Interval in the Theory of Relativity

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It is emphasized that the 4-interval as a Lorentzian scalar does not change when transiting from one inertial reference frame to another one, which provided by its independence of velocity. The "contracted interval" answering the generally accepted (Einstein's) definition of moving rod length does not satisfy the indicated demand. Therefore the representation of the contraction of moving bodies must be rejected. Since the space-like interval is defined by the length of a resting rod, its space part in a moving frame (the rod in motion) because of the negative sign (pseudo-Euclidean-ness) is always greater than the interval itself. And it means that bodies elongate when moving.

... in their opinion, all the quantities having the physical sense must be the scalars or components of tensors.

A.Einstein [1]

Introduction

In the theory of relativity (TR) an interval (pseudo-distance) takes the place of the previous "pre-relativistic" invariant-distance (length). Therefore, for example, one should say more correctly about the (space-like) interval of a rod, *i.e.*, in essence, in the non-relativistic limit, their values coincide which ensures the succession of corresponding theories and the necessary uniqueness of the interval. Taking into account interval Lorenz invariance, in a moving reference frame it leads only to the "radar definition" of the moving rod length (see, *e.g.*,[2]).

It should be emphasized that we deal with one the fundamental problems of physics here. Space dimensions or, in general, space correlations, parallel with time ones, serve as the basis for the description of all natural phenomena (by means of physical theories, in particular). At the same time, we come across a highly strange phenomenon just in TR. The thing is that two statements, namely the demand of interval invariance and the condition of the uniqueness of physical notion, are turned out to be violated in the framework of its traditional interpretation.

Earlier, we have considered these cases in detail (see, *e.g.* [3]). But the full incomprehension of the very problem and its importance makes us return to it again.

The relativistic interval is a four-dimensional quantity, an analogue (and, one can say, the successor) of three dimensional distance(length). Or as one says, the metrics of Minkowski's space is defined by the interval squared:

$$s^2 = \vec{x} - c^2 t^2 \tag{1}$$

The 4-interval (pseudodistance) is the main invariant of TR, and so it is also named the fundamental invariant. By definition, the invariant is a quantity which does not change when transitting from one inertial reference frame to another one. Since this transition is related to changing the motion velocity, interval invariance must mean its independence of velocity, *i.e.*, constancy (see, *e.g.*, [4,5]). The material representatives of the space-like interval are scales (rods) and clocks for the time-like one. Remark that the interval also uniquely allows one to classify rods as it was made before by the "nonrelativistic invariant"—the length.

Non-relativism: rod length. In the pre-relativistic physics which lean vitally upon the Euclidean geometry, the space distance (unlike, say, its projections) is considered as an invariant. Its material realization was a rigid rod. Otherwise, one can say that the length was the unique characteristic of a rod. In general, the demand of uniqueness of the physical notion definition is in essence a a necessary condition of its fitness.

In analytic geometry, the length of a rod is defined by the first term on the right side of eq.(1). In this case it is evident that the values of the projections (terms) are always smaller than the very length (its sum). The length coincides with the "maximum projection" only for a rod oriented along one of the coordinate axes. One can say that this situation takes place, for example on the plane if the rotation angle $\phi = 0$.

However TR has found that the usual rod is physically not a spatial thing but a space-time configuration, *i.e.*, in general case it is necessary to add a time component to its three space ones. Moreover, this addition must occur taking into account the demand of Lorentz covariance, *i.e.*, the formed four-component quantity must be a space-like 4-vector. As a result, the distance (the first term in eq.(1)) loses the properties of the invariant; the interval undertakes this function.

Relativism: the rod interval. Thus, according to TR, one should say more correctly about the interval of a rod. For this, the values of space projections are always larger than the value of the very interval because of the negative sign in the expression for interval (its pseudo-Euclidean-ness). Therefore, by analogy with the previous reasoning, the angle $\psi = 0$ of Lorentzian turn must correspond to the "minimum (space) projection" now. Recall then the Lorentzian angle or the rapidity

$$\psi = Arth\beta = \frac{1}{2}\ln\frac{1+\beta}{1-\beta} \tag{2}$$

where βc is the motion velocity, vanishing ψ leads automatically to the equality $\beta = 0$, which answers the rest frame (S*), Whence it follows that the "minimum projection" is simply defined by the length of a resting rod. In other words, an immovable scale measures a space-like interval [6]:

$$s = l^*. \tag{3}$$

This means that the "previous" and "new" invariants coincide in the rest frame. This ensures the succession of corresponding theories as the 3-vector is a 4-vector with a zero *ct*-component.

It should be noted that, strictly speaking, the very representation of the velocity dependence of the moving rod length leads with necessity to the previous result. Indeed, in accordance with the Lorentz invariance demand, only a constant (independent of velocity) quantity can define the rod interval. But such is solely the length of a resting rod. Remark also that the equality $t^* = 0$ ensures the fulfilment of the interval uniqueness demand here. If a resting clock measures a time-like interval, then on immovable rod measures a space-like one.

Based on the aforesaid, the four-component quantity

$$l_*^i = (ct^*, x^*, 0,0) = (0, l^*, 0,0) \tag{4}$$

corresponds to the resting rod oriented along the X-axis. The same result can be obtained in the "physical way" by relativization of the radar procedure of measuring the the length of a resting rod [7]. Using the Lorentz transformation, for the length of a moving rod we have

$$l = l^* (1-\beta^2)^{-1/2} = l^* \gamma \qquad (5)$$

At one time, the elongation formula was obtained in the framework of the concept of covariant (radar) length (see, *e.g.*,[2]), leaning upon the nontraditional "radar definition" of the moving rod length.

Since, as we have found out, the unique interval (as one length) corresponds to the given resting rod, elongation formula (5), following from eq.(4), defines the behaviour of the length of a moving rod. Therfore, the generally accepted representation of the contraction of moving bodies must be removed.

Non-invariance of the "contracted interval". On the other hand, the four-component quantity answering the traditional (Einstein's) definition of moving rod length, in particular, takes the form

$$l_E^n = (0, l_E, o, o) = (0, l^* \gamma^{-1}, 0, 0) \qquad (6)$$

whence it follows that the corresponding interval

$$s_E = l^* (1-\beta^2)^{-1/2} \qquad (7)$$

depends evidently on motion velocity. And it means that we enter into contradiction with the basic theorem of TR (see, *e.g.*,[4]), according to which the interval is Lorentz-invariant if it is velocity-independent. Certainly, it is surprise that a similar verification of interval invariance corrsponding to the traditional definition of length (leading to the contraction of moving bodies) has not been verified earlier*. However, the aforesaid eq. (2), expressing that the "immovable scale measures the space-like interval"[6], can be considered as the first indirect indication that the traditional definition is out order.

It should be noted that the contraction formula, introduced by Fitzgerald [9] and Lorentz [10] in the theory of ether, was uncritically carried by Einstein to TR (in particular, without taking into account that the notions of resting and moving frames change in fact their places in these theories). By the way, Einstein appeals to the interval [8] only five years after his famous work.

J.S. Bell's problem [11] (see also [12]). Its gist consists in the following. Two rockets B and C are set in motion (say, to the velocity βc) so that the distance between them may remains constant and equal to the starting one (l_0) from the viewpoint of an external observer A. One can simpler imaging here that the observer A operates the flight of the moving-away rockets and has a radar, by means of which he controls the constancy of the distance between them.

We deal in this example with the variable interval between the rockets which changes from $s_{BC}^0 = l_0$ to $s_{BC} = l_0 \gamma^{-1}$. Since the latter expression defines also the distance between the moving rockets in their proper frame, then the thread connected them, despite the generally accepted opinion, sags and does not break. It is the right answer.

Conclusion

In TR, the interval (pseudodistance) takes the place of the previous "pre-relativistic" invariant—distance (length). In the rest frame, their values coincide; in particular, the space-like interval is defined by the length of an immovable rod. Its constancy (velocity independence) ensures automatically the fulfilment of the Lorentz invariance condition. In a moving frame, the space part of the interval the rod length in motion) is always greater than the interval itself because of the negative sign (pseudo-Euclidean-ness). This means that bodies elongate (and do not contract) when moving.

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