

The Yilmaz Cosmology

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Abstract. We investigate the Yilmaz Cosmology by working out the Friedmann-type equations generated by assuming the Cosmological metric for a flat space for the Yilmaz theory of gravitation. In the case of matter-conserving Cosmologies we find the theory demands that the total energy density of the Cosmological fluid is zero. No configuration of vacuum radiation and inert matter can be found that is both compatible with this constraint and with observation. The steady-state Cosmology may, however, be viable.

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INTRODUCTION

Yilmaz [1-6] has published a theory of gravitation which is Einstein GR supplemented with a novel stress energy term. Writing

$$T_{\mu}^{\nu} = \tau_{\mu}^{\nu} + t_{\mu}^{\nu}, \quad (1)$$

then τ_{μ}^{ν} is the traditional stress-energy tensor of the sources (matter and fields). The additional term t_{μ}^{ν} , which is a function of the metric tensor, stands for the stress-energy of the gravitational field, alleged by Yilmaz to have been improperly omitted in the Einstein theory. As a consequence of the latter, in the case of a mass singularity the Yilmaz theory gives the line element

$$ds^2 = \exp(-2\varphi) dt^2 - \exp(2\varphi) d\mathbf{x}^2, \quad (2)$$

where $\varphi = Gm/r$. The corresponding metric has been called ‘Yilmaz exponential metric’ [7-9]. By contrast, in isotropic coordinates the Schwarzschild (GR) line element is

$$ds^2 = \left(\frac{1 - \varphi/2}{1 + \varphi/2} \right)^2 dt^2 - (1 + \varphi/2)^4 d\mathbf{x}^2. \quad (3)$$

Comparing the two one finds for the PPN expansions

$$\text{Yilmaz: } g_{00} = 1 - 2\varphi + 2\varphi^2 - \frac{4}{3}\varphi^3 \dots \quad g_{11} = -1 - 2\varphi - 2\varphi^2 + \dots \quad (4)$$

$$\text{GR: } g_{00} = 1 - 2\varphi + 2\varphi^2 - \frac{3}{2}\varphi^3 \dots \quad g_{11} = -1 - 2\varphi - \frac{3}{2}\varphi^2 + \dots \quad (5)$$

Observational tests of GR are currently insufficiently sensitive to probe the Schwarzschild metric beyond the second order in g_{00} and beyond the first order in g_{11} . Consequently the Yilmaz theory gives predictions that are observationally indistinguishable from GR for the standard tests of light bending, perihelion advance, and Shapiro time delay. Because the Yilmaz theory is a metric theory, EM fields are red-shifted in the vicinity of large masses in accord with GR and the ‘standard test’. The Yilmaz-predicted radiation rate for systems with a changing quadrupole moment has not, to date, been computed. No comparison is possible therefore between the Yilmaz and GR predictions of, for example, the rate of orbital decay of binary pulsars. Because g_{00} is never zero the Yilmaz metric does not give rise to an event horizon. The theory admits therefore, the possibility of arbitrarily large gravitational red-shifts in static stable conditions, i.e. not subject to, nor in the process of, gravitational collapse. Robertson [7,8] has suggested that some Neutron stars and black hole candidates may be such ‘Yilmaz stars’, and Clapp [9] has suggested that a significant component of Quasar red-shift may be gravitational. Confirmation of either of these hypotheses would certainly favor the Yilmaz theory of Einstein GR.

The merits and demerits of the theory have been hotly debated in the literature, primarily from a mathematical viewpoint [10-20]. A general discussion of the issues is given in [21]. The main issue is whether or not the quantity t_{μ}^{ν} in Eq. (1) as defined by Yilmaz in terms of the metric tensor, is really a tensor. In the opinion of the author the most persuasive argument in favor of the Yilmaz theory is the argument by Lo [20]: that the formula traditionally used to describe gravitational radiation cannot be derived from Einstein's equations without a (novel) stress-energy tensor for the gravitational field. Though he does not endorse it, Lo cites the Yilmaz theory as a possible example.

In this paper we briefly explore the Cosmological predictions of the Yilmaz theory, which provides an opportunity for additional independent observational tests whilst circumventing the debate on the mathematical status of the theory.

YILMAZ THEORY

A characteristic of the Yilmaz theory is the central role played by an intermediate tensor field $\hat{\phi}$ which is related to the metric through ([1,22])

$$g_{ab} = (\eta \exp(2(\phi \mathbf{1} - 2\hat{\phi})))_{ab}, \quad (6)$$

where η is the Minkowski metric, $\mathbf{1}$ is the mixed unit tensor, $\hat{\phi}$ is a mixed symmetric tensor and ϕ its trace. The field is the solution of the second order equation

$$\partial_B^2 \phi_a^b - \frac{1}{\sqrt{-g}} \partial_c (\sqrt{-g} \partial^b \phi_a^c) = 4\pi G \tau_a^b \quad (7)$$

where, as stated above, τ is the 'standard' stress energy tensor of matter, vacuum, and radiation. The Yilmaz theory is effectively characterized by Eqs. (6) and (7). They are consistent with the Einstein equations with the decomposition (1), i.e.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\tau_{\mu\nu} + t_{\mu\nu}) \quad (8)$$

(a cosmological / vacuum term, if present, is implicit in τ) provided t has the form, allegedly that of the stress energy tensor of the gravitational field, specified by Yilmaz. That is, the Einstein equations collapse to (7) in the event that t is so specified and given the relation (6).

In this document we will consider only the flat space metrics i.e. having Friedmann-Robertson-Walker (FRW) line element

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \quad (9)$$

(with $a(0) = 1$). Accordingly any conclusions reached are applicable only in that context. The published Yilmaz theory [1] is written in terms of Harmonic coordinates, (ζ, x, y, z) say, defined by

$$\partial_a (g^{ab} \sqrt{-g}) = 0. \quad (10)$$

Eq. (10) is satisfied after making the coordinate transformation of the time variable

$$dt = d\zeta a^3 \quad (11)$$

where ζ is the harmonic time, whence the line element (9) becomes

$$ds^2 = a^6 d\zeta^2 - a^2 d\mathbf{x}^2 \quad (12)$$

and $\sqrt{-g} = a^6$. In an early paper Yilmaz [22] proposed a different line element

$$ds^2 = dt^2 \exp(-\alpha^2 r^2) - \exp(\alpha^2 r^2) d\mathbf{x}^2 \quad (13)$$

which he regarded as better suited to the application of his theory to Cosmology, and which was subsequently adopted by other authors [23-25]. However, as discussed in [21], Eq. (13) is incompatible with the Cosmological Principle and will not be considered here.

YILMAZ-FRIEDMANN EQUATIONS

First we solve for the tensor $\hat{\phi}$ in terms of the scale factor $a(\zeta)$. Comparing (12) and (6) we have

$$\{g_{ab}\} = \text{diag}\left(a^6(\zeta), -a^2(\zeta), -a^2(\zeta), -a^2(\zeta)\right), \quad (14)$$

and therefore $\hat{\phi}$ is diagonal, isotropic, and having each element exclusively a function of harmonic time ζ . The trace is

$$\phi = \phi_t^t + 3\phi_x^x \quad (15)$$

and the exponent in (6) is therefore

$$\phi \mathbf{1} - 2\hat{\phi} = \text{diag}\left(3\phi_x^x - \phi_t^t, \phi_t^t + \phi_x^x, \phi_t^t + \phi_x^x, \phi_t^t + \phi_x^x\right). \quad (16)$$

Inserting this into (6) and using (14) one obtains the two equations

$$\exp\left(2\left(3\phi_x^x - \phi_t^t\right)\right) = a^6, \quad \exp\left(2\left(\phi_t^t + \phi_x^x\right)\right) = a^3 \quad (17)$$

which immediately give

$$\{\phi_b^a\} = \text{diag}\left(0, \log a, \log a, \log a\right). \quad (18)$$

Substituting (18) into (7) it is now possible to write down the equation for the scale factor in terms of the sources. The second term on the left hand side in (7) is

$$\forall a, b: \partial_c \left(\sqrt{-g} \partial^b \phi_a^c \right) = \partial_a \left(\sqrt{-g} g^{bd} \partial_d \phi_a^c \right) = \partial_a \left(\frac{\sqrt{-g}}{g_{bb}} \partial_b \phi_a^a(\zeta) \right) \quad (19)$$

(no sum is implied in the last expression). The final expression in (19) is zero unless $a = b = 0$ because the time derivative ∂_0 is the only non-zero derivative of (any function of) the components of $\hat{\phi}$. But (18) gives $\phi_0^0(\zeta) = 0$, and therefore the full expression in (19) is zero. With the help of (10) the remaining Beltrami operator in (7) is

$$\partial_B^2 = \frac{1}{\sqrt{-g}} \partial_a \sqrt{-g} \partial^a = \frac{1}{\sqrt{-g}} \partial_a g^{ab} \sqrt{-g} \partial_b = g^{ab} \partial_a \partial_b = \frac{1}{a^6} \frac{\partial^2}{\partial \zeta^2} - \frac{1}{a^2} \nabla^2 \rightarrow \frac{1}{a^6} \frac{\partial^2}{\partial \zeta^2}. \quad (20)$$

Combining (20), (19) and (7), and using that the stress-energy tensor for a cosmological fluid is

$$\{\tau_a^b\} = \text{diag}\left(\rho, -p, -p, -p\right), \quad (21)$$

where ρ is a coordinate density, one obtains the two equations

$$\rho = 0 \quad (22)$$

and

$$\frac{d^2 \log a}{d\zeta^2} = -4\pi G a^6 p \quad (23)$$

where ρ is the total energy density and p is the total pressure. These equations correspond respectively to the first and second Friedmann equations of GR.

YILMAZ COSMOLOGY

Decomposing the stress-energy tensor into contributions from matter, radiation, and vacuum, the first Friedmann equation for the Yilmaz theory is

$$\rho = \rho_m + \rho_r + \rho_v = 0. \quad (24)$$

The vacuum energy coordinate density is assumed to be constant in accord with its origin as the 0,0 term in the Cosmological term or, alternatively, in accord with its origin as the net energy density of the zero point fields of QFT. It follows that

$$\rho_m + \rho_r = -\rho_v = \text{constant}. \quad (25)$$

Once matter and radiation have decoupled, barring some coincidence, (25) gives that ρ_m and ρ_r must individually be constant. The observed fact that the matter and radiation have positive energy densities requires that the vacuum must have an overall negative energy density. Since Fermions and Bosons contribute to the overall vacuum energy density with different signs, QFT is compatible with any value in $(-\infty, \infty)$ for the total. A negative value therefore, though unusual, is acceptable.

The equations of state are $p_i = k_i \rho_i$, where $i \in \{m, r, v\}$ and $k_m = 0$, $k_r = 1/3$, $k_v = -1$. It follows from the above that the p_i must be constant. With this (23) can be written

$$\frac{d^2 \log a}{d\zeta^2} = -4\pi G a^6 \left(\frac{1}{3} \rho_r - \rho_v \right) = -4\pi G a^6 \left(\rho_m + \frac{4}{3} \rho_r \right) \quad (26)$$

where we used (25) and where the expression in parentheses is independent of harmonic time. Returning to FRW time using (11), this is

$$a^3 \frac{d}{dt} \left(a^3 \frac{d}{dt} \log a \right) = -4\pi G a^6 \left(\rho_m + \frac{4}{3} \rho_r \right). \quad (27)$$

This equation is most easily solved by substituting $b = a^3$, whereupon

$$\begin{aligned} \frac{1}{3} b \frac{d}{dt} \left(b \frac{d}{dt} \log b \right) &= -4\pi G b^2 \left(\rho_m + \frac{4}{3} \rho_r \right) \\ \Rightarrow \ddot{b} &= -12\pi G b \left(\rho_m + \frac{4}{3} \rho_r \right) \end{aligned} \quad (28)$$

The solutions are

$$\begin{aligned} b &= \alpha \sin(\omega t + \phi); \quad \omega = \sqrt{12\pi G \left(\rho_m + \frac{4}{3} \rho_r \right)} \\ \Rightarrow a &= \alpha^{1/3} \sin^{1/3}(\omega t + \phi) \end{aligned} \quad (29)$$

where α and ϕ are arbitrary constants. It is concluded that the theory predicts an oscillating universe. Clearly ϕ simply offsets the time of the initial singularity and can be ignored. Taking into account that the sign of the scale factor is unobservable, Figure 1 is a plot of its modulus.

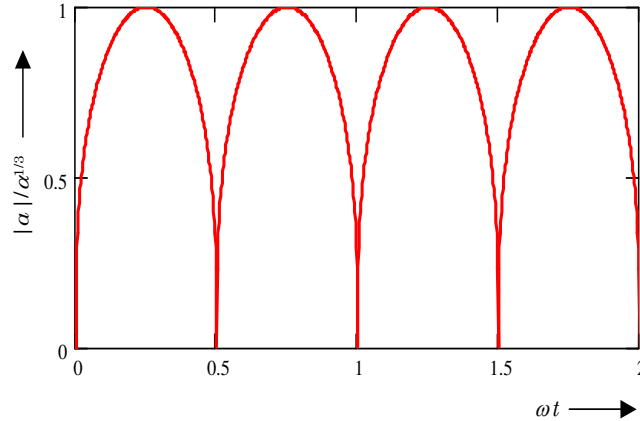


FIGURE 1. Plot of the modulus of the scale factor as given by Eq. (29)

The deceleration parameter can be computed using (29) to give

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{3}{\cos^2(\omega t + \phi)} - 1. \quad (30)$$

Note that as a result of this definition, q is independent of the sign of the scale factor and therefore, except perhaps at the points where a changes sign, the result (30) is unchanged if one were to use $|a|$ instead of a in the definition. From (30) it is deduced that the deceleration parameter is always positive and greater than 2. In terms of the Hubble parameter

$$H \equiv \frac{\dot{a}}{a} = \frac{\omega}{3} \cot(\omega t + \phi) \quad (31)$$

the deceleration parameter is

$$q = 3 \left(1 + \left(\frac{\omega}{3H} \right)^2 \right) - 1 = 2 + \frac{4\pi G}{H^2} \left(\rho_m + \frac{4}{3} \rho_r \right) = 2 + \frac{3}{2} \Omega_m + 2\Omega_r \quad (32)$$

where

$$\Omega_i = \frac{8\pi G}{3H^2} \rho_i \quad (33)$$

are functions of time. The SN1a data permit estimates of the deceleration parameter that are fairly independent of the cosmological model. Currently the data from nearby supernovae ($z < 1$) consistently indicate a negative value of q_0 [26-28], at variance with the prediction above that $q > 2$ for all time.

CONCLUSION AND REMARKS

Assuming that space is flat, the Yilmaz theory of gravity demands that the total energy density of the universe is zero. If this is achieved by balancing the observed positive energy density of matter with a negative vacuum, then the theory predicts an oscillating scale factor with angular frequency $\sqrt{12\pi G(\rho_m + (4/3)\rho_r)}$. The coordinate density of the total of matter and radiation energy is predicted to remain constant at all times. During times when these are decoupled (e.g. post recombination during the expansion phase) the coordinate density of both matter and radiation are predicted to remain independently constant. An implication is that the negative vacuum can be regarded as giving birth to matter and radiation at exactly the right rate during expansion, and absorbing matter and radiation during contraction, so as to maintain their coordinate densities constant at all times. Consequently the proper energy density of matter and radiation increases at the expense of an increase in the magnitude of the negatively-signed proper energy density of the vacuum. Conversely, during contraction the coordinate density of matter and radiation remain constant with the result that the proper energy density of matter and radiation decreases promoting a decrease in the magnitude of the negatively-signed proper energy density of the vacuum.

The Yilmaz universe is always decelerating with $q > 2$. That prediction is at variance with the curve fit to the supernovae data, so that, to the degree these data can be trusted, the Yilmaz Cosmology fails this observational test. The Yilmaz prediction of deceleration is changed if one admits a vacuum-like term with a different equation of state. For example, a term of the kind advocated by Hoyle et al [29] to model matter creation wherein the pressure has the same sign as the energy density, can be made to cause the Yilmaz theory to predict an accelerating universe [21].

As a result of the stipulation of the metric (9), these conclusions are applicable exclusively to the case of flat space. The cases of spherical and hyperbolic space warrant their own investigation.

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REFERENCES

1. H. Yilmaz. *Il Nuovo Cimento* **107 B**, 941–960 (1992).
2. H. Yilmaz. *Annals of Physics* **101**, 413–432 (1976).
3. H. Yilmaz, *Int. J. Theor. Phys.* **21**, 871–902 (1982).
4. H. Yilmaz. *Lettere al Nuovo Cimento* **7**, 337–340 (1973).
5. H. Yilmaz. *Lettere al Nuovo Cimento* **20**, 681–685 (1977).

6. H. Yilmaz. Towards a Comprehensible Physical Theory: Gravity and Quantum Mechanics in *The Present Status of the Quantum Theory of Light*, edited by S. Jeffers and et al, Kluwer Academic, Dordrecht, 1997.
7. S. L. Robertson. *Astro. J. Lett.* **517**, 117–119 (1999).
8. S. L. Robertson. *Astrophys. J.* **515**, 365–380 (1999).
9. R. E. Clapp. *Phys. Rev. D* **7**, 345–355 (1973).
10. C. O. Alley. Investigations with Lasers, Atomic Clocks and Computer Calculations of Curved Spacetime and of the Differences Between the Gravitation Theories of Yilmaz and Einstein, in *Frontiers of Fundamental Physics*, edited by M. Barone and F. Selleri, Plenum Press, New York, 1994.
11. C. O. Alley. The Yilmaz Theory of Gravity and its Compatibility with Quantum Theory in *Fundamental Problems in Quantum Theory: A Conference Held in Honor of Professor John A. Wheeler*, edited by D. M. Greenberger and A. Zeilinger, New York Academy of Sciences, New York, 1995.
12. I. Peterson. *Science News* **146**, 376–378 (1994).
13. F. I. Cooperstock and D. N. Vollick. *Il Nuovo Cimento* **111 B**, 265–268 (1996).
14. E. D. Fackerell. Remarks on the Yilmaz and Alley papers, in *Proceedings of the First Australian Conference on General Relativity and Gravitation*, edited by D. L. Wiltshire, University of Adelaide, Adelaide, 1996, 117–131.
15. H. Yilmaz. Did the apple fall?, in *Frontiers of Fundamental Physics*, edited by M. Barone and F. Selleri, Plenum, New York 1994.
16. Alley, C. O. and Yilmaz, H., *gr-qc/0008040* (2000).
17. W. Wyss and C. W. Misner. *Nuovo Cimento B* **114**, 1099–1101 (1999).
18. C. W. Misner. *Nuovo Cimento B* **114**, 1079–1085 (1999).
19. C. O. Alley and H. Yilmaz. *Nuovo Cimento B* **114**, 1087–1098 (1999).
20. C. Y. Lo. *Astrophys. J.* **455**, 421–428 (1995).
21. M. Ibson. *Class. Quant. Grav.* To be published (2005)
22. H. Yilmaz. *Phys. Rev.* **111**, 1417–1426 (1958).
23. Y. Mizobuchi. *Hadronic Journal* **8**, 193–219 (1985).
24. Y. Mizobuchi. *Hadronic Journal* **9**, 191 (1986).
25. A. Bjornson. *A Universe That We Can Believe*, Addison Press, Woburn, 2000.
26. A. Goobar et al. *Physica Scripta* **T85**, 47–58 (2000).
27. S. Perlmutter. *Int. J. Theor. Phys. A* **15 suppl. 1B**, 715–739 (2000).
28. B. P. Schmidt. *Class. Quant. Grav.* **19**, 3487–3503 (2002).
29. F. Hoyle, G. Burbidge, and J. V. Narlikar. *New Cosmological Models*, in *A Different Approach to Cosmology*, University Press, Cambridge, 2000.