Gravity out of Inertia

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An appropriate consideration of the relationship between the concept of relativistic inertial-mass (rest-mass), and the law of four-momentum conservation, leads to a natural two-fold partition of the class of all relativistic collision processes into those which are, in our own terminology, *inertially determined processes*, and the rest. We find that the 'inertially determined process' can be given a geometric representation with a formal structure which suggests that gravitation might be, in effect, a particular case of such a process and therefore a phenomenon of inertia.

Introduction

An intuitive sense of inertia is, perhaps, one of our most primitive and profound perceptions and it is probably no accident that the first of our modern theories, Newtonian Mechanics, deals precisely with the associated phenomena. Newtonian theory introduces a concept of 'inertial frame', defined as a metaphor for the 'inertial property' of material particles, and then uses this concept to describe a particle property which only manifests itself under dynamic conditions and is designated 'inertial mass'. The introduction of the 'inertial frame' concept, together with a developing technology, led to the further perception—which some see as also profound and others as incidental—that it is apparently possible to define an 'average frame of rest' relative to the distribution of Universal material and then, having done this, to define a class of inertial frames relative to this particular rest-frame; in this way, Universal material defines a unique class of local inertial frames, $\mathcal{F}_{mat}$ say, and consequently appears to be inducing, in some sense, inertial properties in local material. Subsequently, the modern discovery of the cosmic background, with an associated 'frame of rest' and a corresponding unique class of locally defined inertial frames, $\mathcal{F}_{rad}$, with the apparent equivalence $\mathcal{F}_{mat} \equiv \mathcal{F}_{rad}$, has served to emphasize this point of view.

A common conclusion derived from these arguments, that Universal matter somehow appears to induce local inertial effects, is known, in a phrase coined by Einstein, as 'Mach's Principle'. We do not take any particular position with respect to Mach's Principle in any of its forms but, at one time, such arguments were considered sufficiently powerful to justify the opinion that General Relativity—accepted as the best available theory of gravitation—was only a good 'first approximation', and that a 'better' theory, incorporating some recognizable interpretation of Mach's Principle, could be obtained by a suitably constructed modification of GR; in this way, various additional scalar, vector and tensor fields became attached to Einstein's basic theory of gravitation. However, the discovery of the binary-pulsar objects in the 1970's, which appear to represent sources of relatively strong gravitational fields, and the subsequent observations on such systems, rapidly led to the formulation of new requirements to be imposed on gravitation theories, and these have effectively led to the necessary rejection of all additional fields within GR proposed as possible means of incorporating inertial effects into general gravitation theory. As a consequence of the apparent finality of the implications of the binary pulsar observations, the general set of arguments associated with the formulation of Mach's Principle have been effectively marginalized within modern physics, even though the only *objective* conclusion that can be reasonably drawn from these observations within this context is that Mach's Principle itself—interpreted as some kind of theory for the global induction of inertia—and General Relativity will not be easily reconciled.

Whilst we adopt no particular position with respect to any of the various conceptions of Mach's Principle, we are of the opinion that the apparently irreconcilable nature of modern gravitation theory with respect to general notions of 'inertial induction'—represented under the collective heading of 'Mach's Principle'—really has its roots in the conceptual distinction that is made between gravitational and inertial mass. Correspondingly, we believe that if this distinction did not exist then the phenomenology represented in Mach's Principle would be far more accessible to theoretical understanding than it is at present. This general viewpoint has motivated the fairly comprehensive appraisal of the distinction, which is given in the following sections.

Inertial-Mass Versus Gravitational-Mass

The notions of inertial-mass and gravitational-mass are conceptually quite distinct, and one of the enduring mysteries of classical and modern physics arises from the fact that the value obtained for the gravitational-mass ratio of two particles compared in a weighing experiment is identical to that obtained for the inertial-mass ratio of the same two particles compared in a collision experiment (to within experimental error). It is for this reason that we speak of the equivalence between inertial and gravitational mass, and tend to use the concepts interchangeably.

However, when we reflect upon the fact that it is impossible to make determinations of gravitational-mass in conditions of free-fall, but perfectly possible to make determinations of inertial-mass in these same conditions, we begin to realize that the concept of inertial-mass must be prior to the concept of gravitational-mass and—by implication—that the concept of inertia must be prior to the concept of gravitation. For this reason, we focus our efforts on the concept of 'inertial-mass', believing that an appropriate perspective will reveal the derivative nature of the 'gravitational-mass' concept.

Our general approach is simple, and consists, in the first instance, of a review of Newton's Third Law which leads us to a statement of its dual role in physics: on the one hand, it is primary in providing for an operational definition of inertial-mass (when, strictly speaking, it is not functioning as a law), whilst on the other it acts as a constraint imposed on dynamical variables through a physical process (when it is functioning as a law). This distinction is fundamental, and leads us to a natural two-fold partition of all Newtonian collision-processes into those which are, in our own terminology, *inertially determined*, and those which are not.

When this simple analysis is generalized to consider the nature of the relationship between the concept of relativistic inertial-mass (rest-mass) and the law of four-momentum conservation, we are led to a similar natural partition of relativistic collision-processes into those which are *inertially determined*, and those which are not.

We subsequently find that the inertially determined relativistic collision process can be given a geometric representation with a formal structure which suggests that gravitational process might be, in effect, a particular case of an inertially determined process. This view is confirmed in additional work. The material of this paper is based on a much more detailed paper which is to be published, along with the additional work, in a late spring/summer issue of '*Galilean Electrodynamics*', 1991

General Geometrical Idea Used

It is within the nature of the quantitative modelling of physical processes that mathematical language must be introduced; consequently, in order that the general reader can keep in touch (hopefully!) with the gist of the basic argument used here, I have included this section to explain, in terms of relatively basic mathematics, the nature of this argument.

Imagine two straight lines drawn in ordinary three-dimensional space, and suppose that these lines intersect at a single point. In this case, it is always possible to position a sphere such that *both* of these lines are tangential to the sphere at their point of intersection; basically, the two lines are said to define a *plane* which is tangential to the sphere at the point in question.

Now imagine that there are three straight lines drawn in the three-dimensional space which also intersect at a single point. This time, it is generally impossible to position a sphere such that all three lines are tangential to the sphere at the point of intersection—for this to be possible, a condition must be placed on the third line (any line can be designated as the third line), and this condition is *the third line must lie in the plane defined by the other two lines*; that is, if three lines, which intersect at a single point, all lie in the same plane, then it is always possible to position a sphere so that the three lines are all tangential to the sphere at the point of their intersection.

We see that the case of three intersecting lines is different in principle from the case of two intersecting lines since, in this latter case, it is always possible to position a sphere so that the lines are tangential to it at their of intersection, whereas in the former case, a condition—or constraint—must be imposed upon the relative disposition of the lines. In mathematical language, it is said that each of the lines lies within a three-dimensional coordinate space, but for three lines intersecting at a single point to be tangential to a sphere at the point of intersection (or any curved surface, in fact), then each one of the lines *must lie in the two-dimensional sub-space defined by the other two (i.e., the plane defined by the other two*).

The relation of the foregoing discussion to the physics of the collision process is as follows: for the Newtonian case, the straight lines are taken to represent inertial trajectories (forces are absent) and the point of intersection is taken to represent a collision event; so, if three particles collide, then the trajectory of each particle undergoes a change, which can be represented as a velocity change. If we denote these three velocity changes as $d\underline{v}_1$, $d\underline{v}_2$ and $d\underline{v}_3$ respectively and note that they are 3-vectors (*i.e.*, lines in a three-dimensional *velocity-change* space with direction and magnitude) then it turns out that inertial mass constrains these three velocity-change vectors to be co-planar.

An analogous situation exists when the Newtonian 3-velocities and inertial-mass are replaced by the 4-velocities and rest-mass of special relativity except, instead of velocity-changes being confined to two-dimensional planes in a three-dimensional coordinate space, we find that particle-worldlines are confined to three-dimensional ‘planes’ in four-dimensional coordinate space-time.

The question arises, ‘where does all this get us?’ and we answer as follows: basically, the physics of rest-mass (*i.e.*, relativistic inertial-mass) tells us that the particle-worldlines into and out-of any collision (the arcane details inform us that we can only consider four or less colliding particles in the special relativistic case, and three or less in the Newtonian case) are confined to a particular surface defined within the coordinate-space; this then raises the possibility that the *geometry* of such surfaces can be used to categorize collision processes, and this is the possibility pursued within this article.

Finally, before diving into the arcania, let me defuse a potential source of confusion for the non-specialist. We speak of *scalar-functions* and of *level-surfaces to scalar-functions*, and we can illustrate the distinction as follows: an example of a scalar-function is given by the square of the Euclidean distance of a point (x,y,z) from some fixed origin, $f(x,y,z) \equiv x^2+y^2+z^2$. An example of a level-surface to this scalar-function is given by the sphere of radius 2, centre the origin, *i.e.*, $f(x,y,z) \equiv x^2+y^2+z^2 = 4$. So, a scalar-function $f(x,y,z)$ is simply an object which has a numerical value for any chosen values of (x,y,z) whereas the level-surface $f(x,y,z)=k$, for some fixed value k, is the set of *all* those coordinates (x,y,z) for which $f(x,y,z)$ has the specific value k. In terms of this language, we say, typically, that rest-mass constrains four-velocities to lie 'in a three-dimensional invariant sub-space (*i.e.*, a plane) of coordinate space ... and that such a sub-space is tangent to a level-surface of some scalar-function defined everywhere on the coordinate space. These latter words could have been used to describe three ordinary lines being confined to an ordinary plane, and noting that such a plane can always be considered as tangential to a sphere (sphere = level-surface of the scalar-function $f(x,y,z) \equiv x^2+y^2+z^2$).

The Newtonian Inertial-Collision

This section is included, not to say anything new about the Newtonian collision, but to illustrate more explicitly the nature of the arguments to be used in the relativistic case.

Consider a closed system of N material particles having respective velocities $\underline{v}_1..\underline{v}_N$ at time t, and $\underline{v}_1+d\underline{v}_1..\underline{v}_N+d\underline{v}_N$ at time $t+dt$ measured from within some inertial frame, and suppose that these particles have pre-determined masses given by $m^1..m^N$. Then Newton's Third Law, expressed in momentum-conservation form, can be expressed as

$$\sum_{r=1}^{N} m^r d\underline{v}_r = 0 \qquad (1)$$

In this form, and with the given interpretation of the $m^1..m^N$, it represents a general constraint imposed on dynamical variables through a physical process.

However, suppose, for the sake of argument, we do not possess any concept of mass so that the coefficients $m^1..m^N$ are uninterpreted, but that we do know values of these coefficients can always be found such that (1) is true for any measured set of velocity changes. What can be deduced about the physics of the Newtonian collision process on the basis of this knowledge alone? The answer to this question varies according to $N > 3$ or $N \leq 3$.

Since classical velocity is a three-vector then, in the absence of any other information, it is a matter of basic linear algebra that for $N > 3$, and for any measured set of velocity-change vectors, $d\underline{v}_r$, $r = 1..N$, it is always possible to determine a non-unique set of coefficient ratios $\underline{m}^T \equiv (1, m^2/m^1 \,..\, m^N/m^1)$ satisfying (1). In other words, (1) is trivially true for this case, and carries no physical significance.

However, if $N \leq 3$, the situation is quite different: in this case, it is a matter of basic linear algebra that (1) can only be true if the velocity-change vectors are constrained to be linearly dependent. This is a strong statement about a physical process which can be experimentally tested and it is found that the observed velocity changes in $N \leq 3$ collision experiments do appear constrained in precisely the way predicted and where, of course, the coefficient ratios, $\underline{m}$—which can be uniquely determined in any such experiment and are found to be independent of the initial velocities, $\underline{v}_r$, $r = 1..N$—are designated as the 'inertial-masses'.

The foregoing analysis serves to remind us of the duality inherent in the relationship (1):

- It is primary in providing for an operational definition of inertial-mass, but can only do this for cases $N \leq 4$.
- In cases for which particle masses have an assumed prior determination, it operates as a general constraint (conservation of linear-momentum) on the collision process, but can only be considered non-trivially true when either the masses have been gravitationally pre-determined, in which case it is a non-trivial statement for all N, or when the masses have been inertially pre-determined, in which case it is a non-trivial statement only for $N > 3$.

This simple analysis reminds us that the $N \leq 4$ collisions are uniquely those in which the property of *inertial-mass* explicitly manifests itself as a fundamental physical property. For this reason, we distinguish such collisions from all others, and refer to them as *inertially determined Newtonian collisions*.

This analysis of the Newtonian case prepares us for a similar analysis of the relativistic case, given in the following sections.

The Relativistic Inertial-collision

In the case of a relativistic collision, it is no longer possible to suppose that any given particle into a collision can be identified with some particle out of the collision and the analysis of the Newtonian collision must be generalized in the following way. Consider an arbitrarily defined collision event, **e**, involving a total of N particles; consequently, if there are n particles into **e**, having respective four-velocities $\underline{V}_r$, $r = 1..n$, then there will be $N - n$ particles out of **e**, having respective four-velocities $\underline{\mathrm{V}}_r$, $r = n + 1..N$. Now suppose that the particles have pre-determined rest-masses given by $m^1..m^N$ respectively then, if four-momentum is to be conserved, we have

$$\sum_{r=n+1}^{N} m^r \underline{V}_r - \sum_{r=1}^{n} m^r \underline{V}_r = 0 \tag{2}$$

In this form, and with the given interpretation of the $m^1..m^N$, it represents a general constraint imposed on a particular physical process.

However, suppose, for the sake of argument, we do not possess any concept of rest-mass so that the coefficients $m^1..m^{\mathrm{N}}$ are uninterpreted, but that we do know values of these coefficients can always be found such that (2) is true for any measured set of four-velocities. What can be deduced about the physics of the relativistic collision process on the basis of this knowledge alone?

The answer to this question varies according to $N > 4$ or $N \leq 4$, and we easily conclude, using arguments which exactly parallel those of the Newtonian analysis, that, if $N > 4$, then (2) is without physical significance in this case. By contrast, if $N \leq 4$, the asserted truth of (2) for this case amounts to the statement that the four-velocities are constrained to be *linearly dependent*. This is a strong statement about a physical process which, as with the Newtonian case, can be experimentally tested and, upon testing, appears to be verified.

Furthermore, when such experiments are repeated using similar incoming particles for all trials, but with varied incoming four-velocities, it is found that the coefficient ratios $(1, m^2/m^1, m^3/m^1, m^4/m^1)$ are identical (to within expected errors) over all trials. These coefficient ratios are, of course, defined as the rest-masses of the particles concerned.

The foregoing analysis serves to remind us of the duality inherent in the relationship (2) which parallels that inherent to (1):

- It is primary in providing for an operational definition of rest-mass (relativistic inertial-mass), but can only do this for cases $N \leq 4$;
- In cases for which the rest-masses have an assumed prior determination, it operates as a general constraint (conservation of four-momentum) on the collision process, but can only be non-trivially true for the cases $N > 4$.

This simple analysis reminds us that the $N \leq 4$ collisions are uniquely those in which the property of *relativistic inertial-mass* explicitly manifests itself as a fundamental physical property. For this reason, we distinguish such collisions from all others, and refer to them simply as *inertially determined collisions*.

Geometry and the Inertially Determined Collision

We have seen that the property of relativistic inertial-mass is uniquely identified with the particular class of collisions categorized as 'inertially determined relativistic collisions'. In the following, we shall show that any such collision can be given a covariant geometric representation so that, effectively, relativistic inertial-mass becomes geometrized in the same way. It is this representation which, ultimately, provides the link between inertial and gravitational processes and identifies gravitational process as a particular case of inertial process.

Since, in any inertially determined relativistic collision, $N \leq 4$, then, according to (2), the trajectories, represented by the four-velocities $\underline{V}_1 \ldots \underline{V}_N$, of the (up to) 4 particles involved in any such collision must lie in some relativistically invariant timelike connected 3-dimensional subspace of the coordinate space. Any such subspace can, in turn, be considered to define a tangent-plane to a spacelike level-surface of some scalar function, U say, and the concept of a geometry on such a level-surface is implicit in the concept of U as an invariant scalar function; consequently, since the trajectories of massive particles are constrained to tangent-planes of such level-surfaces through arbitrary collisions, then we are led to the possibility of a geometric description of the changes between inertial trajectories which occur during inertially determined collision processes.

To derive this *collision-geometry* representation of the inertially determined collision, we first note that, since $y_a \equiv \nabla_a U$ defines normal-vectors to the level-surfaces of U, then

$$g_{ab} \equiv \nabla_a \nabla_b U \tag{3}$$

defines the covariant rate of change of any such normal-vector in the region of its point of application. If this rate of change is integrated along a path which lies within any given level-surface of U and between any two given points, then the result will describe the total variation in the orientation of the level-surface between the two points; consequently, g_{ab}—used in this way—provides a complete prescription of the geometrical properties of level-surfaces in U. However, the integrated result will also describe the total variation in the magnitude of the normal-vector, and this naturally leads us to enquire if purely geometric considerations are sufficient for the purpose of describing changes between inertial trajectories during inertially determined collision processes—in which case, we must replace the normal-vector by the unit normal-vector—or if it is necessary to include the magnitude information which is implicit to g_{ab} defined at (5).

We can answer this question, and gain a further insight into the foregoing considerations, by considering the most simple non-trivial case of $N = 2$ which corresponds to one particle entering an event, and an identical particle exiting the event, where the 'event' is an arbitrarily chosen point on the trajectory of a non-interacting inertial particle; the simplest solution of (5) corresponding to this case is given by

$$U\left(\underline{x}\right) \equiv \Phi\left(\underline{x} - \underline{x}_0\right)$$

where

$$\Phi \equiv \frac{1}{2}\left(x^i - x_o^i\right)\left(x^j - x_o^j\right)g_{ij} \tag{4}$$

$$g_{ab} \equiv g_{ab} = \frac{\P^2\Phi}{\P x^a \P x^b}$$

for some arbitrarily chosen origin x_0^a.

To understand the significance of the spacelike level-surfaces $\Phi = l < 0$, we first note that

$$y_a \equiv \frac{\P\Phi}{\P x^a} = \left(x^j - x_0^i\right)g_{aj} \tag{5}$$

defines the normal-vector to the surface $\Phi = l$ at any point x^a, and so any inertial trajectory which is orthogonal to y_a and passes through x^a lies in the tangent-plane to the surface at this point. Consequently, the surface itself can be considered as a representation of all those inertial trajectories which potentially pass through x^a, and which are orthogonal to y_a. In this way, an infinite sub-class of all possible inertial trajectories is made into a single equivalence class, and each distinct level-surface of Φ will then correspond to a similar, but likewise distinct, equivalence class. Furthermore, the arbitrary choice of origin, x_0^a, allows the direction of the normal-vector, y_a, to be varied arbitrarily so that Φ-functions can be found with level-surfaces which accommodate all possible inertial trajectories. In this way, finally, all possible inertial trajectories can be reduced to a set of equivalence classes, $\Im(l, \underline{x}_0)$, where $\underline{x}_0$ defines the origin for a particular Φ-function, and l defines a particular level-surface for this function.

If we now calculate the magnitude of the normal-vector y, defined at (5), on the level-surface $\Phi = l < 0$, we find $\left|\underline{y}\right| = \sqrt{\left|y_i y_j g^{ij}\right|} = \left|l\right|$, so that any change in the magnitude of the normal-vector on a level-surface of U reflects a change between inertial trajectories which belong in distinct equivalence classes, $\Im(l, \underline{x}_0)$. It follows that if we restrict our attentions to purely geometric considerations—so that only variations in the unit normal-vector are considered—then we exclude the possibility of the most general kinds of change between inertial trajectories during inertially determined collision processes. Since there is no reason a-priori to restrict inertially determined collisions in this way, we retain magnitude information, and use g_{ab} given at (3) as a definition of the geometry on the locally defined *collision-manifold* which provides a geometric description of the general inertially determined collision process.

The collision-manifold associated with g_{ab} is, of course, not yet sufficiently specified since we have not considered the nature of the connections on it. A determination of these requires a discussion of the most general cases of N= 3,4. Consider two distinct points P_1 and P_2 separated by a timelike interval, and let us consider the nature of the geodesics which might connect them in any circumstance; in the simplest case of N=2 they can only be connected by a conventionally defined geodesic in an inertial space-time and this remains a possibility in the more general cases of N = 3,4. However, in these latter cases it is also possible to locate the non-trivial inertially determined collision event between P_1 and P_2 such that these two points can be connected by a 'dog's leg' trajectory passing through the event; consequently, geodesics on the collision-manifold defined by (3) for the $N = 3,4$ cases exist as the union of piecewise inertial segments which are connected at non-trivial inertially determined collision events. It is quite clear that such geodesics can only arise on collision-manifolds which are non-trivially connected so that (3) can be more explicitly presented as

$$g_{ab} = \frac{\partial^2 U}{\partial x^a \partial x^b} - \Gamma^k_{ab} \frac{\partial U}{\partial x^k} \tag{6}$$

where Γ^c_{ab} are the non-trivial connection coefficients. Since the geodesics in this system are inertial between branching events, then the system is everywhere locally flat—except at some countable set of branch points—which implies that the Ricci condition, $g_{ab;c} = 0$, is satisfied everywhere except possibly at this countable set. If we ignore these excluded points—a step which must ultimately be justified—this implies that the connection coefficients must be the Christoffel symbols, and we tentatively accept this to be the case.

Finally, because (6) is expressed in terms of a language normally associated with the concepts of curved spacetime manifolds used in conventional metric theories of gravitation, it is re-assuring to know it can be shown that the manifold described by (6) specified with the Christoffel symbols can only admit solutions which are either globally flat (the trivial collision-manifold which contains no collisions) or are not everywhere locally flat (local non-flatness occurring at inertially determined collision events).

Gravitation 1. Definite Conclusions

The significance of the collision-manifold geometry to gravitational effects appears in the following way:

The notion of a non-interacting inertial particle, potentially realizable at an arbitrarily defined place, is formally equivalent to the notion of the class of globally defined scalar fields defined at (4). It turns out that a spherically symmetric disturbance of any member of this class, which is constrained to satisfy the general collision manifold geometry defined at (6), necessarily generates a particular class of spherically symmetric advanced or retarded potentials. If we restrict our attention to the retarded potentials, it can be shown that the passage of any such potential through any test-particle causes the test-particle to register a disturbed infinitesimal proper-time which, effectively, means that the retarded potential has transferred momentum to the test-particle.

We conclude that the disturbing source and the test-particle have undergone an inertial "at-a-distance" interaction which is arbitrated by a retarded potential and that, effectively, this retarded potential acts exactly as we might expect a "gravitational wave" to act, if such a thing existed.

This view of the retarded inertially-determined potential, as the generator of gravitational effects, is confirmed when we find that it has a multipole representation in which the effect of the monopole component on the infinitesimal proper-time of any test-particle can be made identical (at $O(1/R^3)$) to the corresponding effect described by the Eddington form of the Schwarzschild metric of GR, so that all the standard tests of a gravitation theory are satisfied.

The differences between GR and the presented description emerge when the the neglected $O(1/R^3)$ terms generated by the monopole component are included: We find that the Schwarzschild boundary at $R_s = 2gM/c^2$ still exists as a 'one-way membrane' for test-particles, but the essential singularity which exists in the GR model at $R = 0$ does *not* exist here. Instead, what happens is that at some $R = R^*$, satisfying $0 < R^* < R_s$, gravitational attraction 'turns off', and becomes a repulsion for $0 < R < R^*$; the origin, $R = 0$, then becomes the top of a 'potential hill'. The effect of this on test-particles is that, instead of collapsing into a singularity at $R = 0$, they orbit the origin inside $R = R_s$, oscillating between $R = 0$ and $R = R_s$. This behaviour on the interior on the Schwarzschild boundary has obvious implications for the physics of gravitational collapse, and in the following section we enter in a tentative discussion of what these might be.

When we go on to consider higher order terms in the multipole expansion of the model, we find that dipole terms are *explicitly absent*. This is significant since an independence of dipole terms is the only objective requirement imposed by the binary pulsar observations on any multipole radiation model of a gravitation theory (Will 1981). Consequently, the presented view is capable of describing gravitational phenomena up to, and including, those associated with the binary pulsar so that all the standard modern tests of a gravitation theory are satisfied.

Significant consequences of gravitation-from-inertia are: concepts of curved space-time are made redundant to gravitational physics; the essential singularities at gravitational origins, which are characteristic of both Newtonian gravitation and GR, do not exist; gravitational process becomes a retarded particle/particle interaction of a conventional kind.

Gravitation 2. The Physics of Attraction/Repulsion

It is perhaps helpful to understand how ‘gravitational attraction’ can become ‘gravitational repulsion’ in the presented description: According to this description, gravitational phenomena arise when retarded inertial disturbances, originating at some massive source, interact with test-particles, and result in a transfer of linear-momentum. If such a test-particle is in the unbounded region $R > R^*$, then the transfer of linear-momentum *from* the test-particle *to* the outward-going disturbances. This results in the inwardly directed radial acceleration of the test-particle, that we conventionally attribute to 'gravitational attraction'. However, if the test-partice is in the interior region, $0 < R < R^*$, then non-linear effects cause a reversal in the transfer mechanism so that the transfer of linear-momentum is *to* the test-particle *from* the disturbance, giving rise to a gravitational repulsion in this interior region.

Gravitation 3. Tentative Discussion

The gravitational-repulsion phenomenon in the test-particle/point-massive-source case is not observable since the test-particle is confined within the Schwarzschild boundary of the massive-source. However, suppose the idealized model of the test-particle/point-massive-source interaction is replaced by the less idealized model of the point-massive-source/point-massive-source interaction: We can visualize this as two point massive sources, each sitting within its own Schwarzschild boundary, being attracted to the other. Suppose their respective orbits are such that the two particles fall into each other; the two particles will then reside within a joint Schwarzschild boundary which is a connected surface formed as the union of the two original surfaces - we can think of two soap bubbles coming into contact. The two particles will continue to fall towards each other until attraction becomes repulsion, and a 'bounce' occurs. Since the geometry of the joint Schwarzschild boundary must depend on the relative displacements of the two sources it contains, we can easily imagine that it might revert into two disconnected surfaces under the action of the bounce. In this case, the bounce-phenomenon will be observable to an external observer.

The question then arises, does any evidence exist which is consistent with this kind of view? There is evidence, which Victor

Clube has been largely responsible for putting before us: Victor's studies on the motions of globular clusters (used as characteristic marker particles) within our own galaxy indicate that the class of all these clusters appears to be in a state of outward radial motion away from the galactic centre. In his opinion, this outflow indicates that all of the material in the galaxy must be considered to share the same state of motion, so that a picture emerges of our galaxy being, at present, in an expanding state (a state which is not to be confused with the supposed Big-Bang driven expansion of the Universe). Victor sees this present state as simply the expanding phase of a general in-out pulsation of the galaxy and, as he points out in his article, it is very difficult to understand this condition on the basis of either Newtonian or Einsteinien gravitation. As a consequence, he uses the phenomenon to make hypotheses about the possible behaviour of matter in very dense conglomerations (his idea of temporary mass inflation), which would go some way towards accounting for the observations. However, Victor's image of our galaxy, as a system undergoing cyclic expansion/contraction, is entirely consistent with the gravitational model discussed here, in which collapsing matter eventually goes into a 'bounce' phase.

A Speculation: Quantized Redshifts

The presented work is predicated upon the recognition that inertial effects are only explicitly manifested in the particular class of collision we have termed as an "inertially determined collision", and I have indicated how the resulting theoretical structure supports retarded inertial interactive effects which cannot be distinguished from gravitational effects. What I have not done—although I have tried—is to extend the notion of the inertially determined collision between massive particles to interactions between photons, or electromagnetic waves. I have a strong suspicion (but no evidence!) that such interactions will account for the redshift quantization phenomenon, uncovered by Tifft (1986), and recently substantiated in analyses by Napier (1991).

However, my own technical inadequacies apart, my abortive attempts to implicate the inertial interaction in redshift quantization suggested to me that the problems encountered were rooted more in a general lack of understanding concerning the nature of what a photon actually is, (for example, is it really massless, or does it possess a very small mass as Vigier (1990) believes?), and seemed to draw me into a view of the photon which was strangely echoed in Jacques Trempe's posthumous article in *APEIRON*.

I concluded that the study required so much speculation on my part, about the nature of the photon, that it was, at best, premature and that the problem of redshift quantization is perhaps not yet ripe for resolution (at least, not by me!).

Conclusions

We have analysed the class of all collisions between massive particles to show that this class could be partitioned into those collisions involving four particles, or less—which we have called the subclass of *inertially determined collisions*—and the rest, and we have subsequently shown that the inertially determined collisions can be given a representation in terms of the geometry on a *collision manifold*.

In additional work, we show that this collision manifold geometry supports not only the inertially determined interactions between colliding particles, considered here, but also inertially determined interactions between non-colliding particles which are arbitrated by retarded potential fields. We subsequently show that these retarded inertial processes cannot be distinguished from gravitational processes, up to and including those associated with the binary pulsar.

In this way, the equivalence between inertial and gravitational mass can be finally understood, and General Relativity is made superfluous to the needs of physics.

This view of gravitation, as an inertial process, also implies that there are no essential singularities involved in gravitational phenomenon (singularities = bad physics?) and that, associated with the non-existence of such singularities, gravitational attraction becomes gravitational repulsion for very near interactions. There is evidence, in galaxy motions, which lends credence to the existence of such phenomena.

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