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On the Redward Shift of Spectral Lines of Nebulae*

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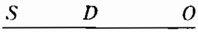
The characteristics of a wave traveling in a medium in which the velocity of light is a function of time is considered and it is shown that one is able to account for the redward shift of spectral lines from distant nebulae on this basis. For simplicity, the function $c_1 = c(1-\alpha t)$ is assumed and numerical calculations are made yielding a value of $\alpha = 1.81 \times 10^{-17}$ cm per sec. as being necessary to account for observed redward shift.

THE redward shift of spectral lines of nebulae has been the basis of much speculation in recent years. Next in importance to the experimental fact of the shift itself is the shift-distance relationship announced by Hubble and Humason,[1] according to which the redward shift is proportional to the distance to the nebula.

The theory most commonly heard for explaining these observations is that in which the shift is interpreted in terms of the Doppler effect, implying a recession of the nebulae with velocities proportional to their distances. Other attempts have been made to account for the phenomenon. For example, Zwicky[2] suggests that there may be a "gravitational drag" on light; Milne[3] proposes a certain kinematical model and MacMillan suggests that there may be, in some unaccounted manner, a loss of energy by the photon in its long journey through space. Another explanation is offered herewith as one which has the advantage of considerable simplicity and avoids the necessity of an "expanding universe." It is based on the assumption that the velocity of light throughout space changes slowly as a function of time.[4]

At least in the special relativity, the velocity of light has been considered as a fixed and constant property of space, but this is itself an assumption and we have not had nearly sufficient time over which to take observations to either prove or disprove such an assumption. In the general relativity theory, there is less ground for assuming the constancy of the velocity of light with time, as pointed out by Vrkljan.[5] And, finally, in developing the theory below, it is not necessary to consider that the velocity of light in a matter free, field free space is changing, for it will be sufficient if we can think of the matter content or the field content as changing and so producing, in some manner, a change in velocity. With this preliminary statement the line of reasoning would run along as follows:

Let us consider the case of a source of light and an observer, separated by a distance D.



We will allow light to pass from S to O with constant velocity establishing a train of waves, the number of these waves being given by $N = D/\lambda = \nu t$, where ν is the frequency at the source and t is the time for light to travel the distance D. Let us assume that in some way the velocity of light suddenly changes from c to c_1, where for definiteness c_1 is less than c. In this process the length of the waves between S and O is not changed for the velocity of each element of the wave changes at the same time by the same amount. The rate, however, at which they thereafter proceed towards the observer, is reduced and the number of waves which will reach the observer per unit of time is reduced. The time required for the N waves to sweep through to the observer will be increased to t_1 where $t_1 = D/c_1$ and the rate of arrival of waves will be $N/t_1 = \nu t/t_1 = \nu c_1/c$. This new rate of arrival of waves will persist until the train of waves standing in the space at the moment of change of velocity has swept out, and there would be a redward shift of spectral lines. After

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[1] Hubble and Humason, Astrophys. J. **74**, 43 (1931).

[2] F. Zwicky, Proc. Nat. Acad. Sci. **15**, 773 (1929).

[3] Milne, Nature **130**, 9 (1932).

[4] The change of velocity of light here contemplated is not related to the alleged changes discussed by de Bray in Nature **127**, 522, 739, 892 (1931) and elsewhere, but is of an entirely different order of magnitude, being very much smaller.

[5] Vrkljan, Zeits. f. Physik **63**, 688 (1930).

this interval, however, the rate of arrival will be restored to the original rate, although the wavelength will be shorter.

A more important case is that in which the velocity changes continuously rather than suddenly and for this discussion the more interesting as well as the simplest case is that in which the velocity changes linearly with respect to time. For simplicity let us consider that this change is a decrease so that the velocity of light at any time is given by $c_1 = c(1-\alpha t)$ where c is the velocity of light throughout space when $t=0$. Let frequency of the source $=\nu$. Then the number of waves, at $t=0$, in 1 cm at source is $n = 1/\lambda = \nu/c$.

Let $t_1 =$ time to travel distance D to observer O with the changing velocity. At that time, then, the velocity is $c_1 = c(1-\alpha t_1)$. The length of any particular wave, having once started, is not changed and the waves in the original 1 cm train $= n = \nu/c$. The time for this group to pass the observer is $1/c_1$. $\therefore$ the new rate of arrival is $\nu_1 = n/(1/c_1) = nc(1-\alpha t_1) = \nu(1-\alpha t_1)$ or $(\nu-\nu_1)/\nu = \alpha t$ and thus there is a redward shift which is proportional to the time since the wave left its origin. If α is small $t_1 = D/C$ approximately and $(\nu-\nu_1)/\nu = \alpha D/c$. Thus it is seen that the original simple assumption leads one definitely to a redward shift which is proportional to the distance to the source and this is in agreement with the Hubble Humason observations.

The shift has been considered above in terms of change in frequency of the arriving wave. It may be argued that in our optical work, as with gratings, we measure wavelength and not frequency. Consideration from this point of view leads to the same redward shift. Thus the sodium line emitted by a source in our laboratory has a certain wavelength λ'. Sodium light from a star a million years away was emitted at a time when the velocity of light was greater by the factor $(1+\alpha t)$ and the wavelength λ'' was therefore longer, given by $\lambda'' = \lambda'(1+\alpha t)$. Since this wave, having once been started on its journey, remains constant in length, it arrives at the grating longer than the locally produced wave by the factor $(1+\alpha t)$. Thus $(\lambda''-\lambda')/\lambda' = \alpha t = \alpha D/c$ as before.

Let us now estimate the value of α for light from the nebulae. Hubble and Humason conclude from their Mt. Wilson observations that there is an apparent velocity of recession of 558 km/sec. for a nebula 1 million parsecs. (3,259,000 light years) away. This is calculated from what amounts to a redward shift of 12.2A for the H_α line of 6562.8A. But $(\nu-\nu_1)/\nu = (\lambda_1-\lambda)/\lambda_1 = \alpha t$, $\therefore$ $\alpha t = 12.2/6575 = 1.87\times10^{-3}$ so that $\alpha = 1.87\times10^{-3}$ for a unit of time corresponding to 10^6 parsec.

For other units of time α is given as follows:

Unit of time	α
3.259 years	1.87×10^{-9}
1 year	5.72×10^{-10}
1 sec.	1.81×10^{-17}

If we accept the work of Hubble and Humason as showing that the redward shift is proportional to the distance, then, on the basis of the present theory, we must conclude that the uniform decrease in velocity has been going on at least as recently as the time to the nearest star or nebula for which redward shift has been determined and for as long a time as that to the most remote nebula for which the redward shift has been observed, i.e., for some 150 million years.

It will be noted that if this rate of decrease continues, the velocity of light will be zero after about 1.7×10^9 years. The difficulty is reduced if we assume that the velocity of light is a periodic function of time and that for a long period we have been on a nearly linear portion of the velocity-time curve. While it does not appear now it may develop, on more complete measurements, that the redward shift is not strictly proportional to the distance and in that event it would not be necessary to assume that the velocity has been changing at a constant rate over the past 150 million years.

It has been suggested that it would be helpful in accounting for the redward shift if we could find a "photon leak." There has, however, been no satisfactory proposal as to how this leak might occur. The theory above seems to offer a simple explanation of how a photon may lose energy. For if the frequency of emission is ν then the photon energy is $h\nu$. As a result of the decrease of velocity during transit the rate of receipt of waves becomes ν_1. The wavelength for that train remains constant and the apparent photon energy is $h\nu_1$. The corresponding loss in photon energy is associated with the loss of velocity, for $h\nu = hc/\lambda$ and $h\nu_1 = hc_1/\lambda$ and thus a decrease in velocity appears to carry with it a "photon leak."

It may be argued that if one desires some other explanation than the Doppler principle for the redward shift there is no reason for selecting the velocity of light as the "victim" but that one might equally well assume that our units of length or our units of time are changing. So far as the unit of length is concerned it should be noted that it is not a case where we measure the wavelength of nebular light at the time of its emission, say 10^8 years ago, and compare it with the wavelength of light emitted now during which 10^8 years our meter stick may have changed but that we are comparing two wavelengths, one an "old" one and the other a "young" one and are doing this today with the same meter stick and the outstanding experimental fact is that the wavelengths are different.

The same argument applies to a possible change in time units. We are comparing the frequency or the time of vibration *today* of two waves, one of ancient origin and one of recent origin and we are determining these with the same time unit. Again the outstanding observational fact is that the frequencies, as measured today against any unit of time, are different. One may, of course, contemplate some more subtle change in unit of time than that above but this could lead only to the conclusion that the frequency of oscillation, corresponding, say, to a Na atom, has changed in the meantime; but to abandon the idea of the constancy of frequency as a characteristic of an atom and as a reliable unit of time is a very difficult thing indeed.

It must be admitted that the theory given above as an explanation of redward shift is subject to criticism because of its high degree of ad-hocness and can be justified only if it finally connects up with other phenomena or suggests useful experiments. As a step in this direction it may be pointed out that a change in velocity of light in space as a function of time does not necessarily carry with it the idea that the velocity in matter free, field free space is variable.[6] It occurs to one, for example, that the permeability and the dielectric constant, and therefore the velocity, may be dependent slightly on radiation density. No suggestion of such dependence is indicated by Maxwell's equations but this, of course, is because no physical concept corresponding to this was included in the original framework of his problem and therefore cannot be present in the final result. We have become quite accustomed to the idea of matter turning into radiation and in recent years we are becoming familiar with the thought of radiation going into matter. If these processes are going on it is not at all probable that they are proceeding at the same rate and thus we conclude that radiation density throughout space is, on the whole, increasing or decreasing. If, then, it should develop that the velocity is a function of radiation density we would have a satisfactory basis for the explanation of the redward shift of nebular lines. It is not, of course, to be assumed that such changes would be exactly the same in all parts of the universe; but any departure from uniformity would imply a departure from the Hubble Humason law. Present uncertainties, however, as to the distances to nebulae do not permit an answer on this point.*

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[6] The suggestion has been made that such changes in velocity as indicated above might be due to thinly dispersed matter. This view is not tenable for it would mean that the amount of such matter has been increasing for the past 10^8 years or more. During that time the "index of refraction" of space on this theory has changed by the amount $5.72 \times 10^{-10} \times 10^8 = 5.72 \times 10^{-2}$. This is about 200 times as great as the change in index due to a normal atmosphere of air.

* Since preparing this article my attention has been called to an article by Gramatzki, Zeits. f. Astrophys., Apr. 1, 1934, in which he endeavors to derive the equation of a wave in such a medium as is here considered. Unfortunately the equation he obtains seems to be incorrect. This will be taken up in an article to appear soon.