the classical thinking. The reason is as follows.

The work done by an electron against the electric field of the laser field is emitted as laser radiation which propagates in the positive axial direction. From Newton's third law of motion (the law of action and reaction) and that the work done against a force f is expressed as $W = -\mathbf{F} \cdot d\mathbf{r}$, the magnitude of the reactive force f_{rad} acting on the electron due to the laser radiation is given by

$$F_{rad} = e\vec{\mathcal{E}} \cdot \frac{\mathbf{v}}{v_z \hat{z}} \sim e\mathcal{E}_l \vec{z}\ , \tag{1}$$

where $\vec{\mathcal{E}}$ is the laser electric field and the fact $v \approx v_z$ in the FEL is used. $\mathcal{E}_l \cdot \mathbf{v}$ is dependent on the electron phase with respect to the laser wave. Since the laser field forces the electrons bunched in the classical FEL theory, the radiative-reaction force whose magnitude is comparable to that of the laser field and depend on the electron phase with respect to the laser wave should not be ignored in the bunching calculation. This means that the classical FEL theory is not even the classical thinking.

Now we will reveal how the claim (a) is made. Let the laser electric field be expressed as

$$\vec{\mathcal{E}}_\ell(\mathbf{r}, t) = \mathcal{E}_o \cos(k_\ell z - \omega t)\hat{x}\ , \tag{2}$$

and the vector potential of the magnetic field as

$$\mathbf{A}_m(r, t) = A_{mo} \cos(k_m z)\hat{x}\ . \tag{3}$$

where $k_m = 2\pi / \lambda_m$ and $A_{mo} = B_o / k_m$ with B_o and λ_m being the amplitude of the magnetic induction and wavelength of the magnetic Wiggler, respectively. Then, the transverse velocity of an electron is given by (Kim 1992a)

$$\mathbf{v}_\perp(t) \cong \frac{eA_{mo} \cos(k_m v_z t + \phi)}{\gamma mc}\hat{x}\quad , \tag{4}$$

where $\gamma = E / mc^2$ is the Lorentz factor of the electron and ϕ is the electron phase with respect to the laser wave which is very slowly varying compared to the Wiggler and the laser field. From eqs. (2) and (4), we find that the electron transverse velocity is phase-locked with the laser field if the z-velocity is equal to the phase velocity v_p of the so-called ponderomotive wave which is given by $v_p = \omega / (k_\ell + k_m)$. The lasing electron means the electron whose z-velocity is equal to v_p. Since a lasing electron continuously works against or is worked by the laser electric field, a lasing electron is similar to an electron in a uniform electric field. First, let us assume that an electron with an initial energy of 1 MeV travels in the direction anti-parallel to a uniform electric field of 100 V / cm by 1 cm. Then, the electron obviously loses its kinetic energy by 100 eV. In the classical FEL theory, it is conceived that the electron potential energy is increased by 100 eV. So the electron does not lose its total energy, and thus no radiation can be emitted from this electron. Secondly, let us consider a lasing electron in the magnetic Wiggler and the laser wave given by eqs. (2) and (3) respectively. The lasing electron is in a different situation from the above even though it continuously works against the laser electric field. The lasing electron wiggles in the x-direction with velocity given by $v_x = eA_{mo}\cos(k_m v_p t + \phi) / \gamma\ mc$ and experiences the laser field whose potential given by $\mathbf{A}_\ell = A_{\ell o}$ $\cos(k_m v_p t)$ and the magnetic Wiggler whose potential is given by $\mathbf{A}_m = A_{mo} \cos(k_m v_p t)\,\hat{x}$. Since the transverse wiggling motion and the total field (*i.e.*, the laser field plus the magnetic Wiggler) which the electron experiences are synchronized (or phase-locked), the electron returns to the same field just after many cycles of the transverse wiggling. So the potential energy does not change just after the electron completes many cycles of the transverse wiggling. Since the lasing electron net-works over many cycles of the transverse wiggling, the energy amount of the work, which is equal to the decrease in the kinetic energy, must go to infinity, that is, should be emitted as radiation, since it does not transform to the potential energy.

The above thinking is nothing but spurious. The thinking in the first situation is valid only when the electron velocity is infinitely small. In reality, since the electron moves with non-zero velocity and acceleration corresponding to the electron kinetic energy being 1 MeV and its acceleration being 100 $\mathrm{eV/cm}/\gamma$ m, the electron radiates with power given by the Liénard formula $P = 2e^2\gamma^6\left[\left(\dot{\vec{\beta}}\right)^2 - \left(\vec{\beta}\times\dot{\vec{\beta}}\right)^2\right]$, where P is the radiation power, $\gamma = E/mc^2$ is the Lorentz factor, and $\vec{\beta} = \mathbf{v}/c$. The radiation energy PL/v during $L = 1$ cm travel distance is on the order of 10^{-13} eV and extremely small. Nevertheless, we should not ignore radiation in the qualitative argument. The decrease in the electron kinetic energy is 100 eV. The energy amount equal to this decrease in the electron kinetic energy minus the above radiated energy, which is 100 eV in the practical sense, must be released from the electron as non-radiation. In classical electrodynamics, a free electron releases its kinetic energy only through velocity and acceleration electromagnetic (EM) fields. Acceleration EM field is radiation. Through velocity EM field, the electron leaves a part of its kinetic energy whose amount is equal to the decrease in the kinetic energy behind its trajectory as space-field energy when it is decelerated, and picks up space-field energy whose amount is equal to the increase in its kinetic energy when it is accelerated. Thus, the above 100 eV, which was dragged in association with velocity EM field by the electron in the beginning of the 1 cm travel, is left behind the electron trajectory as space field energy which can not arrive to any practical detector. If the electron is at rest after the travel of 1 cm, the 100 eV energy eventually becomes the electrostatic field energy around the rest electron. The concept of Liénard-Wiechert potentials applies to the lasing electron too. Since the electron velocity and acceleration oscillate in the x-direction, the radiation calculated with Liénard-Wiechert potentials is practically monochromatic and propagates in the positive axial direction as spontaneous emission. The power of the radiation is still of the order of the power of spontaneous emission which is an extremely small fraction of the work done by the electron. Thus, the energy equal to the work done by the electron in the practical sense is emitted through velocity EM field.

One may claim that the classical FEL theory is a new classical theory which is not based on the presently existing classical electrodynamics so that it may contradict the concept of Liénard-Wiechert potentials. Therefore, no matter how nonsensical is the classical FEL theory from the viewpoint of existing classical electrodynamics, we just accept it as a new theory and calculate the power of induced emission with it, and compare the result with the measured one.

The first FEL evidently shows that the classical FEL theory is nothing but a groundless wishful thinking, namely, a theory which has been falsely believed to agree the FEL since the concept is wished to do so. We will first show that the measured power of induced emission in the first FEL is incomprehensible from the concept of the classical FEL theory in the following.

In the first FEL, the average inter-electron distance L_e of the bunch electrons is given by $L_e \approx n^{-1/3} \gtrsim J_p/evS)^{-1/3}$, where n is the electron density, J_p is the peak current ($J_p = 70$ mA), and S is the cross section of the electron beam. Assuming $S \approx 1\ \mathrm{mm}^2$, which is sufficiently reasonable, we find that L_e is longer or about equal to the laser wavelength $\lambda = 10.6\ \mu$m. Since there is no physical mechanism to control the electron phase with respect to the laser wave, the bunch electrons enter the FEL cavity with random phases with respect to the laser wave. Thus, the initial phase distribution is stochastic; for example, the initial phases may be uniformly distributed in the whole phase region between 0 and 2π, or may be packed in the region between $\pi/10$ and $\pi/10.001$. The initially and 2π, or may be packed in the region between $\pi/10$ and $\pi/10.001$. The initially random phase distribution keeps its randomness in phase distribution regardless of whether or not there is a phase bunching mechanism which transforms the initial phase distribution to a bunched phase distribution if the initial phase distribution is the uniform distribution as the classical FEL theory claims. Thus, the net work done by all the electrons in the cavity fluctuates with time about the zero value. Since the net work completely transforms to radiation in the classical FEL theory, the power of induced emission should fluctuate with time about the zero value if the classical FEL theory agrees with experiment. However, the measured power of induced emission is constantly equal to 7% of the power of the externally launched laser field. Furthermore, we will show that the magnitude of the measured power is even greater than the work which would be done by all the electrons in the cavity when they are bunched into a single point of the phase at which a point charge can work at maximum against the laser field

In the first FEL the laser intensity $\mathcal{S}$ can be approximated as being constant since it increases only by 7% over the total length of the cavity. The average work per unit time done by all the electrons in the first FEL cavity against the laser field over a time scale much longer than the laser wave is given by

$$\begin{aligned}\mathcal{P} &= N_c \int_0^{2\pi} \lim_{T\to\infty} \frac{e\int_0^T \mathbf{v}_\perp(t)\cdot\boldsymbol{\mathcal{E}}_e(v_z t, t)\,\mathrm{d}t}{T} f(\phi)\,\mathrm{d}\phi \\ &= N_c \frac{r_e B_o \lambda_m}{\gamma}\left(\frac{\mathcal{S}}{2\pi}\right)^{1/2} \int_0^{2\pi} \cos(\phi) f(\phi)\,\mathrm{d}\phi = \mathcal{P}_{\max}\mathcal{F}_{bunching}\ ,\end{aligned} \tag{5}$$

where $\mathcal{P}_{\max} = N_c(r_e B_o \lambda_m/\gamma)(\mathcal{S}/2\pi)^{1/2}$ is equal to the work by all the electrons in the cavity when they are bunched into a single point of the phase at which a point charge can work at maximum against the laser field, and $f_{bunching} = \int_0^{2\pi}\cos(\phi) f(\phi)\,\mathrm{d}\phi$ is called here the bunching factor. In eq. (5), $r_e = e^2/mc^2 = 2.818\times10^{-13}$ cm is the classical electron radius, N_c is the total number of the electrons in the cavity, $f(\phi)$ is the normalized phase distribution function of the electrons in the FEL cavity and $\mathcal{S} = c\mathcal{E}_o^{\,2}/8\pi$ has been used.

Since $N_c < J_p L_c/\mathrm{ev}$ for pre-bunched electron beams, where L_c is the cavity length, and $f_{bunching} \ll 1$ in reality, we have

$$\mathcal{P} \ll \mathcal{P}_{\max} < \mathcal{P}_c = \frac{c r_e J_p L_c B_o \lambda_m}{e\gamma v}\sqrt{\frac{\mathcal{S}}{2\pi c}}\ . \tag{6}$$

Inserting the following experimental values° $J_p = 70\times10^{-3}\ A$, $L_c = 5.2\times10^2$ cm, $B_o = 2.4\times10^3$ gauss, $\lambda_m = 3.2$ cm, $v \approx c$, $\gamma = 48$, and $\mathcal{S} = 1.4\times10^{12}$ erg / sec / cm^2, into the expression of $\mathcal{P}_c$ in eq. (6), we have $\mathcal{P}_c = 2.8\times10^3\ \times$ which is less than the measured radiation power $\mathcal{P}_\ell = 4\times10^3\ W$. From this fact we cannot but conclude that the measured radiation power is greater than the work done against the laser field per unit time which would be done by all the electrons in the cavity even under the assumption that they are bunched into a single position of the phase at which a point charge can work at maximum.

We estimate the total number of the electrons by another way. Both calculations of the spontaneous emission power from an electron in a magnetic Wiggler with classical electrodynamics (Hofman 1978) and with the so-called semi-classical treatment of quantum mechanics (Kim 1993a), which is identical with QWD treatment for the magnetic Wiggler, render exactly the same formula [*cf* eq. (2.3) of Kim 1993b

$$P_{spon.em.} = \frac{cr_e^2 B_o^2 \gamma^2}{12}\ . \qquad (7)$$

According to the classical concept (Hofmann 1978, Marshall 1985) which views even the spontaneous radiation in the first FEL as an EM wave, most of spontaneous radiation is emitted in a cone of solid angle $\Omega_{classical\ theory} = \pi / \gamma^2 N_w$ about the axis where N_w is the number of the Wigglers. However, a quantum-mechanical calculation (Kim 1993a) shows that most of spontaneous power is emitted in a cone of solid angle $\Omega = \pi / \gamma^2$. The number of spontaneously emitted photons during an emission period is given by $\mathcal{P}^{measured}_{spon.em.} \lambda^2 / N_c hc^2$, where $\mathcal{P}^{measured}_{spon.em.}$ is the measure power of spontaneous emission, and is on the order of 10^{-6} in the first FEL. Thus, spontaneous emission in the case of the first FEL is a quantum mechanical emission since an electron gives off only one photon per one hundred thousand or a million emission periods. Thus, the emitted photons do not behave as wave. Accordingly, we reject the classical concept and opt the quantum mechanical concept. The adoption of the quantum mechanical concept over the classical one is also supported by the following fact. According to the classical concept, the measured spontaneous power into the cone of solid angle 5×10^{-6} sufficiently represents the total spontaneous emission power since the solid angle is on the same order of magnitude as $\pi / \gamma^2 N_w = 8.7\times10^{-6}$. Therefore, the total number of the electrons is $N_c = \mathcal{P}^{measured}_{spon.em.} / P_{spon.em.}$. From $\mathcal{P}^{measured}_{spon.em.}$ being 4×10^{-6} W and eq. (7), we find that $N_c = 1.6\times10^7$. Then, when the laser light is externally launched, an average electron whose initial energy is 24 MeV loses its energy by $\Delta = (\mathcal{P}_\ell / N_c)(L_c / v) = 27$ MeV, which is certainly unreasonable. Therefore, we dismiss the claim of the classical theory for the emission cone, and adopt the quantum mechanical one. If the power measurement is done at a distance not far greater than (*e.g.*, 100 times) the cavity length 5.2 m from the outlet of the FEL cavity, the total number of the electrons calculated under the assumption that most of photons is emitted into the cone of solid angle π / γ^2 about the axis with its apex at the inlet point of the FEL cavity is much larger than the actual total number. Therefore, according to the quantum-mechanical theory, the total number of the electrons in the FEL cavity, N_c, is much less than $(4\times10^{-6}\ \text{W} / \sec / P_{spon.em.})[(\pi / \gamma^2) / 5\times10^5] = 4.6\times10^9$. This number is less than but on the same order as the one calculated from the peak current. The fact that N_c ¼ 4.6×10^9 indicates that the measured power is larger than the work which would be done by all the electrons in the cavity when they are hypothesized to merge into a single point charge and the transverse velocity of the single point charge is phase-locked with the laser field at the phase where the single point charge can work at maximum so that the bunching factor is equal to one, *i.e.*, $f_{bunching} = 1$. Since $f_{bunching} \ll 1$ in reality, the power radiated through induced emission is far larger than the net work done by the electrons in the cavity per unit time. Since the net work per unit time is too small compared to the radiation power, we could not see any expected fluctuation in the radiation power or gain even if the net work were entirely transformed to laser radiation.

Since the measured power of induced emission is even greater than the maximum work that can be done by a point charge whose charge is equal to the total charge of the electrons in the cavity, we can see by the same argument of eq. (1) that the radiative reaction force due to induced emission is even greater than the Lorentz force of the laser field inducing the emission (Kim 1994a). Such force which cannot be accounted for with the Lorentz force equation must be generated by a non-classical radiation mechanism. We investigate into the radiation mechanism which is responsible for the measured power of induced emission.

3. Free-Electron Lasing Mechanism in the First Free-Electron Laser

Let $f(E,E_b)$ be the energy distribution function for the electrons in the FEL cavity when an electron beam with energy distribution being centered at E_b (`beam energy') is injected into the FEL cavity. The electron beam can be of mono- energy before being injected into the FEL cavity. The power $\mathcal{P}_{spon.\ em.}(E_b)$ of spontaneous emission from the electron beam of E_b is proportional to $f(E_e,E_b)$ where E_e is uniquely determined by the momentum of the incident photons and the Wiggler wavelength. If the gain is due to net stimulated emission, the gain function $\mathcal{G}(E_b)$ is proportional to $(\partial f(E,E_b) / \partial E)_{E=Ee}$ (Kim 1992a, c). If the shape of the electron distribution function $f(E,E_b)$ does not drastically change as E_b slightly varies such that

$$f\left(E, E_b + \Delta E\right) - f\left(E + \Delta E, E_b\right) = O\left(\Delta E\right)^2, \tag{8}$$

where $O[(\Delta E)^2]$ stands for a quantity on the same order of $(\Delta E)^2$, the following equation is satisfied:

$$\left.\frac{\partial f\left(E, E_b\right)}{\partial E}\right|_{E=E_e} = \frac{\partial f\left(E_e, E_b\right)}{\partial E_b}. \tag{9}$$

Therefore, if equation (8) holds for $f(E,E_b)$ of the first FEL, and the induced emission is due to the quantum-mechanical stimulated emission, we expect

$$\mathcal{G}\left(E_b\right) \propto \frac{\mathrm{d}P_{spon.em.}\left(E_b\right)}{\mathrm{d}E_b}, \tag{10}$$

for the first FEL.

Figure 2 of the article by Elias *et al.* (1976) shows that in the first FEL the relationship between the functional shape of the measured gain *vs.* E_b and that of the measured power of spontaneous emission *vs.* E_b satisfies eq. (10). Since we cannot think of any reason that eq. (8) does not hold in the first FEL, the induced emission in the first FEL is due to the quantum-mechanical net stimulated emission. We will determine whether net stimulated magnetic *bremsstrahlung* (Appendix of Kim 1992a) is the responsible mechanism.

From $\mathcal{G} = nP_{net\,st.\ em.} / \mathcal{S}$, the power of net stimulated emission by all the electrons in the FEL cavity, is expressed by

$$\mathcal{P}_{net\,st.em.} = N_c P_{net\,st.em.} = N_c \mathcal{S} \sigma_{net\,st.em.}, \tag{11}$$

where $\sigma_{net\,st.\ em.} = \mathcal{G} / n$ is the equivalent microscopic cross section of net stimulated emission. If the measured stimulated power is due to net stimulated magnetic *bremsstrahlung*, the equivalent microscopic cross section is expressed by

$$\sigma_{net\ st.mag.brem.} = \frac{\pi^2\left(1 + K^2\right) J_1^2\left(\zeta_o\right) e^4 A_{mo}^2 k}{4c^3 p_o k_m^2 \gamma^2} \frac{\mathrm{d}f_o\left(p_o\right)}{\mathrm{d}p_z}, \tag{12}$$

where the meanings of all notations are found in Appendix of Kim (1992a) and this paper. If the momentum distribution function $f(p)$ is Gaussian, the maximum of $\mathrm{d}f(p) / \mathrm{d}p$ is approximately equal to $1 / (\Delta p) \approx 1 / (\xi\gamma mc)^2$, where ΔP and ξ are the full width at half maximum (FWHM) and the ratio of ΔP to the center of the momentum distribution, respectively. Assuming that the momentum distribution can be approximated as a Gaussian one in the case of the first FEL, the maximum of the equivalent microscopic cross section of net stimulated magnetic *bremsstrahlung* can be written as

$$\sigma_{net\ st.mag.brem.}^{\max} = \frac{\pi K^2\left(1 + K^2\right) J_1^2\left(\zeta_o\right)}{8\gamma^2 \xi^2} \frac{r_e \lambda_m^2}{\lambda}. \tag{13}$$

All quantities except N_c and ξ which are needed to calculate the stimulated power using the above equations are directly measured. We can take $\xi = 0.02$ from Fig. 2 (a) of Elias *et al.* (1976) and the practical assumption that equation (8) holds for the case of the first FEL, and $N_c = 10^9$ as is previously discussed. Inserting these values into eqs. (11) and (13), we find that

$$\frac{\text{Measured Peak Stimulated Power}}{\text{Maximum Power of Net Stimulated Magnetic } Bremsstrahlung} = 3\times 10^4 . \tag{14}$$

Since the maximum power of net stimulated magnetic *bremsstrahlung* is proportional to N_c / ξ^2 and the above values of N_c and ξ are most likely accurate within a factor of 10, it is unfeasible for that the above ratio can be equal to one in reality. This means that net stimulated magnetic *bremsstrahlung* is not responsible for free-electron lasing in the first FEL. We should look for other net stimulated emission mechanism.

The beam electron sees the wavelength of the Wiggler being contracted by a factor of $1/\gamma$ (`FitzGerald-Lorentz contraction'). Thus, if the bunch length is larger than λ_m/γ, the magnetic Wiggler exerts spatial effect of wavelength λ_m/γ on the beam bunches. From eq. (4), we find that the amplitude of the transverse wiggling is $\lambda_m K / 2\pi\gamma = 0.07$ mm, which is much smaller than the transverse size of the electron beam. Thus, the transverse wiggling does not make the column of the electron beam bend but produces an axially propagating ES wave of the same wavelength as the magnetic Wiggler if the the density varies in the transverse direction which is intrinsically true for any beam (Kim 1989, 1994c). Both a kinetic theory (Kim 1989) and a hydrodynamics theory (Kim 1994c) render the same result for the ES wave. The result is derived under the assumption $K/\gamma \ll 1$, which is satisfied for the first FEL. The potential amplitude ϕ_o of the ES wave derived from these theories is approximately [*cf* eq. (20) of Kim 1989]

$$\phi_p = \frac{1}{\sqrt{2}} \frac{\omega_p^2 A_{mo} d\lambda_m}{\pi^2 c^2 \left(4d^2/\lambda_m^2 + 1\right)} , \tag{15}$$

where $\omega_p^2 = 4\pi ne^2 / m\gamma$. Thus, we find that $\phi_o \propto 1/\gamma$. The transverse wiggling motion drives net stimulated FETQS emission in this ES wave (Kim 1993c). The ratio of the power of net stimulated transverse-motion-driven FETQS emission (TMFETQS) in the ES wave to the power of net stimulated magnetic *bremsstrahlung* in the magnetic Wiggler is given by [*cf* eq. (17) of Kim 1993c]

$$\frac{\text{Power of Stimulated TMFETQS}}{\text{Power of Stimulated Magnetic } Bremsstrahlung} \sim \frac{1}{4\pi^2}\left(\frac{\lambda_m^2}{r_e a_o}\right)\left[\frac{K^2}{J_1^2(\zeta_o)}\right]\left(\frac{\phi_o}{A_{mo}}\right)^2 , \tag{16}$$

where a_o is the Bohr radius. Assuming that the the cross sectional area S of the electron beam is 1 cm^2, and inserting $n = J_p / evS \approx 10^7$ cm^{-3} and other values into the above equations we get the theoretically estimated power of net stimulated TMFETQS emission is 7 times larger than the measured stimulated power, that is, the former is on the same order of magnitude as the latter. Thus, under the present knowledge, free-electron lasing in the first FEL should be attributed to net stimulated FETQS emission driven by transverse wiggling, which is produced by the magnetic Wiggler, in the ES wave generated by the same magnetic Wiggler within the electron beam.

From the above we can convince ourselves of that the classical FEL theory is nothing but a groundless wishful thinking in the first FEL, and free-electron lasing in the first FEL is most likely net stimulated FETQS emission driven by the transverse motion in the ES wave produced by the magnetic Wiggler. Then, the question logically arising from this fact is why do the results calculated with the classical FEL theory agree with experimental ones in Raman FELs. The answer to this question will be published elsewhere. In this paper we only concentrate to confirm that FETQS

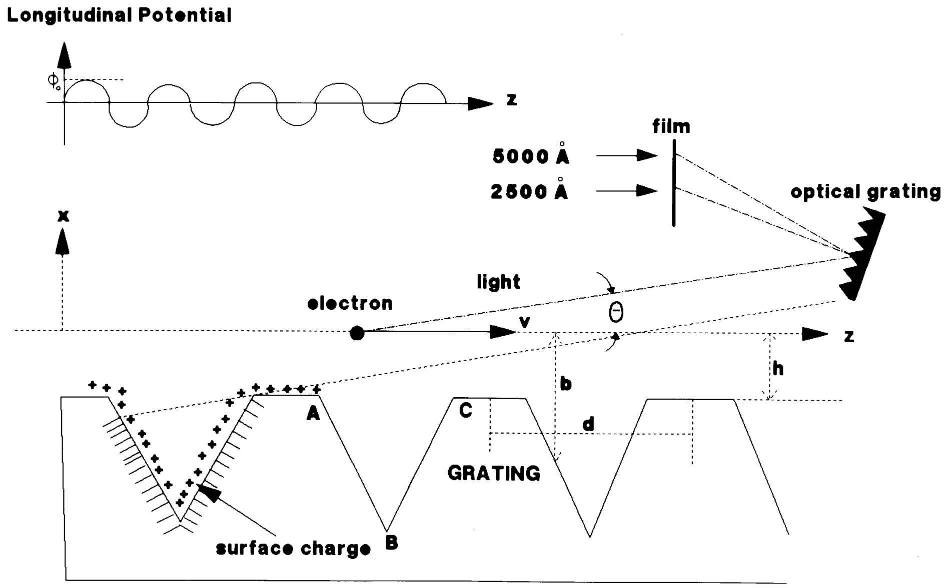

Fig. 1. Smith-Purcell experiment. The electron travelling on a straight trajectory induces the distributed charge on the grating surface. The radiation from the portion of the surface charge in the shaded region cannot be measured by the spectrometer installed at angle θ.

emission occurs as QWD describes. For this we need another evidence of FETQS emission. The radiation in the first SP configuration (Smith and Purcell 1953) has been claimed to be an FETQS emission by an analysis which is mostly qualitative but less quantitative (Kim 1993b). The electric Wiggler is generated by a much simpler process in the first SP configuration than in the first FEL so that the amplitude and wavelength of the electric Wiggler in the former is readily calculated without using hydrodynamics or kinetics as in the latter. Further, since the SP radiation cannot be any stimulated emission, which will be explained later, the calculation of the power of the SP radiation does not involve the value of $(\partial f(E,E_b) / \partial E)\_E = E_b$, which is estimated by the assumption that eq. (8) holds, which is reasonable but not quite obvious. We will study more rigorously than before the radiation in the first SP configuration, and show that the experimentally estimated power and wavelength can be explained only by identifying the SP ration with an FETQS emission.

4. Brief Review of the Smith-Purcell Radiation and Free-Electron Two-Quantum Stark Emission Driven by Macroscopic Axial Motion

The radiation observed in configurations similar to that shown in Figure 1 is called the SP radiation. In the original SP experiment (Smith and Purcell 1953), the inter-electron distance is 10 times longer than the gap distance d of the grating (Kim 1993b), which is longer than the wavelength λ of the observed radiation since the latter can be written by the Smith-Purcell (SP) formula (Smith and Purcell 1953)

$$\lambda = \mathrm{d}\left(\frac{1}{\beta} - \cos\theta\right), \tag{17}$$

and $(1/\beta - \cos\theta) < 1$ in the SP experiment, where $\beta = v/c$ with v the electron velocity and θ is the angle between the electron path and the emission direction as shown in Figure 1. Since the radiation is observed in the oblique direction, *i.e.*, $\theta \neq 0$, which has zero amplification distance [*cf* Figure 1], the observed radiation cannot be a radiation which is amplified by stimulated emission or any other amplifying process. Accordingly, any classical theory which conceives that amplification of a radiation in a Wiggler is due to bunching by the Wiggler and the radiation field cannot apply to the radiation in the SP configuration even if this kind of theory were valid.

The measured radiation in the SP experiment (Smith and Purcell 1953) is an incoherent sum of the emissions which are originated from the individual electrons and are not amplified during the passage from the electrons to the spectrograph. If the measured radiation were originated by a classical process, the estimated power based on the photographic emulsion (Smith and Purcell 1953) should at least agree with the power calculated by using six equations of classical electrodynamics (Maxwell's four equation, the Lorentz force equation, and Newton's second law of motion) in the order of magnitude. We will calculate the power by using the six equations and show that the classically calculated power is several orders of magnitude smaller than the estimated power based on the photographic emulsion. We first briefly review the SP experiment.

Purcell (Smith and Purcell 1953) noticed that Larmor's formula for the radiation by a nonrelativistic charged particle $P = 2e^2a^2\omega^4/3c^3$, where a is the radius of the rotation or the root-mean-square (rms) value of the oscillating amplitude in position, and ω is angular frequency, does not include the mass of the particle. He also figured that if an electron passes close to the surface of a metal diffraction grating, moving at right angles to the rulings as shown in Figure 1, the electron will induces the charge on the surface of the grating. Purcell thought that the surface charge can be represented by a point charge ("image") whose mass cannot be designated, and Larmor's formula can be applied to the image (since mass does not appear in Larmor's formula). Both the electron and its image repeat their periodic motions with exactly the same frequency. Since the amplitude of the oscillating vertical position of the image is several orders of magnitude larger than that of the electron, and Larmor's formula prescribes that $P \propto a^2$ for a given frequency, Purcell guessed that the power radiated from the surface charge will be larger than that from the electron by a factor of many powers of ten. Further, Purcell found that the wavelength of the radiation from the electron is given by eq. (17). Since the wavelength given by eq. (7) is also independent of the mass, Purcell conceived that the wavelength of the radiation from the image follows the SP formula.

Smith and Purcell set a configuration as shown in Figure 1, and observed that (i) the wavelength of the radiation follows the SP formula, and (ii) its power is equal to the power of a Larmor radiation with the same frequency as the electron experiences in the periodic structure of the grating, and with $a \approx 0.1\, d$, which is equal to 10^{14} times greater than the power directly radiated by the electron according to the classical concept (Kim 1993b). Since the experimental result well agreed with his theoretical prediction and there was no incentive to question seriously Purcell's concept, Purcell's concept had been let alone without making a scrutiny into it until the nature of the so-called wake field became a matter of concern (Kim, Chen and Yang 1990). Doucas *et al.* verified that the wavelength of the SP radiation follows the SP formula (Doucas *et al.* 1992).

It is not valid that the origin of the radiation energy can be traced back only to the surface charge. It is obvious that the origin is traced back to the electron beyond the surface charge. From the common sense, the power of the SP radiation will not follow the Larmor formula since the amplitude of the transverse wiggling of the surface charge can be made arbitrarily large [*e.g.*, 10 cm for a pill-box cavity) by making the depth of the grating valley very large [*e.g.*, a pill-box structure

(Kim, Chen and Yang 1990)], and then we should see a spectacularly ultra strong emission which is many orders of magnitude larger than any presently known emission. When the physics of the wake-field acceleration concept (Bane, Wilson and Weiland 1983) was proven as generally valid by experiments (Figueroa *et al.* 1988), the nature of the wake field became of great interest. Kim for the first time clarified that the wake field is mostly a radiation field converted at the conducting surface from the so-called velocity field (Jackson 1975), which is emitted by a moving charge (Kim, Chen and Yang 1990). Further, Kim clarified that Purcell's concept is caused from the misunderstanding on the wake field (Kim 1994b). We briefly review the wake field.

An electron moving in free space induces the electromagnetic (EM) fields of Liénard-Wiechert which consist of acceleration (or radiation) and velocity fields (Jackson 1975). The velocity EM field can be viewed as the modification of the Coulomâ field due to the electron motion. The energy density of velocity field at a position scales as $1 / R^4$, where R is the retarded distance between the electron and the observation position. Thus, the velocity field is completely negligible at any detector installed at an open space, and the detector can measure only the acceleration EM field in the practical sense. However, the velocity field from a charged particle or bunch in practically uniform motion on a straight line in a disk-loaded radio-frequency (RF) cavity or on a grating produces a measurable effect at a detector. This is because the velocity field induces distributed charge and current at the metallic surface, which in turn produce electric and magnetic fields, which consists of fields falling off as $1 / R$ and $1 / R^2$; the former, which is called the transverse (or electromagnetic) wake, can arrive at the detector. We can see it in detail as follows.

For simplicity, let us assume that a surface near the electron path $\mathbf{r}_e = \mathbf{r}_e(t)$ is made with a perfect conductor. When R is small, the acceleration EM field can be ignored compared to the velocity EM field. The surface-charge density σ and current $\mathbf{K}$ at a position $\mathbf{r}_s$ on the surface, induced by the velocity EM field, are related to the electric field and magnetic induction at $\mathbf{r}_s$ by the equations:

$$\hat{m} \cdot \vec{\mathcal{E}} = 4\pi\sigma \,, \tag{18a}$$

$$\vec{m} \times \mathbf{B} = \frac{4\pi}{c}\mathbf{K} \,. \tag{18b}$$

The scalar and vector potentials at $\mathbf{r}$ and t due to these surface-charge density and current at $\mathbf{r}_s$ can be written as

$$A_\mu(\mathbf{r},t) = \frac{1}{c}\iint \frac{K_\mu(\mathbf{r}_s,t')}{R_s}\delta\left(t' + \frac{R_s}{c} - t\right)\mathrm{d}S\mathrm{d}t' \,. \tag{19}$$

where $K_\mu = (\mathbf{K}, ic\sigma)$, $A_\mu = (\mathbf{A}, i\phi)$, $R_s = |\mathbf{r}_s|$ with $\mathbf{r}_s = \mathbf{r} - \mathbf{r}_s$, and d$S$ is the infinitesimal surface area around $\mathbf{r}_s$ (Jackson 1975). Accordingly, we find readily

$$\vec{\mathcal{E}}(\mathbf{r},t) = \int\left[\frac{\sigma\left(\mathbf{r}_s, t - R_s/c\right)\hat{n}_s}{R_s^2} + \frac{1}{cR_s}\left(\frac{\mathrm{d}\sigma\left(\mathbf{r}, t - R_s/c\right)}{\mathrm{d}t'}\hat{n}_s - \frac{\mathrm{d}\mathbf{K}\left(\mathbf{r}_s, t - R_s/c\right)}{\mathrm{d}t'}\right)\right]\mathrm{d}S \tag{20a}$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\int\left[\frac{\mathbf{K}\left(\mathbf{r}_s, t - R_s/c\right)\hat{n}_s}{R_s^2} + \frac{1}{cR_s}\left(\frac{\mathrm{d}\mathbf{K}\left(\mathbf{r}, t - R_s/c\right)}{\mathrm{d}t'}\right)\times\hat{n}_s\right]\mathrm{d}S \approx \left(\vec{\mathcal{E}} \times \hat{\boldsymbol{u}}_e\right)_{ret} \,, \tag{20b}$$

where $\hat{n}_s = \mathbf{r}_s / R_s$ and $\hat{n}_s = (\mathbf{r} - \mathbf{r}_e) / |\mathbf{r} - \mathbf{r}_e|$, and the subscript '*ret*' means the value at the retarded time.

Equations (20a) and (20b) show that some of the velocity field from a moving electron converts to a radiation which falls off as $1 / R_s \doteq 1 / |\mathbf{r} - \mathbf{r}_e(t_{ret})|$ after the velocity field hits the conducting surface. Since the surface charge density σ and and the surface current density **K** induced by the velocity EM field is enormously large compared to those induced by the acceleration EM field when the conducting surface is very near the electron path, the radiation field converted from the velocity EM field is much stronger than the acceleration EM field, which comes directly from the moving electron. The radiation field converted from the velocity EM field is specifically called the transverse wake. From eqs. (20a) and (20b), we find that the surface charge and current directly induced only by the fields from the electron does not critically depend on the curvature of the surface. The actual surface charge and current distribution depends on the curvature since the charge and current induced by the field from the electron exerts self-consistently (with Maxwell's equations) a new field on itself.

Once the nature of the wake field was clarified, two concepts had been arisen to identify the SP radiation. The first was that the SP radiation is a wake-field phenomenon (Doucas *et al.* 1992). However, it had been shown that the theoretical wavelength of the wake does not follow the SP formula (Kim 1994b; Kim, Chen and Yang 1990) and was confirmed by a wake-field acceleration experiment (Figueroa *et al.* 1988). The other one was that the SP radiation is FETQS emission due to electron spin interacting with the axial component of the force acting on the electron from the surface charge (Kim 1993e). The wavelength of FETQS emission due to electron spin is exactly given by the SP formula (Kim 1993e). However, the measured power of the SP radiation (Kim 1993b) is many powers of ten larger than the power of FETQS emission due to electron spin interacting with the axial component of the force from the surface charge. These mean that the SP radiation is neither a wake nor an FETQS emission due to electron spin. In order to solve the dilemma, Kim claimed that FETQS emission can be produced even by practically uniform axial motion, and that the SP radiation is that kind of FETQS emission (Kim 1993b, c). Kim's calculation of the power of FETQS emission driven by macroscopic axial motion is based on the concept that the phase of the probability amplitude of a transition route involving an interaction with a Wiggler is random so that the probability amplitudes of the two routes of a two-quantum process are incoherently added to make the total transition probability (Kim 1993d), which is designated as the third law of QWD (Kim 1994c). If we calculate the probability of a transition through a two-quantum process involving interaction with an electric Wiggler in accordance with the third law of QWD, the probability scales as $1 / \hbar^2$ so that the emission power becomes extraordinarily large. If we calculate such a transition probability based on the concept that the phase of the probability amplitude of a transition route involving an interaction with a Wiggler is coherent so that the probability amplitudes of the two routes of a two-quantum process cancel each other, the lowest-order terms in $\hbar$ of the transition probability are the terms of the order of $\hbar^0$, which are generated by the electron spin. (Kim 1993c, *e*). Before the advent of QWD, it had been believed that high-energy electrons exhibit only the phenomena associated with quantities which do not diverge to infinity in the classical limit ($\hbar \to 0$), which is misleadingly called the correspondence principle. Hence, the latter concept had been adopted before the advent of QWD. Since there is no difference between FETQS emission due to electron spin and that driven by macroscopic motion from the kinematic viewpoint, the wavelength of FETQS emission driven by macroscopic motion is given by the SP formula as that of FETQS emission due to electron spin is so (Kim 1994b).

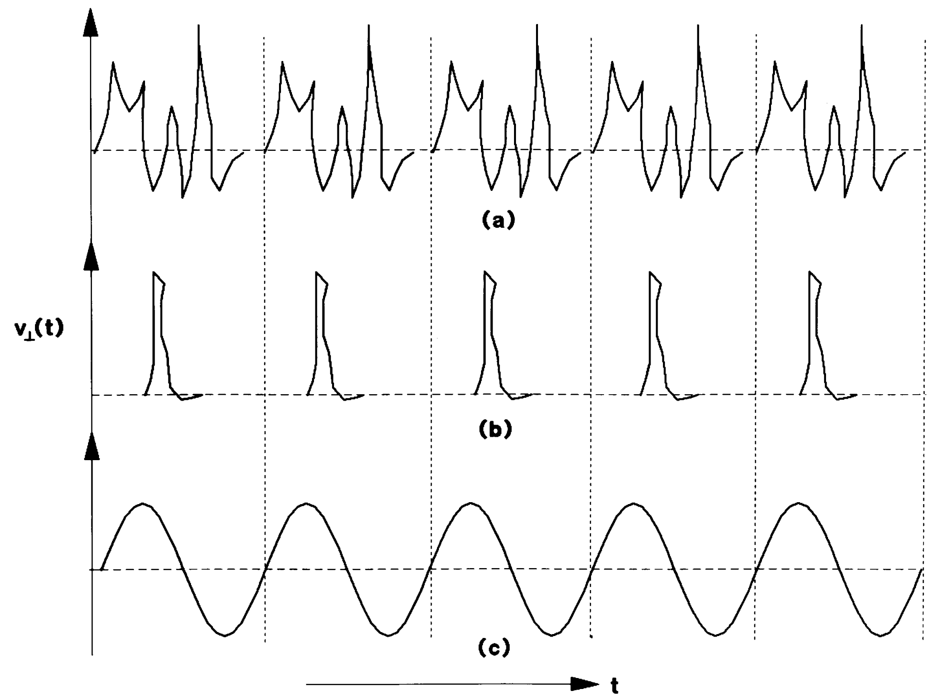

Fig. 2 Temporal profiles of the transverse velocities of three electrons in different periodic motions with the same period and *z* velocity.

5. Wavelength of the Electromagnetic Wake

Let us consider three electrons moving in the *x–z* plane. All three electrons have the same *z* velocity, but different periodic transverse velocities v_x with the same period as shown in Figure 2. In the concept based on classical electrodynamics, all three electrons cannot emit electromagnetic waves of the same wavelength. Furthermore, the electrons in (a) and (b) cases cannot emit a line-spectrum emission since their transverse velocities are not synchronized with the electric field of an electromagnetic wave (Kim 1992a, b).

For two point charges to emit radiations of the same wave vector, their transverse velocities must be synchronized with the electric field of a monochromatic EM wave with the same wave vector. Thus, the electron transverse velocities must be synchronized with each other. Accordingly, for both an electron and an element of the surface charge induced by the electron to emit monochromatic electromagnetic waves of the same wave vector, the transverse velocity $v_\perp^e(t)$ of the electron must be related with the transverse velocity $v_\perp^s(t)$ of the fluid element of the surface charge as seen from the detector by

$$v_\perp^e(t - t_o) = a v_\perp^s(t)\,, \tag{21}$$

where a and t_o are constants. Equation (21) cannot be satisfied for the electron and a surface-charge fluid element. This is readily seen by a following dramatic example. When the charge element moves into the shaded region as shown in Fig. 1, any radiation from this element cannot be directly seen by the spectrometer installed at angle θ. Thus, from the viewpoint of the spectrometer, the charge element disappears when it is in the shaded region. Accordingly, the transverse velocity of the electron as seen by the observer at angle θ is continuous, while that of the surface-charge element is discontinuous as depicted by Fig. 2 (b). Hence, the radiation from the surface-charge element is periodically pulsating with an interval time. By Fourier analysis, the wavelength of the dominant mode of such radiation cannot be expressed by the SP formula, *i.e.*, eq. (17). Accordingly, the radiations which arrived at the observation instrument directly from all elements of the surface charge (without any reflection at the grating surface) cannot sum to a radiation which has the unique wavelength given by the SP formula. Thus, the spectrum of such radiation cannot be of line-shape in the frequency and direction but should be broad.

From the above, we can conclude that the observed SP radiation, which satisfies the SP formula, cannot originate from the surface charge, and there is no contradiction in the claim that the SP radiation is FETQS emission as far as the observed wavelength and spectral width is concerned since FETQS emission is of monochromatic and satisfies the SP formula (Kim 1993b).

6. The Radiation Power from the Surface Charge in the Smith-Purcell Configuration

Smith and Purcell (1953) and Doucas *et al.* (1992) measured radiations at angle $\theta \neq 0$. Thus, the observed radiations cannot be the induced emission since there is practically no another electron which can be induced by a radiation emitted in such oblique direction from an electron. We will also show that the observed radiation does not come from the surface charge induced by the electrons by analyzing the radiation power.

The force from the surface charge to the electron cannot be larger than the force from the electron to the surface charge, and the latter cannot be larger than e^2 / r^2 where r is the distance between the electron and its positive image charge, which represents the surface charge. Since the average of the latter force is of the order of $e^2 / b^2 \sim e^2 / d^2$, where b is the average of the perpendicular distance from the electron path to the grating surface as shown in Fig. 1, the relativistic ($v \sim c$) electron moves in the transverse (x) direction by a distance

$$(\Delta x)_e \sim \left(\frac{e^2}{mb^2}\right)\left(\frac{d}{v}\right)^2 \sim \mathrm{d}\frac{d}{b}\frac{\left(e^2/b\right)}{mv^2} \sim r_e \, , \tag{22}$$

when it travels a distance d in the z direction, where $\mathrm{r}_e = \mathrm{e}^2 / mc^2$ is the classical electron radius.

Equation (22) indicates that the electron travels on a practically straight path since the average downward slope is $r_e / d \sim 10^{-8}$ in the SP experiment. Smith and Purcell (1953) estimated the total radiation power by means of a photographic emulsion as being equal to a Larmor radiation with $a = 0.1\, d$ and $\omega = \gamma v / d$ within an error margin of the order of one thousand. The kinetic energy loss as an electron travels a grating period d is equal to the estimated radiation power multiplied by the time ($t = d / c\beta$) for the electron to travel a grating period (Kim 1993b), which is expressed by

$$E^{measured} = 10^{14} r_e \frac{e^2}{d^2} \, . \tag{23}$$

This kinetic energy loss is about one ten thousandth of the initial electron kinetic energy (~ 300 keV). Thus, in the zeroth-order approximation, the electron can be considered to be in uniform motion on a straight line. If the grating surface were flat, there would be no line-emission even in the classical concept. Thus, the SP radiation, which is of line spectrum, must be produced by the grooves in the classical concept, that is, the SP radiation is derived from the energy hitting the valley

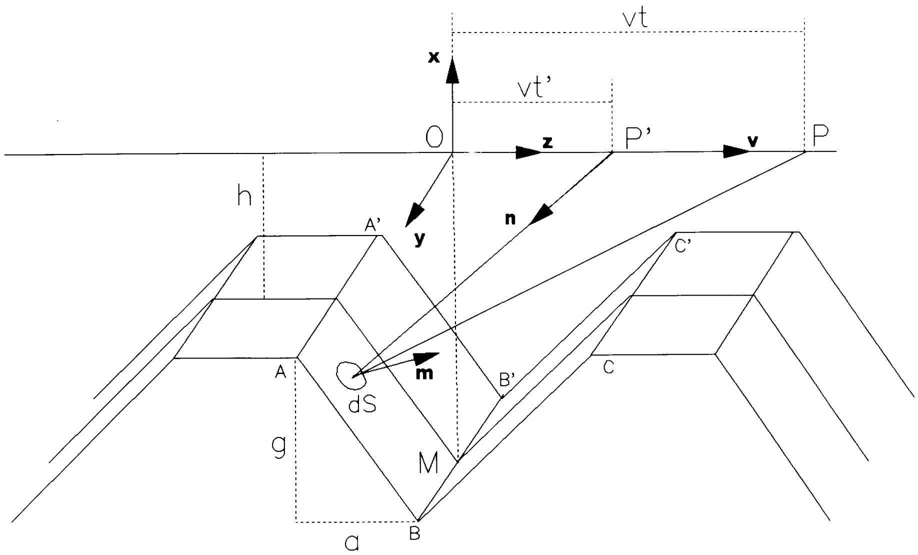

Fig. 3 Schematic showing the field from a moving electron at what position arrives at an infinitesimal area dS at the valley surface.

surface of the grooves. Therefore, if we show that the energy hitting the valley surface of a groove is far smaller than the measured value of the energy radiated, the classical concept should be dismissed as invalid.

Let the electron be on the point O at $t = 0$ as shown in Figure 3. From the Liénard formula (Jackson 1975) we readily find that the energy arrived at the unit area with normal vector $\hat{m} = m_x\hat{x} + m_y\hat{y} + m_z\hat{z}$, which is H distant from the electron path, in any direction is

$$E = \int_{-\infty}^{\infty} \frac{c}{4\pi}\left|\left(\vec{\mathcal{E}} \times \mathbf{B}\right)\cdot\hat{m}\right|_{ret} \mathrm{d}t = \frac{ce^2\beta\left(1-\beta^2\right)^2}{4\pi}\int_{-\infty}^{\infty}\left[\frac{\left|n_x\left(m_z n_x + m_x\beta - m_x n_z\right)\right|}{\left(1-n_z\beta\right)^6 R^4}\right]_{ret}\mathrm{d}t << \frac{e^e}{H^3}\left(\frac{3\gamma|m_z|}{32}+\frac{|m_x|}{8\pi}\right), \tag{24}$$

In the SP experiment (Smith and Purcell 1953), the radius R_b of the electron beam is 0.007μ cm, and the grating period is 1.6×10^{-4} cm. We assume very conservatively, *i.e.*, in much favor of the classical concept that the lowest altitude h in Fig. 1 for the electron to pass over the grating without any collision is the classical electron radius r_e, and the beam center grazes the grating so that the energy deposit in the grating valley becomes maximum. Then, the magnitude of the energy hitting on the surface $S = AA'B'\hat{A} + BB'C'C$ of a grating valley as shown in Fig. 3 by an average electron should satisfy

$$E^{surface} < e^2\left\langle \iint_S \frac{(3\gamma/32)\mathrm{d}x\mathrm{d}y + (1/8\pi)\mathrm{d}z\mathrm{d}y}{\left[(h+x)^2+y^2\right]^{\frac{3}{2}}}\right\rangle < e^2\left(\frac{3\gamma d}{16D}+\frac{1}{2\pi}\right)\left\langle\frac{1}{h}\right\rangle \approx \frac{e^2\gamma}{4R_b}\ln\left(\frac{2R_b}{r_e}\right), \tag{25}$$

where $\langle\ \rangle$ means the average over the beam electrons, D is the maximum valley depth, $D \sim d$ is assumed, and

$$\left\langle \frac{1}{h} \right\rangle = \left(\frac{4}{\pi R_b^2} \right) \int_{r_e}^{R_b} \frac{\sqrt{R_b^2 - h^2}}{h} \mathrm{d}h \doteq \frac{4}{\pi R_b} \left[\ln\left(\frac{2 R_b}{r_e} \right) - 1 \right] \tag{26}$$

has been used. Thus, even if we assume that all energy hitting the valley surface is transformed into the transverse wake, the ratio of the wake power to the measured power in the SP experiment ($\beta = 0.8$) is limited by

$$R = \frac{\text{Wake Power}}{\text{Measured Radiation Power}} \ll \frac{E^{surface}}{E^{measured}} < 10^{-6} \,. \tag{27}$$

Accordingly, the power of the measured SP radiation cannot be accounted for with the wake-field concept.

7. The Power of Free-Electron Two-Quantum Stark Emission in the First Smith-Purcell Configuration

Since the SP radiation does not come from the surface charge, it must come directly from the electron. The field as seen by the electrons in the SP experiment is a periodic field whose potential amplitude, which is obviously less than 10 eV ($e^2 / a_o \sim 10$ eV, where a_o is the Bohr radius), is very small compared to the electron kinetic energy. Thus, the field can be considered as a quantum in kinematics. It was shown from kinematics that the wavelength of the emission directly from the electron satisfies the SP formula (Kim 1994b). Thus, as far as wavelength is concerned, the SP radiation can be an emission which directly comes from the electron. In classical electrodynamics, for a given magnitude of the applied force the radiation emitted with a transverse acceleration is a factor of γ^2 larger than with a parallel acceleration (p. 665 of Jackson 1975). However, even the power of the emission driven by the transverse component of the force from the surface charge, which can be calculated with either quantum mechanical (Kim 1993a) or classical (Hofmann 1978) formula, is a factor of 10^{-14} smaller than the measured power in the SP experiment (Kim 1993b). Therefore, the SP radiation must be an emission directly from the electrons through a non-classical process. Since the electron moves by a distance of the order of r_e in the transverse direction as it travels one Wiggler period in the axial direction, the transverse motion does not produces any macroscopically observable phenomenon. In our present knowledge we can only identify such non-classical emission with FETQS emission due to axial motion in the axial component of the force from the surface charge (Kim 1993b). We will show that the power of the FETQS emission well agrees with the estimated power based on the photographic emulsion by Smith and Purcell (1953) in the following.

The power of FETQS emission due to axial motion of an electron traveling in an electric Wiggler with potential amplitude ϕ_o is given by [*cf* eq. (4.13) of Kim 1993b

$$P^{FETQS} = \frac{(e\phi_o)^2 \gamma^2}{4mca_o} = \frac{(e\phi_o)^2 e^2 \gamma^2}{4c\hbar^2} \,. \tag{28}$$

Since $\phi_o \sim e / \mathrm{h}$ in the SP configuration, the radiation power averaged over the beam electrons in the Smith-Purcell configuration is given by

$$\left\langle P^{FETQS} \right\rangle = \alpha^2 r_e m c^3 \gamma^2 \left\langle \frac{1}{h^2} \right\rangle , \tag{29}$$

where

$$\left\langle \frac{1}{h^2} \right\rangle = \frac{4}{\pi R_b^2} \int_{r_e}^{R_b} \frac{\sqrt{R_b^2 - h^2}}{h^2} \mathrm{d}h \doteq \frac{4}{\pi R_b r_e} . \tag{30}$$

From eqs. (28)–(30), the energy radiated through FETQS emission as the electron travels a grating period is

$$E^{FETQS} = \frac{P^{FETQS} d}{c\beta} \approx \left(\frac{4\alpha\gamma^2}{\pi\beta} \right) \left(\frac{d^3}{R_b \vec{\lambda}_c} \right) \frac{e^2}{d^2} \approx 10^{13} r_e \frac{e^2}{d^2} \tag{31}$$

where $\bar{\lambda}_c = \hbar / mc$ is the Compton wavelength. The numerical value of eq. (30) is smaller by a factor 0.1 than the measured value expressed by eq. (23). Considering that Smith and Purcell's estimation for the radiation power based on photographic emulsion is valid only within a factor of hundreds, we can claim that the power of the SP radiation reasonably agrees with the power of FETQS emission.

Besides the above fact, Kim discovered that in both Smith-Purcell (1953) and Doucas *et al.* (1992) configurations the DC (non-alternating) force which the electron experiences due to spontaneous emission is stronger than the oscillating Lorentz force acting on the electron from the surface charge which drives the emission (Kim 1994a, b). This can be readily verified with the foregoing results in the case of the SP configuration as follows: The amplitude of the Lorentz force acting on the electron is less than $e^2 \langle 1/h^2 \rangle$, while the DC force acting on the electron is $E^{measured} / d$. From eqs. (13) and (20), we find that the latter is larger than the former. Accordingly, if the SP radiation is some emission that can be explained with classical electrodynamics, the observed fact that the force due to spontaneous emission is larger than the Lorentz force which drives the emission must be explained with classical electrodynamics which is based on the tenet that there is no macroscopically measurable force acting on a charged particle from electric, magnetic and / or electromagnetic fields other than the Lorentz force.

8. Discussion and Conclusions

We have revealed that the classical FEL concept is groundless by showing that the measured radiation power of the first FEL in the presence of the externally launched laser light is much larger than the power which would be calculated with the classical FEL theory. The measured power is far greater than the power of net stimulated magnetic *bremsstrahlung*. It is shown that in the present knowledge, only free-electron two-quantum Stark (FETQS) emission driven by the transverse wiggling, which is generated by the magnetic Wiggler, in the electric Wiggler produced within the electron bunch by the magnetic Wiggler has such power. Such FETQS emission can be conceivable only in Quantum Wiggler Electrodynamics (QWD). In order to check the validity of QWD, we have thoroughly studied the so-called Smith-Purcell (SP) radiation.

In the SP experiment (Smith and Purcell 1953) the energy release through acceleration EM field per unit time is only a negligible portion of (*i.e.*, 10^{-14} times) the observed radiation power (Kim 1993b). The velocity EM field cannot arrive at any practical detector. However, when a rugged surface like the surface of a grating is near the emitting electron, velocity EM field converts to acceleration EM field by generating a surface charge on the surface.

The prevalent concept on the SP radiation is that the SP radiation is the radiation from the surface charge produced on the grating surface by the electrons passing above the grating surface, which is commonly called a wake. In order to disprove this concept, we have shown that the wavelength of the radiation from the surface charge in the SP configurations can by no way satisfy the SP formula which is satisfied by the measured radiation and cannot but be of continuous spectrum. Further, we have proven that the magnitude of the radiation power from the surface charge must be a factor of 10^{-7} smaller than the measured radiation power in the SP experiment. Therefore, the SP radiation must be to come from an emission process that cannot be explained with classical electrodynamics. The ordinary quantum mechanics before the advent of QWD does not describe any spontaneous emission process which cannot be described with classical electrodynamics; it describes a stimulated emission which does not exist in the framework of classical electrodynamics and corresponds to a spontaneous emission that can be described with classical electrodynamics. The observed radiation in the SP type experiment (*i.e.*, the SP radiation) cannot come from any stimulated emission since it is observed in the oblique direction with the electron beam. Therefore, the SP radiation must be an emission that cannot be described even the ordinary quantum mechanics before the advent of QWD. The SP radiation cannot but be identified as free-electron two-quantum Stark (FETQS) emission due to macroscopic axial motion in the axial component of the periodic force from the surface charge which is conceivable only with QWD. Indeed, the wavelength of the FETQS emission is given by the SP formula.

Further, by taking into account a fact that the electron beam radius is about 50 times longer than the grating period in the calculation of the strength of the electric field acting on an average electron from the surface charge in the case of the first SP experiment (Smith and Purcell 1953), we have found that the power of FETQS emission due to axial motion in the axial component of the electric field from the surface charge well agrees with the observed power. Thus, we cannot but conclude that the SP radiation is such FETQS emission. This means that at least the third law of QWD, which can be stated as the probability amplitudes of the two routes of a two-quantum process are incoherently added to make the total transition probability, must be valid.

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