Measuring Time and other Spatio-Temporal Quantities

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Ordinary clocks do not measure time in the common and Newtonian sense, and there is a similar problem for spatial measurements due to effects of motion and gravitation. Einstein's theories of relativity are based on the denial of the possibility of the 'absolute' measurements that would be required. Nevertheless, here it is shown how such measurements can be performed. For this purpose, a "light clock" (or equivalent) is linked with a "space-time odometer" that counts the zero crossings in the field of the cosmic microwave background radiation. The readings of these two instruments allow to calculate the time interval and the length of their path in Euclidean space even in the presence of local variations in gravitational potential.

Introduction

In a famous experiment by Hafele und Keating (1972), cesium clocks were flown eastward and westward around the world and the clock that was flown eastward was observed to be late by 59±10 ns when it returned, while the one that was flown westward was early by 273±7 ns in comparison with a clock at rest. Hafele and Keating (1972) further showed that these results were in agreement with calculations based on Einstein's general theory of relativity, and they concluded that ."..these results provide an unambiguous empirical resolution of the famous clock 'paradox' with macroscopic clocks." However, it would appear more appropriate to say that the results demonstrated the nature of this paradox in a vivid fashion, rather than that they 'resolved' it. When we are confronted with such a finding, common sense tells us that these clocks are no good clocks since their pace is influenced by some factors in addition to time, otherwise, their readings should have been the same. For the common people, "time" is an abstract quantity that is immune against factors such as gravitational fields and velocity of motion, which may affect the pace of clocks. Therefore, such clocks are not ideally suited for the measurement of *time* (common sense notion), although it may well be that they measure *something else* with high precision.

However, according to currently accepted doctrine in physics, these clocks are said to measure time, and nothing else, but time is considered a relative concept. Time itself is said to be 'dilated' by motion and gravitation. This conception is due to Einstein (1905), who abolished the traditional notion of "absolute time," considering it as immeasurable and irrelevant to physics. The new conception of space-time was subsequently further clarified by Minkowski (1909) and Reichenbach (1928). Since this conception is incompatible with the common notion of time, which Galileo and Newton adhered to, relativity theory appears paradoxical to those who have not been adequately indoctrinated. Paradoxes arise when common sense notions are applied instead of the cranky notions required by the theory. Such paradoxes can never be resolved by any physical experiments, but only by an analysis of how people reason.

Some further confusion has been caused by the widespread practice of referring to the pseudo-space-time of Minkowski just as "space-time." This misleads one to think of it as a four dimensional manifold with three spatial dimensions to which time is added as a fourth dimension. However, the fourth dimension in Minkowski space-time does not represent any real time, but an imaginary one, while the other dimensions are the three real dimensions of space.

Of course, the conceptual revolution initiated by Einstein would need a perestroika if it, unexpectedly, could be shown how time and spatial distances can be measured in agreement with common sense, *i.e.*, without distortion by motion and gravitation. In the following it will be shown that the kind of measurements required for this can be performed by conceptually quite simple means.

2. Effects of motion

In order to reason about measurements of temporal and spatial distances, we shall use two measuring instruments: a light clock and a space-time odometer. Since both of these are intended for *Gedanken* experiments rather than for practical measurements, we shall not be concerned with engineering problems. To begin with, we shall consider the behavior of the two devices for cases in which the effects of local gravitational fields can be neglected.

2.1 The space-time odometer

The space-time odometer utilizes the fact that the Universe is homogeneously filled with electromagnetic microwave radiation, commonly referred to as the "background radiation" [5]. It has the properties of a thermal blackbody radiation with a temperature of 2.73 K. This radiation provides us with something like a four-dimensional cosmic coordinate grid that we can read off if we want to measure the length of our path through space-time. *A space-time odometer can be imagined as a device that detects the electric potential of the background radiation at a point in space and counts its zero-crossings with respect to the average potential in the spatio-temporal vicinity of the measuring point.* Since these counts will be affected by the statistic fluctuations of the background radiation, measurements of short intervals would not be very accurate, but the fractional accuracy can be decreased at will by choosing longer intervals (and by other means) so that we can neglect them for our purely conceptual ends. However, the space-time seen by such an odometer is not that of Minkowski, but it is Euclidean to the extent that the effects of local gravitational fields can be neglected.

For reasons to be motivated below, we are going to refer to the quantity measured by the space-time odometer as the *path time* t_s. For linear motions, this is just the linear distance

$$\Delta t_s = \left(\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2\right)^{1/2} \tag{1}$$

in a reference system that is at rest with respect to the background radiation, *i.e.*, in which this radiation appears to be isotropic. From the perspective of this instrument, there is no difference between temporal and spatial distances. Therefore, we can do without the velocity of light in this equation. The common absolute unit, the zero-crossing period, corresponds to roughly 1.8 ps and 0.53 mm.

When at rest with respect to the background radiation, a space-time odometer measures just time. Since the forward movement in time is obligatory, the instrument functions always more like a clock than like a spatial odometer. Only when it reaches the velocity of light, its motion in space contributes to the same extent to the accumulation of zero-crossings as its movement in time. The measured path time can never be shorter than the temporal distance and never longer than $2^{1/2}$ times as much. This is the limiting value that would be obtained if the instrument was moving at the velocity of light.

2.2 The light clock

While the space-time odometer is a new invention, the light clock and its response to motion has been described already by Larmor (1900) within the frame of an ether theory. *A light clock can be imagined as consisting of two mirrors facing each other in a rigid housing. A short light pulse propagates between the mirrors, and at one of them, there is a detector that counts the number of arriving reflections.*

Suppose that the light clock is moving with constant velocity in the direction of the coordinate *x*. We orient the ray in a direction transversal to that of the motion as seen by a co-moving observer. It is easy to see that the path length *L* of the light pulse increases with the velocity of the clock. Since

$$L^2 = (2\Delta y)^2 + \Delta x^2 , \tag{2}$$

where

$$\Delta x = L\frac{v}{c} , \tag{3}$$

we obtain

$$L = \frac{2\Delta y}{\left(1 - v^2/c^2\right)^{1/2}} , \tag{4}$$

The unit of measurement increases as a consequence of this increase in path length. In Einstein's theories, this phenomenon is called time dilation. It is known that the pace of an atomic clock depends on motion and gravitation in the same way as that of a light clock, and experiments such as those by Hafele and Keating (1972) have shown that the behavior of such clocks is adequately described by the (general) theory of relativity. Nevertheless, if we want to avoid a conflict with the common people's concept of time, we have to say that these clocks do not measure *time*, but a related quantity that we shall refer to as the *proper time* t_0. This expression has some tradition within the frame of relativity theory, but in the present frame it has a wider range of application. (The expression "path-time" is analogous to this - and it is even more adequate to speak of a 'time' here, since path-time can never deviate so much from common time as proper time can.) For linear motions, it holds in general that

$$\Delta t_0 = \left(\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2\right)^{1/2} , \tag{5}$$

which resembles equation (1). Comparing these equations, we see that the passage of time affects the readings of the two instruments in the same way, while motion in space affects them in opposite senses. This holds provided that the behavior of the light clock is analyzed in a reference system in which the background radiation appears isotropic. The space-time odometer does not give us any choice in this matter.

If the distance between the mirrors is constant, the path length *L* of the light pulse increases when the clock is oriented differently, *e.g.*, with the path in parallel with the coordinate *x*. In this case, the pace of the clock would slow down, since

$$L' = \Delta x \frac{1}{1 - v/c} + \frac{1}{1 + v/c} , \tag{6}$$

i.e.,

$$L' = \frac{2\Delta x}{1 - v^2/c^2} . \tag{7}$$

However, the distance between the mirrors does *not* remain constant when the clock is rotated. The length of a rigid body, such as the housing of the mirrors, is given by the interactions between the particles of which it consists, mainly by the electrical interactions among electrons and protons, and it has been shown that the distances in such a system of particles decrease in the direction of motion when it moves through a stationary 'aether' (Lorentz, 1904) so that

$$\Delta x' = \Delta x\left(1 - v^2/c^2\right)^{1/2} . \tag{8}$$

Due to this phenomenon, known as the FitzGerald-Lorentz contraction, the pace of the light clock remains unaffected by rotation.

2.3 Combined measurements

Neither the light clock nor the space-time odometer allows an immediate measurement of spatial distances, and time can be measured with any one of the instruments when these are at rest with respect to the background radiation, but not when they are in motion. If its velocity is known, spatial as well as temporal distances can be determined with any one of the two instruments. This velocity can be obtained by analysis of the isotropy deviations of the background radiation. Such measurements (Smoot, Gorenstein, and Muller, 1977) have shown that our solar system moves with a velocity of about 400 km/s in the direction of the constellation of Leo.

However, for linear motion and for sufficiently linear pieces of more complex motions, temporal (Δt) as well as spatial distances (Δs) can be calculated quite simply from the path time

$$\Delta t_s = \left(\Delta t^2 + \Delta s^2\right)^{1/2} \tag{9}$$

measured with a space-time odometer, and the proper time

$$\Delta t_0 = \left(\Delta t^2 - \Delta s^2\right)^{1/2} \tag{10}$$

measured with a light clock linked to it, as

$$\Delta t = \left(\frac{\Delta t_s^2 + \Delta t_0^2}{2}\right)^{1/2} , \tag{11}$$

and

$$\Delta s = \left(\frac{\Delta t_s^2 - \Delta t_0^2}{2}\right)^{1/2} . \tag{12}$$

Thus, for cases in which the effects of gravitation can be neglected, it is evident that time and spatial distances can be measured in agreement with common sense. A *good clock*, that gives us a valid measurement of t can be realized by implementing equ. (11). This implies also that good clocks can be synchronized with any number of other good clocks, irrespective of their mutual distances and velocities, by comparison with a good clock that is moved around to each place.

Effects of gravitation

The effect of the gravitational field of a body on the motion of small particles is known to be independent of the mass of these particles down to zero. Consider now that any local systems of reference in which the background radiation appears isotropic are linked to the space-time grid defined by this radiation, which can be considered to consist of such particles. It follows that these systems of reference will be in accelerated motion towards the center of gravity. At each point in space, the magnitude of the velocity of this motion is the same as that of the radial velocity of escape v_{esc}. At a distance r from the center of gravity of a spherical non-rotating body with mass M, this can be calculated, according to Newton, as

$$v_{esc} = \left(\frac{2GM}{r}\right)^{1/2} . \tag{13}$$

It follows that a space-time odometer that remains at a constant distance from the center of gravity in a local gravitational field will be affected as if it was moving at v_{esc}, and its pace will increase so that

$$\Delta t_s = \Delta t\left(1+\frac{v_{esc}^2}{c^2}\right)^{1/2} . \tag{14}$$

A light clock in the same place will also be affected as if it was moving at v_{esc}, and its pace will decrease accordingly. The detailed analysis of this involves also length effects, as in the case of a moving light clock. The general theory of relativity which, as far as we know, correctly describes the behavior of the light clock, predicts according to Schwarzschild (1916)

$$\Delta t_0 = \Delta t\left(1-\frac{2GM}{rc^2}\right)^{1/2} . \tag{15}$$

Since we can substitute vesc2 for 2GM/r, this gives us

$$\Delta t_0 = \Delta t\left(1-\frac{v_{esc}^2}{c^2}\right)^{1/2} . \tag{16}$$

The equations (14) and (16) can be seen to be equivalent with (9) and (10). Due to this equivalence of the effects of gravitation and velocity of motion, the equations (11) and (12) remain valid also in this case. If this holds in general, space-time can be surveyed in agreement with common sense without any need of knowing how much of an observed discrepancy in pace between the two instruments should be ascribed to gravitation. Good clocks, as defined at the end of section 2.3 remain good clocks even if brought into and/or out of local gravitational fields. However, initially we have to calibrate each space-time odometer and light clock so that they run at the same pace when they are outside any strong local gravitational fields and at rest with respect to the background radiation.

4. Conclusions

Einstein, Minkowski, and Reichenbach can hardly be blamed for not having considered the possibilities that we have exploited here, since the background radiation was only discovered by Penzias and Wilson in 1965. However, since we can now conceive of measuring common time t in a physically objective way, Einstein's redefinition of the concept of time appears no longer admissible. The common notion of time has no place at all in his theories, which only deal with proper times t_0 and how these appear in other reference systems. The special theory of relativity does not disallow to describe events (like Prokhovnik, 1985) in a reference system in which the background radiation appears isotropic and the matter of the Universe at large is at rest, and in this case we can equate t_0 with t (and t_s), but in all other cases it is essential to distinguish these quantities in order to avoid misunderstandings and paradoxes.

In the general theory of relativity, the notion of gravitation has been re-interpreted in addition to that of time. Unlike the other types of interaction, it is treated as a property of space-time. Due to the universality of gravitation, it may be possible to describe the world in this fashion, but if we survey space-time in the way described here, we obtain a more readily intelligible Euclidean description that calls for treating gravitation more like the other fundamental interactions.

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