The Perfect Cosmological Principle and the Hubble Effect

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A wide range of astronomical data are inconsistent with the expanding Universe theory. A more adequate framework is provided by the Seeliger-Einstein steady state cosmological model, in which the Hubble effect is a consequence of the energy degradation of photons due to the asymmetry of gravitational interaction of photons with the matter distributed in the front and back half-spaces. Gravitational drag of photons and neutrinos is caused by the equality of velocity of propagation of gravitation and the speed of light, such that the gravitational signal does not pass into the front half-space. The background 3° K microwave radiation is regarded as resulting from the energy degradation of neutrinos. Different hypotheses concerning the aging of photons are considered.

The Steady-State Seeliger-Einstein Universe and the Perfect Cosmological Principle

In a footnote to a paper on the non-uniformity of lunar motion, A. Einstein (1919) noted Seeliger's important cosmological research

(Seeliger 1909) and compared it with his own work 'Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (1917). '*What is presented there in Section 1 comprises Seeliger's idea*', Einstein emphasized. The cosmological term Λ in Einstein's equation was necessary to assure the steady state of the Universe. According to Seeliger, the gravitational paradox is removed by the introduction of the cosmological constant, with the law of gravitation assuming the form

$$F = \frac{m_1 m_2}{r^2} G \exp\left(-r\Lambda\right)$$

By combining these two approaches, we can treat the radius $R = \Lambda^{-1} = c/H$ as the 'effective radius' of the Universe, which, as Seeliger assumed, actually has an infinitely large volume and is not expanding (H is the Hubble constant and c is the speed of light). To explain the Hubble effect in the absence of expansion of the Universe, we must adopt the hypothesis of the 'aging of photons', proposed in 1929 by F. Zwicky (1929a,b). In different versions of this hypothesis, the decrease in the energy of a photon with distance or with time t is described by the same equation

$$E_t = E_0 \exp\left(-Ht\right)$$

According to this formula, the redshift is

$$z = \frac{l_t - l_0}{l_0} = \exp\left(Ht\right) - 1$$

Louis de Broglie suggested a version of this hypothesis based on the postulate that a photon has a nonzero rest mass (de Broglie 1962, 1966), This assumption was later developed by J.-P.Vigier, J. C. Pecker and co-workers (Jaakkola *et al.* 1975), W. Yourgrau (Yourgau

& Woodward 1971, 1975), A. Gerasim (1965, 1975) and other physicists and astronomers.

The simple empirical relations between the cosmological and physical constants, to which K.P. Stanyukovich, R. Dicke and P. Dirac drew the attention of physicists, represent the primary evidence in favor of the hypothesis of a steady-state Universe. In the first instance, the observed density ρ of matter and the radius of curvature $R = c/H$ are form a ratio that corresponds to Mach's principle and a quasi-Euclidean topology of space. Since the gravitational radius of the Metagalaxy is $R_g = 2GM/c^2$, while its 'effective' mass is $M = 4pR^3\rho/3$, the total energy Mc^2 is approximately equal to the potential energy $U = -Gm^2/R$. Since the mass is constant in accordance with the law of conservation of matter, we thus obtain $G/R \approx c^2/M =$ constant.

P.A. Dirac drew attention to the approximate equality of the 'Large numbers', close to 10^{40}, *i.e.*, $(R/r_e) \approx (m/m_p)(e^2/Gm^2) \approx N^{1/2}$ (Dirac 1974). Here $r_e = e^2/mc^2$ is the classical radius of the electron, m_p is the mass of the proton, and $N = (M/m_p) \approx (10^{40})^2$ is the number of nucleons in the Metagalaxy of radius R. P. Jordan combined this with the ratio between the density of matter in the state of maximum concentration, *i.e.* the nuclear density $r_{max} = 10^{14}$ g/cm^3 and the density which corresponds to a uniform distribution of matter in space,

$$(r_{max} : r) = (10^{14} : 10^{-29}) = 10^{43} \approx (e^2/Gm^2) = 4.2\times10^{42}.$$

From the approximate equality of the 'Large numbers' Dirac concluded that $GR =$ constant. From the ratio of densities we find $GR^3 = 3Me^2/4pm^2 r_{max} =$ constant. Together with Dirac's relation, this leads to the conclusion that $R =$ constant and $G =$ constant. The same conclusion of constancy (perhaps with small fluctuations about the mean value) of the size of the Metagalaxy and the gravitational constant is obtained when the empirical relations are combined, G/R = constant and GR = constant.

Replacing the factor (m/m_p) in Dirac's equations by the fine structure constant $a = 2pe^2/hc = 1/137$, from the equation $(R/r_e) = a(e^2/Gm^2)$ we obtain the value $H = 108$ km/sec. Mpc $= 3.5\times10^{-18}$ sec^{-1} as the upper limit corresponding to the minimum distances from a light source (Kropotkin 1971). This value corresponds to the estimates of de Vaucouleurs, D. Hanes ($H = 100\pm12$ km/sec. Mpc), and D. Lynden-Bell ($H = 110\pm10$ km/sec. Mpc) (Hanes 1982, Lynden-Bell 1977). According to the exponential formula for photon energy degradation, the value $H_t = (E_0 - E_t)/E_0t = 91$ km/sec. Mpc if the distance $r = ct = 1000$ Mpc.

The very simple relation $RL^2 = r_e^3$ connects three natural units of length: R, r_e, and Planck's fundamental length $L = \sqrt{G\hbar/c^3}$. The products $GR = a^3\left(\hbar^2/m^3\right)$ = constant and $G^2M \approx a(e^4/m^3)$ = constant also acquire a simple form.

For $H = \left(m^3c^4G\hbar/e^6\right) = 108$ km/sec. Mpc and $E_t = E_0\exp(-Ht)$, the energy loss of each photon per oscillation is the same in radiation of any frequency and can be expressed as:

$$E_{\min} = 2p\,\hbar H = 2pa^{-2}\left(Gm^2/r_e\right) = m_{\min}c^2$$

Here $m_{min} = 2.5\times10^{-65}$ g is the mass of a graviton (according to D.D. Ivanenko), or a value close to the rest mass of a photon (Yourgau & Woodward 1971, 1975, Gerasim 1965). It is possible that the slow decay of a photon occurs with the separation of one graviton in each vibration and a corresponding decrease in the energy and mass of the photon (Kropotkin 1988, 1989). The factor inside the parentheses in our equation represents the simplest expression for a quantum of potential gravitational energy, constructed from fundamental constants. The construction of the formula shows that a quantum-mechanical process occurs in photons with the participation of gravity, which leads to a decrease in photon energy, *i.e.*, to the cosmological redshift.

As was pointed by Zwicky and Holmberg, a study of spatial distribution of galaxies and their velocities in the clusters using statistical-mechanical methods yields estimates of their ages of approximately 10^{12}–10^{18} years (Zwicky 1942, 1947). Such estimations are incompatible with the inferred time of expansion of the Universe $T < 2\times10^{10}$ years.

Major predictions made on the grounds of the expansion theory are not confirmed by the astronomical data. It has been suggested, for example, that microwave ‘ relict’ radiation retains information about the heterogeneities in the matter distribution which existed at the earlier stage of the expansion of the Universe. However, observations made during 1975-1980 with the RATAN-600 (USSR) and Kitt Peak (USA) radiotelescopes show that the expected fluctuations are absent. As Partridge has noted, these observations introduce restrictions on the formation conditions of bound systems in expanding Universe models (Partridge 1980). It is even more difficult to reconcile the results of latest investigations with the cosmological expansion models (Parijskij *et al.* 1982).

Nor are the assumed evolutionary cosmological effects supported by the astronomical observations (Jaakkola *et al.* 1979, Jaakkola 1989, La Violette 1986). Successive refinements in techniques of optical research have uncovered quasars with values of redshift z increasing from $z = 2.22$ in 1950, 2.8 in the beginning of the 1970s, 3.4 in 1973, 3.82 in 1982, up to $z = 4.43$ in 1987. Five quasars with $z > 4$ are presently known. Apparently the space density of bright quasars does not decrease at high redshift values (Warren *et al.* 1987).

These data cast doubt on the so-called anthropic principle of cosmology, which has been associated with the Big Bang and the expansion of the Universe, and favour a more natural Perfect Cosmological Principle. The Perfect Cosmological Principle was formulated in the 1950s by Bondi, Hoyle and Gold. This principle states that the Universe, when considered on the large scale, is identical in its fundamental features (the mean density of matter, relative abundance of different chemical elements) not only spatially, everywhere, but also temporally, always. The requirement of homogeneity and a steady state leads to the conclusion that space is Euclidean.

Attempts to combine the perfect cosmological principle with an interpretation of the redshift as a Doppler effect led to the hypothesis of continuous creation of matted (Bondi, Dirac). However, this contradicts the law of conservation of matter. The tests based on astronomical and geological data have shown the inadequacy of this hypothesis (Towe 1975). The perfect cosmological principle may reasonably be combined only with the hypothesis of spontaneous loss of photon energy, the so-called 'aging of photons' proposed by Zwicky and de Broglie.

The Nature of the Cosmological Redshift.

In 1929 E.Hubble formulated a well-known correlation between the radial recession velocities of extragalactic nebulae and the distance to them. In the same year, Zwicky suggested that, assuming the photon has a gravitational mass $2p\hbar n/c^2$, we should expect a gravitational drag on photons from the gravity field of the Metagalaxy. In his opinion, this mechanism would lead to a gradual decrease in photon momentum and energy. The momentum would then be transferred to the masses whose gravitation field is crossed by the ray of light. For

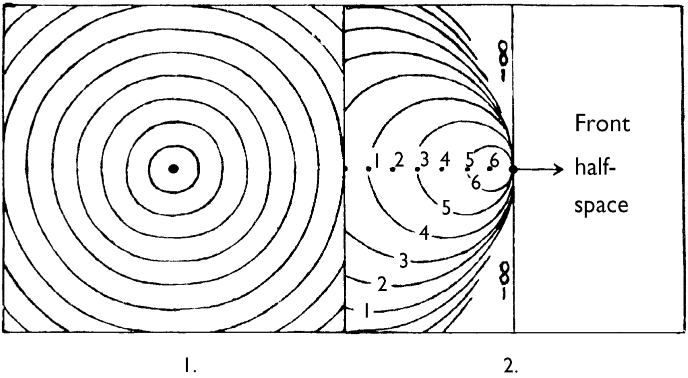

Figure 1 - The propagation of gravitational signals from a fixed mass (1) and from the mass moving with the same velocity as the signal (2). The diagram shows the fronts of propagation of the signals emitted by the mass A at points 1-6 at equal time intervals.

the Hubble constant, Zwicky obtained a value 3×10^{-16} sec$^{-1} > H > 3\times10^{-21}$ sec^{-1} (Zwicky 1929a,b).

The basic idea suggested by Zwicky has retained its significance because gravitational effects are related both to the spatial distribution of masses and potentials of the gravitational field, and to the velocity of the moving mass. One example is the Lense-Thirring effect that arises in the case of a rotating mass. These effects arise because the propagation of gravitational waves and interactions have a finite velocity $c_g = c$. Thus a particle (photon, neutrino) moving with the speed of light undergoes a gravitational interaction with the masses of the Metagalaxy as well as with any immovable body fixed in the cosmological frame of reference (Figure 1).

Let us divide the space into two parts by a plane that crosses the particle and is perpendicular to the direction of its motion. If the particle moves with the same speed as the gravitational signal, its attraction to the masses of the back hemi-space (half-space) creates a drag force acting on the particle (Figure 2).

A similar asymmetry of the cosmic gravity field's influence on a moving mass has been demonstrated by Weinstein and Keeney, and by Ghosh in their concepts based on Mach's principle. A drag force acting on any body moving freely under its own inertia is determined as $F = -pH$, where $p = mv$ is the kinetic momentum of this body (Weinstein & Keeney 1973, 1974). In the case of photons, this drag leads to a gradual loss of energy. This energy loss is proportional to $r^{1/2}$, where r is the mean density of matter (Ghosh 1984).

A gravitational drag on photons as a mechanism for the Hubble effect also occurs in the scalar theory of gravitation proposed by Ferrari (1984).

Let us consider the problem of gravitational drag of photons using two different approaches, but primarily, the approach based on quantum gravitation theory. In such a theory, the interaction is due to an exchange of field quanta, *i.e.*, the exchange of virtual gravitons (gravitational signals). The exchange requires propagation of gravitational signals between the masses *A* and *B* in either direction, that is, from *A* to *B* as well as from *B* to the mass *A*. Since the velocity of gravitational quanta is presumably equal to the speed of light, we may conclude that gravitational interaction occurs on the condition that the mass *B* is 'visible' from the mass *A*, and the mass *A* is visible from *B* at same time instant. If the mass *A* is moving in relation to *B*, the position of mass *A* visible from *B* differs from the actual position of *A* at the moment of signal reception.

If the mass A moves with light velocity, its gravitational signal does not pass into the front hemi-space at all. The situation where both masses *A* and *B* are fixed in the cosmological frame of reference differs essentially from the case of rapid motion of the mass *A*. In both cases signals from *B* reach mass *A*. But in the second case the gravitational signal from A does not arrive at B because it does not pass into the front hemi-space if the velocity of mass A is as high as the velocity of signal propagation. Therefore, in the second case an exchange of gravitational signals does not occur. Mass A is attracted only to the matter in the back hemi-space, since exchange by gravitational signals with these masses is possible.

Let us now consider the difference between photon energy loss in local gravity fields and in the cosmos, where the background potential j_0 is the same all over space. It is determined by the gravitation of all masses in the Universe. Let us designate the potential energy of a photon $U = mj = m(j_0 + j_1)$where j_1 is the gravitational potential of a local field. Because the value of j_0 is identical everywhere, it may be omitted when considering the physical phenomena in the local fields, Only the values of local potentials additional to j_0 are taken into account. The change of photon energy ΔU corresponds to a red or blue gravitational shift of spectral lines.

In the case of cosmological redshift, the change of the frequency ν and energy E of photon is due to a direct decrease of photon mass m in the product $U = mj$. A photon performs work in overcoming the drag force F which is proportional to its mass and caused by its attraction to the matter of the back hemi-space. This leads directly to a decrease in photon energy according to exponential formula.

Analogous conclusions on the gravitational drag of light may be obtained from the Newtonian model, corrected by Seeliger, because the formula of reciprocal gravitational attraction of masses m_1 and m_2 accounts for the square of the distance r between them. As long as the signal from a particle (photon, neutrino) moving with the speed of light does not pass into the front hemi-space, no information will be obtained in that part of space about the geometrical point of the particle’s location at a given moment. Hence, there is no way to define the distance r between a particle and any masses distributed in the front hemi-space at this instant. Consequently, the gravitational attraction between the particle and the matter in the front hemi-space may be equated to zero.

To obtain a rough estimate of the effect of gravitational drag of photons, we shall consider the whole bulk matter in the back hemi-space as the mass $M = 4pR^3\mathbf{r}/3$, distributed with uniform density $\mathbf{r}$, on the volume of a sphere that has radius $R = c/H$, *i.e.* the effective radius of the Universe in Seeliger's model. This postulate permits us to apply the Newton law further. As we know, the attraction of a particle located at the surface of a homogenous sphere on the sum total of masses within the sphere is equal to the attraction on mass M located at the center of the sphere. Thus the gravitfional acceleration creating the drag effect,

$$a = -\frac{GM}{R^2} = \frac{-4p\,G\,r\,R}{3}$$

A photon of a mass $m = E/c^2$ moving through a distance $L = ct$ performs a quantity of work

$$\Delta E = -\Delta U = -maL = 2p\,\hbar\Delta n$$

Therefore, the loss of photon energy during the time t is

$$\Delta E = \left(\frac{E}{c^2}\right)\cdot\left(\frac{4p\,G\,r\,R}{3}\right)ct\ ,$$

with the result, in accordance with Mach's principle, that

$$H = \left(\frac{\Delta E}{Et}\right) = \sqrt{{4p\,G\,r}\big/_{3}}\ .$$

While the density $\mathbf{r} = 10^{-29}$ g/cm^3, the equation corresponds to a value of the Hubble constant $H = 52$ km/sec.Mpc. This value of $\mathbf{r}$

corresponds to the astronomical data that account for the ‘hidden’ (missing) masses in the nuclei and halos of galaxies.

Different Hypotheses for the Energy Degradation of Photons

Apart from the hypothesis of gravitational drag of photons tentatively outlined by Zwicky and subsequently developed in the above mentioned studies by Weinstein and Keeney, Ghosh, Ferrari and myself (Kropotkin 1988, 1989), several hypotheses based on other postulates have been proposed. All of them yield the same exponential formula for a gradual degradation of the energy of photons. More comprehensive studies may possibly show how to combine these versions with the gravitational hypothesis.They surely reflect different aspects of quantum electromagnetic processes occurring in the photon under the influence of cosmic gravity field.

One of the first attempts at a non-Doppler explanation of the cosmological redshift was Bogorodsky’s hypothesis of the ‘aging of photons’ resulting from a specific gravitational self-induction, caused by proper gravitational field of the photon (Bogorodsky 1940).

An interesting approach to the problem of the emission of gravitational waves by a photon was suggested by Brekhovskikh. Using the analogy of the aging of photons with Cherenkov effect and Rosen’s conclusions on the peculiarities of electromagnetic and gravitational waves, he deduced that a spherical electromagnetic wave radiates gravitational waves in the course of its propagation (Brekhovskikh 1945).

Fürth later tried to make an analogy between the photon and the electron, which loses its energy by radiation when it moves along a curved path. He suggested that a photon gradually loses its energy by emission of gravitons or gravitational waves because it moves along a curved trajectory. The radius of curvature of this path corresponds to *R*, *i.e.* radius of Riemann curvature of the Metagalaxy (Fürth 1964). Another advocate of the theory of aging of photons, the Romanian astronomer Gerasim, also proposes the emission of gravitons by photons (Gerasim 1975).

We have already mentioned a version of photon aging suggested by de Broglie (1962, 1966). The same assumption concerning a non-zero rest mass of photon was used by Yourgrau and Woodward (1971, 1975) in combination with the Proca equations of elecrodynamics.

Crawford (1979), Horedt (1973) and Popov (1978) regarded the exponential formula of energy degradation of photons as a formula of spontaneous decay of photons. Popov has defined several features of a hypothetical particles generated in the course of the photon decay.

According to Kipper (1974, 1979), if the vacuum is a complex of electromagnetic oscillators, it must have a minute non-zero electrical conductivity, $s = 10^{-17}$ sec^{-1}. He equates this approximately to the value of the Hubble constant *H*. From this comes the conclusion that photon energy decreases gradually due to the interaction of electromagnetic waves with oscillators in the vacuum. The energy of radiation is transferred slowly to the vacuum. Hornbostel and Marcinkowski (1971), meanwhile, have considered the aging of photons as a spontaneous decrease of frequency in the electromagnetic wave group.

Evolutionary Cycles in a Steady-State Universe

Jaakkola, Moles and Vigier (Jaakkola *et al.* 1979, Jaakkola 1989) and La Violette (1986) have pointed out the absence of evolutionary effects over cosmological distances, which are to be expected in the expanding Universe. The perfect cosmological principle, on the other hand, demands the existence of perpetual eternal cycles of energy transformation.

The concept based on this principle and on the hypothesis of energy degradation of photons, also known as the 'tired Iight' hypothesis, leads to some consequences that can be tested. The gravitational paradox may easily be removed by formulating the law of gravitation according to Seeliger. Olbers' paradox may similarly be removed through the energy degradation of photons by the exponential law (Ferrari 1984). In this case, the photometrical radius of the Universe becomes equal to its effective radius $R = c/H$.

A decrease in momentum due to the asymmetry in the gravitational interaction of a particle with the matter of the front and back hemi-space must also occur when neutrinos move with velocities close to the speed of light. As Glashow has shown, if neutrinos have a non-zero rest mass, they must lose energy in the form of electromagnetic radiation (Rujula & Glashow 1980). This process may be the cause of 3 K microwave radiation in space (Marchant 1984).

While the gravitational and photometrical paradoxes may be easily removed using the cosmological model considered here, the thermodynamic paradox in a Universe that has always existed can be removed only through additional assumptions. The increase of energy in the cosmos may be cancelled by absorption of the energy of electromagnetic and gravitational radiation in black holes which comprise the 'hidden mass' of galaxies, with its subsequent transformation and the regeneration of stars. Processes inside black holes may possibly occur as if the arrow of time were reversed.

There is evidence that cosmological processes have an equilibrium, cyclical nature. Ward came to such a conclusion on the basis of the fact that in large volumes of the Metagalaxy, the amount of energy lost by photons in the optical range through the redshift is equal to the amount of energy that stars emit in the same volume of space per unit time (Ward 1961). This is supported by the latest estimates of radiation density, the value of *H*, and the distribution of stars in space.

In a settled equilibrium state, the number of astronomical objects existing at any definite stage of the cycle must be proportional to the lifetime of this stage in the total duration of the cycle. This principle must underly, for example, the dynamically stable T-associations and O-associations, consisting of very young stars, that have been studied by Ambartsumian.

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