On the Energy-Inertial Mass Relation: I. Dynamical Aspects

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Dynamical aspects of the problem of the connection between energy and inertial mass are considered. Possible new relativistic expressions for the total energy and inertial mass are derived, and are the continuous functions of object velocity in some cases. Conditions for the existence of new classes of objects with real rest mass are found; these objects move with arbitrary velocity. Under certain assumptions the known results follow from the derived relations.

Introduction

The results of experimental investigations of a spectrum of superhigh energy primary cosmic protons points to the conclusion that the Lorentz transformations fail at $(1-v^2/c^2)^{-1/2} \sim 5\cdot 10^{10}$ (Kristianson 1974, Bogoslovsky 1977). This means that the Lorentz-factor $(1-v^2/c^2)^{-1/2}$ in the relativistic energy-velocity relationship

$$\left.\begin{array}{l} m(v)=m_o(1-v^2/c^2)^{-1/2} \\ E(v)=mc^2, \quad (m\neq \text{const.}) \end{array}\right\} \qquad (1)$$

needs further generalization.

There are some different relativistic relations which are determined on an electromagnetic basis from ether theory (see, for example, Jammer 1961). However, all known relativistic relations have a mathematical singularity of the total energy function $E=E(v)$, the essence of which is the lack of continuity at the point $v=c$.

In the case of special relativity, the singularity mentioned above has made it possible to define three classes of objects (see, for example, Bilaniuk and Sudarshan 1969). Ordinary real objects (tardions) with the rest mass $m_o\neq 0$, which cannot reach the velocity of light from below ($0\leq v<c$) fall into class I. Photons and neutrons (luxons) with $m_o=0$, which move with the velocity $v=c$ and have finite values of mass m, energy $E=mc^2$ and momentum $p=mc$, fall under class II. And objects (tachions) with imaginary mass at rest $m_o=im_*$, which move with velocity $c<v<\infty$, are classified as class III.

Thus, generalization of the Lorentz-factor is also interesting when considering the motion of objects with velocity greater than the velocity of light *in vacuo* (or in ether). The geometrical approach is based on the transformation of coordinates and time between inertial systems *in vacuo* (or in ether). The dynamic approach is based on a study of the energy-velocity relationship.

The purpose of this paper is to find—on the basis of the dynamic approach—more general possible relativistic expressions for inertial mass and total energy which do have any mathematical singularities under certain conditions. It presents an opportunity to determine new class objects with real mass at rest moving with arbitrary velocity, and to determine the kind of geometry that corresponds to the physical reality as well.

1. Total relativistic energy and mass inertia

It is well-known that the initial laws in classic and relativistic dynamics have the same form:

$$\frac{d\vec{p}}{dt}=\vec{F} \qquad (2)$$

$$\frac{dE}{dt}=\vec{v}\vec{F}=\vec{v}\frac{d\vec{p}}{dt} \qquad (3)$$

which connects variations of momentum $\vec{p}=m\vec{v}$ and energy E with external force $\vec{F}$.

If mass does not depend on velocity, one can obtain the following values for Newtonian dynamics

$$m(v)=m, \quad E(v)=\frac{1}{2}mv^2, \quad (m=\text{const.}) \qquad (4)$$

If mass depends on velocity, one has relations (1) for the relativistic dynamics of special relativity. If an object does not move ($v=0$) one can use the equalities from (1)

$$m(0)=m_o, \quad E(0)=m_oc^2 \qquad (5)$$

where m_oc^2 is the rest energy.

We now write one of the possible connections between total energy, mass and velocity up to a constant value, as follows:

$$E=a\,mc^2+b\,mv^2+h \qquad (6)$$

where a, b and h are arbitrary, real parameters. In the expression (6) the simplest dependence of energy on m,

v^2 and c is shown, as it exists, correspondingly, in the Newtonian (4) and relativistic (1) theories.

We now substitute the momentum $p = mv$ and energy (1) into (3) and obtain the expression for a variation of inertial mass:

$$\frac{\mathrm{d}m}{m}=\frac{(2e-1)\bar{v}\mathrm{d}\bar{v}}{a\,c^2-e\,v^2}, \quad (e=1-b) \tag{7}$$

Taking an integral of (7) at a $a \neq 0$, one obtains the general mass-velocity relation:

$$m(v)=\frac{m}{\left(1-ev^2/ac^2\right)^{1-\frac{1}{2e}}} \tag{8}$$

where m is an integration constant.

If the object does not move ($v = 0$), we have $m = m(0) = m_o$.

We can use the equality $E(0)=m_oc^2$ as a boundary condition for energy, which is widely corrolorated in nuclear physic. This leads to the value $h = (1 - a)m_oc^2$ and finally we obtain the following expressions:

$$\begin{aligned} m(v)&=\frac{m_o}{\left(1-ev^2/ac^2\right)^{1-1/2e}}, \\ E(v)&=a\,mc^2+(1-e)mv^2+(1-a)m_oc^2 \\ &=\frac{a\,m_oc^2+(1-e)m_ov^2}{\left(1-ev^2/ac^2\right)^{1-1/2e}}+(1-a)m_oc^2, \end{aligned} \tag{9}$$

which depend on two parameters.

Subject to the condition that $(e/a)v^2 \ll c^2$, from the relativistic formula (9) we can derive:

$$\begin{aligned} E&=\left[a\,m_oc^2+(1-e)m_ov^2\right]\left[1-\left(1-\frac{1}{2e}\right)\frac{e\,v^2}{a\,c^2}+\ldots\right] \\ &\cong m_oc^2+\frac{m_ov^2}{2} \end{aligned} \tag{10}$$

This is the Newtonian approximation (4) for energy up to the energy at rest.

2. Functions $m=m(v)$ and $E=E(v)$ with mathematical singularity

Let us consider the case where the functions $m = m(v)$ and $E = E(v)$ loose their continuity under certain conditions. The variation domains of parameters a and e are shown in Figure 1. The parameters can take on positive and negative values. From formula (9), it follows that there are two domains where continuity is absent at $v=c(a/e)^{1/2}$: a) $1/2 < e < \infty$, $a > 0$; b) $-\infty < e < 0$, $a < 0$. The domains mentioned are shown by thin lines in Figure 1.

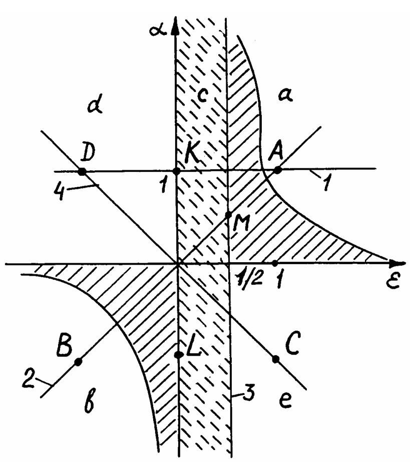

Figure 1. Parameter variation diagram.

By analogy to special relativity, we can determine three classes of objects. Objects with $m_o \neq 0$ moving with velocity $0 \le v \le c(a/e)^{1/2}$ fall under class I. Objects with $m_o = 0$ moving with velocity $v = c(a/e)^{1/2}$ fall under class II. For this class of objects one obtains finite real values m, $E=(1+a-e)mc^2$ and $p = mc$. Objects moving with the velocity $c(a/e)^{1/2} < v < \infty$ are classified as class III. The mass at rest of these objects equals $m_o=(-1)^{(1-1/2e)}m_*$ and it can be a real or complex quantity. For the mass and energy we have:

$$\begin{aligned} m(v)&=\frac{m_*}{\left(\dfrac{ev^2}{ac^2}-1\right)^{1-1/2e}}, \\ E(v)&=a\,mc^2+(1-e)mv^2+(1-a)m_oc^2 \\ &=\frac{a\,m_*c^2+(1-e)m_*v^2}{\left(\dfrac{ev^2}{ac^2}-1\right)^{1-1/2e}}+(1-a)(-1)^{(1-1/2e)}m_*c^2, \end{aligned} \tag{11}$$

Complex values of m_o are admitted at $a = 1$, since only in this case will the total energy (11) assume a real value. This condition presents a part of the straight line 1 in the domain a).

If a mathematical singularity occurs at $v = c$, then we have the equality $a = e$. This condition presents two parts of the straight line 2 in the shaded areas in Figure 1.

For the total energy we have:

$$E=\frac{e\,m_oc^2+(1-e)m_ov^2}{\left(1-\dfrac{v^2}{c^2}\right)^{1-1/2e}}+(1-e)m_oc^2. \tag{12}$$

At $e = 1$ in the domain a), from (9) and (12) we can obtain the relativistic relations $m = m_o\left(1-v^2/c^2\right)^{-1/2}$, $E = mc^2$ for special relativity (3). For instance, we have the formulae at $e = -1$ in the domain b)

$$m=m_o\left(1-v^2/c^2\right)^{-3/2}, \quad E=-mc^2+2mv^2+2m_oc^2 \tag{13}$$

In Figure 1 these two cases are shown, correspondingly, by the points A and B.The point A is a cross of lines $a = 1$ and $a = e$. As expected, tachions with $m = m(v)$ and imaginary mass at rest are permitted only in this case.

3. Functions $m = m(v)$ and $E = E(v)$ without mathematical singularity

Let us consider the case where $m = m(v)$ and $E = E(v)$ are continuous and define new classes of objects. The relativistic total energy against object velocities is plotted in Figure 2. From formula (9), three characteristic domains of parameters follow where a continuity holds: c) $0 < e < ½$, $a > 0$, $a < 0$; d) $-\infty < e < 0$;^ $a > 0$; e) $½ < e < \infty$; $a < 0$. The domain c) is shown by the dashed lines, the rest of the domains are unshaded.

Let us consider domain c) where we have the following expression for the energy:

$$E = \left[a\, m_o c^2 + (1-e) m_o v^2\right]\left[1 - \frac{e\, v^2}{a\, c^2}\right]^{|1-1/2e|} + (1-a) m_o c^2 \qquad (14)$$

In this theoretical case we determine three classes of objects.

Ordinary real objects with the real mass at rest m_o moving with velocity $0 \le v \le c(a/e)^{½}$ ($a > 0$) are classified as class IV. For example, at $e = a = ⅕$ one can obtain:

$$m = m_o\left(1 - v^2/c^2\right)^{3/2},\; E = \frac{1}{5}mc^2 + \frac{4}{5}mv^2 + \frac{4}{5}m_o c^2 \quad (15)$$

Objects moving with the velocity $c(a/e)^{½} < v < \infty$ ($a > 0$) fall under class V. The mass at rest of these objects is equal to $m_o = (-1)^{-|1-1/2e|} m_*$ and it can be a real or complex quantity. We have the formulae for mass and energy:

$$\begin{aligned} m(v) &= m_*\left(\frac{ev^2}{ac^2} - 1\right)^{|1-1/2e|} \\ E(v) &= a\, mc^2 + (1-e)mv^2 + (1-a)m_o c^2 \\ &= \left[a\, m_* c^2 + (1-e)m_* v^2\right]\left(\frac{ev^2}{ac^2} - 1\right)^{|1-1/2e|} \\ &\quad + (1-a)(-1)^{-|1-1/2e|} m_* c^2 \end{aligned} \qquad (16)$$

Complex values of m_o are permitted at $a = 1$ since only in this case will energy (16) have a real value. This condition presents a part of the straight line 1 in the domain c).

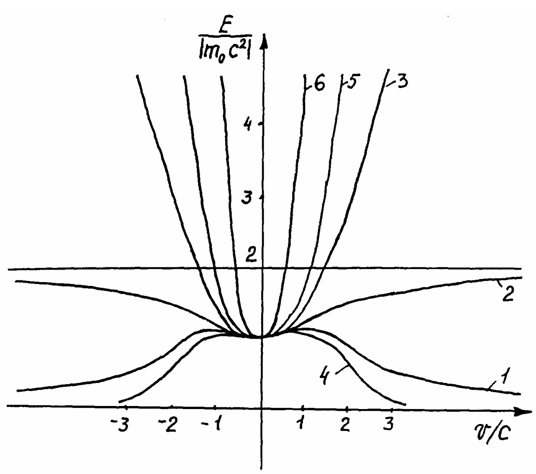

Figure 2. Relativistic total energy-velocity relation.

Ordinary real objects moving with arbitrary velocity $0 \le v \le \infty$ fall under class VI. For instance, at $a = e = ⅙$ we can obtain:

$$m = m_o\left(1 - v^2/c^2\right)^2,\; E = \frac{1}{6}mc^2 + \frac{5}{6}mv^2 + \frac{5}{6}m_o c^2 \quad (17)$$

The domains d) and e) in Figure 1 are classified as objects of class VI as well. Without loss of generality, let us choose two points D and C in these domains.They are in the straight line 4 of the relation $a = -e$ and they correspond to the values $e = -1$ and $e = 1$. From formula (9) we can obtain the relation:

$$E = \frac{-e\, m_o c^2 + (1-e) m_o v^2}{\left(1 + \frac{v^2}{c^2}\right)^{(1-1/2e)}} + (1+e) m_o c^2, \quad (18)$$

from which the expressions follow for the cases under consideration

$$m = m_o\left(1 + \frac{v^2}{c^2}\right)^{-3/2},\quad E = mc^2 + 2mv^2, \qquad (19)$$

$$m = m_o\left(1 + \frac{v^2}{c^2}\right)^{-1/2},\quad E = -mc^2 + 2m_o c^2, \quad (20)$$

In Figure 2 these relations are shown by the curves 1 and 2&

4. Functions $m = m(v)$ and $E = E(v)$ for boundary conditions of parameters

Let us consider the cases $e = 0$ (the ordinate axis in Figure 1) and $e = ½$ (the straight line 3) which are not included in the domains considered. At $e = ½$ from the relations (9) we can obtain the exact formulae $m = m_o$ and $E = m_o c^2 + \frac{1}{2} m_o v^2$ from Newtonian dynamics (4) where the energy contains the energy at rest. Nonrelativistic dependence is represented by curve 3 in Figure 2. At $e = 0$ from (7) we find the expressions for mass and energy

$$m = m_o e^{-\left(v^2/2a\, c^2\right)},\; E = a\, mc^2 + mv^2 + (1-a)mc^2, \; (21)$$

depending on one parameter a. For the points K and L with values $a = 1$ and $a = -1$ (Figure 1) we get from (21)

$$m = m_o e^{-\left(v^2/2c^2\right)},\; E = mc^2 + mv^2, \qquad (22)$$

$$m = m_o e^{v^2/2c^2},\; E = -mc^2 + mv^2 + 2m_o c^2, \quad (23)$$

Corresponding relations are shown in Figure 2 as the curves 4 and 5. Values $e = 0$ and $e = ½$ correspond to objects of class VI.

Let us consider the special case $a = 0$ (the abscissa axis in Figure 1) which does not belong the domains considered. We can write the energy in the form:

$$E = (1-e)mv^2 + h, \qquad (24)$$

where $e \ne 1$. Taking the integral of (7) at $a = 0$ we obtain the relation

$$m(v)=\frac{m}{\left(v^2/c^2\right)^{1-1/2e}}\,, \qquad (25)$$

where m is an integration constant. To find m we will use the value $v = c$. Then we will get $m = m(c) = m_c$. Let us use the equality $E(c) = m_c c^2$ as a boundary condition for the energy. This yields a value $h = em_c c^2$ and we obtain finally:

$$m(v)=\frac{m_c}{\left(v^2/c^2\right)^{1-1/2e}}\,,\ E(v)=\frac{(1-e)m_c v^2}{\left(v^2/c^2\right)^{1-1/2e}}+e\,m_c c^2 \qquad (26)$$

Continuity of mass and energy disappears at the value $v = 0$ in domain b). In domain a) a mathematical singularity at $v = 0$ is satisfied only for mass. We can now define two classes of objects.

1. Objects always moving with $m_c \neq 0$ and velocity $0 < v < \infty$ are classified as class VII.
2. Objects always at rest ($v = 0$) with $m_c = 0$ fall under class VIII. For this class of objects we have finite values $m \neq 0, E = p = 0$.

Continuity of mass and energy holds in domain c) which corresponds to objects of class VI with mass and energy:

$$\begin{aligned} m(v) &= m_c\left(v^2/c^2\right)^{|1-1/2e|}, \\ E &= (1-e)m_c c^2\left(v^2/c^2\right)^{1+|1-1/2e|}+e\,m_o^2 c \end{aligned} \qquad (27)$$

At e = ½ from relations (27) we can get the exact formulae $m = m_c$, $E=\frac{1}{2}m_c c^2+\frac{1}{2}m_c v^2$ of the Newtonian dynamics (4) where the energy contains a constant term equalling $\frac{1}{2}E(c)$. For example, at $e = 1/6$ we have the formulae for mass and energy

$$m(v)=m_c\left(v^2/c^2\right)^2,\quad E=\frac{5}{6}mv^2+\frac{1}{6}m_c c^2 \qquad (28)$$

In Figure 2, curve 6 shows $E/\left|e\,m_c c^2\right|$ related to the velocity for this case.

5. Conclusions

Possible new relativistic expressions for the inertial mass and total energy have been determined, and eight classes of objects where known objects of relativistic dynamics are included have been defined. In the approach developed here, the inertial mass and total energy can increase or decrease as the object's velocity increases. The point of greatest interest is class VI, which describes ordinary real objects with infinite velocity ($0 \le v \le \infty$). Therefore, in Figure 2 the relations for this class only are shown. These relations will be discussed in detail elsewhere. In the present work, the possibility that class VI objects exist is shown.

We note that the results found here include the conclusions of some authors (Podlaha 1978, 1979; Podlaha and Sjödin 1977). On the basis of the kinematic approach they introduce a relativistic factor in the form $\left(1-v^2/c^2\right)^{-g}$ where $g \in [-½,0]$. This factor corresponds to the quantity $\left(1-v^2/c^2\right)^{1-1/2e}$ in our consideration where $e \in [½,1]$. In Figure 1 this case is represented by the segment AM of curve 2.

The approach developed here does not change the Lagrange formalism of analytical mechanics. The Lagrange function $L=\vec{p}\vec{v}-E$ for some cases has the following forms:

$$\begin{aligned} a \neq 0:&\quad L=-a\,m_o c^2\left[1-\frac{ev^2}{ac^2}\right]^{1/2e}-(1-a)m_o c^2\,, \\ a = 0:&\quad L=-e\,m_c c^2\left(v^2/c^2\right)^{1/2e}-e\,m_c c^2\,, \\ e = 0:&\quad L=-a\,m_o c^2 e^{-\frac{v^2}{2ac^2}}-(1-a)m_o c^2\,, \end{aligned} \qquad (29)$$

In later work, kinematic and geometric aspects of the problem of the connection between energy and inertial mass will be discussed in more detail and connections between the results of this paper and another impending publication will be considered.

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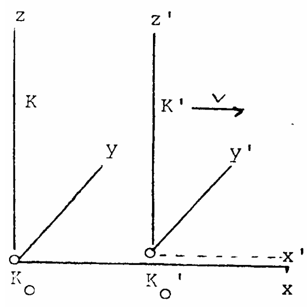

Figure 1 (Szego &Ofner; from p. 34)